

Mach and Ehrenfels: The Foundations of Gestalt Theory¹

§1. Preamble

One important measure of the success of a philosophy of science is the extent to which the clarifications which it yields have positive and fruitful consequences within the sciences themselves. Such success is at least in part a function of the extent to which its examples and problems are taken over from genuine science and are not merely trivial or over-simplified illustrations. The thought of Mach in particular, and of Austrian philosophers of science in general, provides us with striking examples of such interaction. Mach's epistemology and ontology grew out of his investigations, both systematic and historical, in physics and psychology, and they contributed in turn to the further development of his own thinking in these areas and to the work of those, such as Einstein and Ehrenfels, whom he influenced. Similarly, it was the interaction between philosophy and psychology which made possible the seminal work on the notion of Gestalt quality by Ehrenfels, and this work, together with the writings on the logic and ontology of parts, wholes and structures by other members of the Brentano school, led in turn to significant further developments, not only in psychology itself, but also in neighbouring disciplines such as linguistics.²

We shall find in what follows that we can come to terms with the implications of the ideas of Mach and Ehrenfels on the perception of what is complex and on the complexity of perception only by paying especially detailed attention to their respective understandings of the notion of *non-causal dependence*. The clarification of this notion – first effected in a truly systematic way in the writings of these two authors and in those of their contemporaries Brentano and Stumpf – is, we shall argue, one of the great achievements of Austrian philosophy of science. Mach, it will turn out, was unable successfully to incorporate his descriptions of complex

perception within his general atomistic framework in no small part because his understanding of dependence was in a quite specific sense too narrow. The great significance of the work of Ehrenfels and of other members of the Brentano tradition from our point of view is that, because they were more faithful to the structures of what is given in perception, they were able to develop a richer theory of dependence, the implications of which were to extend far beyond the narrow sphere of perceptual psychology.

§2. The Problem of the Perception of Complexes

To talk of a 'perception of what is complex' is, from the atomistic perspective which held sway amongst the majority of 19th century psychologists, already to employ a form of speech that is illegitimate in the sense that it is not grounded in any underlying reality. There is at most, according to the atomistic psychologist, the possibility of a *summation* of simple perceivings, each one of which would have something unitary or non-complex as its object or content.³

Mach, too, embraced an atomism of this kind. For him all complexes, including the ego itself, are mere ideal, practical or provisional 'mental-economic unities'. As he puts it in the *Analyse der Empfindungen*, only the 'elements' (sensations, *Empfindungen*) are real.⁴ But he clearly saw that there is a *problem* of complex perception,⁵ and Ehrenfels, as is well known, was able to take certain passages from this work as the starting-point of his investigation of complex-perception in his classic essay of 1890, "Über 'Gestaltqualitäten'". These passages are not isolated instances of what might be taken to be less than careful thinking on Mach's part. Indeed the examination of Mach's writings reveals that his anticipation of Ehrenfels goes back at least 20 years earlier. On receipt of Ehrenfels' paper, Mach replied in a letter that he had already put forward the main ideas – albeit in a more psychological way, in terms of a theory of 'muscular sensations' – in an earlier paper.

The paper in question is almost certainly his "Bemerkungen zur Lehre vom räumlichen Sehen" of 1865,⁶ a critical discussion of the psychology of Herbart, dealing specifically with the problem of our recognition of perceptual complexes. How, Mach asks, do we recognize different spatial figures ('*Gestalten*') as the same? How does it come about that we

apparently recognize melodies as being alike? How is it that we recognize the form of a melody more easily than the key in which it is played? Why is it that we recognize a rhythm more easily than an absolute duration? Where is the similarity between the individual, unitary qualities presented in the hearing of a melody played on a trumpet in the key of C, and those presented in the hearing of 'the same' melody played on a violin in G? Recognition and likeness here, as Mach points out,

cannot depend on the qualities of the perceptual presentations [*Vorstellungen*], for these are different. On the other hand recognition, according to the principles of psychology, is possible only on the basis of presentations which are the same in quality (Mach 1865, p.122 of repr., Eng. p.391, quoted in Schulzki, p.42).

There is, Mach concludes,

no other alternative but for us to consider the qualitatively dissimilar presentations in the two series as being *necessarily connected* with some sort of qualitatively similar presentations. (*loc. cit.*, our emphasis)

Mach, that is to say, claims that there is a means of solving the problem of complex perception within the atomistic framework by means of an appeal to additional elementary sensations *outside* the sphere of perception, sensations he calls *Muskelempfindungen*. When we hear the same melody in two different keys, our apprehension of this 'sameness' rests on the fact that, for all the differences in tone-sensations, the same feeling-sensations are involved in both cases. On a trivial interpretation, Mach here is presenting a view according to which our experience enjoys a certain sort of double structure, each separate experience of the individual tones in a melody or of the points in a spatial figure is coloured by a certain element of feeling. It remains the case that, on this modified view of 'element', experience is just one damned element after another.

Such a view is indeed able to solve the problem of identity of complex objects of experience, at least for simple cases, but it is not only this problem which an account of our perception of what is complex is called upon to resolve. Such an account must explain also the *unity* of complexes that is given in experience, and it must do justice to the fact that complexes are given in such a way as to be demarcated from other, neighbouring complexes in such a way as to form unified and integral wholes. And Mach's account, on this interpretation, is inadequate to features such as this.

There is, however, another, more subtle interpretation of Mach's

position, the possibility of which we almost certainly owe to Ehrenfels, since it consists in a certain sense in reading back Ehrenfels' ideas on Gestalt qualities into the relevant Machian texts. According to this interpretation, it is not the successive elementary successions, but rather each apparent complex perception that comes to be associated with its own characteristic feeling-sensation or nervous *quale*. The existence of similarities between such *quale* can then explain both how it is that we can enjoy the appearance of what is putatively *the same* complex even where the associated elementary data of perception are in fact distinct, and also how it is that the apparent complex in question is given as something unitary and as something set apart from its environment.

Thus when I see a square, for example, then in addition to the perceived elements (whether these be conceived as points, lines or segments) there is also a peculiar nervous sensation which I have as a result of the innervations of the muscles of my eyes, a sensation that is repeated, spontaneously and without any effort on my part, whenever I see a similar figure. The *body as a whole* we might say, in consort with specific sensory presentations of what is simple, is to do the job of accounting for our apparent presentation of what is complex. And we should, as Mach himself argues, look to the variety of the human organism,

which is provisionally rich enough to cover the outlays of psychology in this regard – and it is high time that we took seriously the talk of 'bodily resonance' in which psychology has so readily engaged. (1865, *loc. cit.*, Eng. p.392)⁷

Now an account of this kind works well enough, on its own terms, in relation to our (apparent) perceptions of congruent but differently coloured spatial shapes (space and shape, we note, are the subject-matter of Mach's 1865 paper). Each such shape can indeed be seen as being associated – 'necessarily connected', as Mach puts it – with its own characteristic muscular innervation, itself derived from corresponding motor processes of the eye and head. (Modern-day psychologists, with their investigation of the role of the kinaesthetic dimension in experience, have at least to some extent vindicated Mach in this regard.) We are interested, however, in a general theory of complex perception. Indeed Mach himself writes:

Just as the same, differently coloured forms, the same muscular sensations, must occur if the forms are to be recognized as the same, so too each and every form,

each and every abstraction, as one might say, must in just the same way be based upon presentations of a quite particular quality. This holds true for space and shape, as well as for time, rhythm, pitch, the form of melodies, intensity, and so on. (*loc. cit.*, Eng. p. 391f.)

Mach assumes, that is to say, that it is possible to generalize the theory of muscular sensations to encompass all sensory dimensions. More, that it is in principle possible to extrapolate from this theory in such a way as to encompass our apparent presentation of all 'Abstraktionen' from what is given.⁸

Ehrenfels, too, recognized the necessity of such a general theory of complex perception.⁹ But he saw also – and this was a significant achievement of "Über 'Gestaltqualitäten'" – that a completely general theory could not be obtained on the basis of an appeal to additional elementary phenomena along the lines of Mach's muscular sensations. For such sensations can at best explain our apparent perception of what is complex only in relation to what is non-temporal, of what is capable of being presented instantaneously, i.e. simple spatial figures, simple smells, simple musical chords. There is no way in which an appeal to extra elementary (and thus instantaneous) sensations alone can solve the ontological problem raised by our (apparent) perception of temporally extended, unitary complexes such as melody and rhythm, and in general of all Gestalten involving change and motion. For there is clearly no answer to the question as to *when* a single elementary feeling-sensation – putatively associated with a plurality of elementary perceptions spread out in time – could become associated with this plurality in the relevant way.¹⁰

The elementary innervation (or what have you) can do service for the perception of what is complex only if it is somehow associated with *all* relevant perceptions. This association can come about, however, only if these perceptions are already *collected together*, e.g. through the operations of memory, to form a single and instantaneous composite perception. But the appeal to such a composite perception clearly signifies a departure from the atomistic perspective. Moreover, once such composites have been accepted, it is difficult to see what explanatory role could remain for any associated muscular innervations.

For reasons to be investigated only later, Mach need not acknowledge that this argument has isolated any inadequacy in his account, since he rejects the notion of time as traditionally conceived; the very concepts of

simultaneity and non-simultaneity are held by him to correspond to no underlying reality.

It is not, however, this inadequacy of Mach's account which will be of interest to us here. Our attention will be directed, rather, toward the nature of the relation between muscular and perceptual *quale* that is presupposed by his theory.

§3. The Analysis of Sensations

The theory of *Muskelempfindungen* of 1865 is not simply abandoned by Mach in his later writings. Many of the same ideas are at work also in the *Analyse der Empfindungen*, though now the theory of muscular sensations has been extended – legitimately or not – to embrace a taxonomy of different kinds of 'space-sensations', 'time-sensations' and in principle also muscular innervations of other sorts – illustrating Mach's faith in the 'power and variety of the human organism'.

Thus consider the following passage quoted by Ehrenfels at the beginning of his paper:

In melodic as well as in harmonic combinations, notes whose rates of vibration bear to one another some simple ratio are distinguished (1) by their agreeableness, and (2) by a sensation characteristic of this ratio. (1886, p.130)¹¹

Such distinctiveness manifests itself also in our forms of expression:

Colours, sounds, temperatures, pressures, spaces, times and so forth are connected with one another in manifold ways; and with them are associated moods of mind, feelings and volitions. Out of this fabric, that which is relatively more fixed and permanent stands prominently forth, engraves itself in the memory, and expresses itself in language. (1886, p.2, Eng. p.2)

What is missing from the *Analyse der Empfindungen* – and this is a crucial development – is any talk of a 'necessary connection' or 'intimate mutual relation' such as we find in the 1865 account.¹² We now learn only that the characteristic sensations are 'connected to' or 'dependent on' the elements with which they are associated. Further, this dependence is seen as being in every case relative to the perspective or point of view adopted by the investigator:

A colour is a physical object as long as we consider its dependence upon its luminous source (other colours, heat, spaces, etc.). But if we consider its dependence upon the retina...then it is a psychological object, a sensation. (1886, p.13, Eng. p.17.)

We shall turn below to the task of examining in detail just what Mach understood by 'dependence' here. For the moment it is sufficient to note that it is not any sort of causal relation. Causality is rejected by Mach as a metaphysical encumbrance, an anthropomorphic notion, properly to be eliminated from any science that is worthy of the name.

§4. On Gestalt Qualities

Ehrenfels, too, employs a notion of non-causal dependence in his theory. But for him it is the Gestalt qualities themselves, certain *sui generis* objects of presentation, which are dependent on the data of sensation which are their foundation.

Ehrenfels seeks to be faithful to the reality (veridicality) of our perception of what is complex. There is something *there*, he insists, which we perceive through specific types of complex networks of acts of presentation (perception, memory and imagination) of what is simple, whenever we perceive a melody, a rhythm, or any other Gestalt quality. And he claims further that, to produce a truly faithful account of our perception of such formations, we have to distinguish objects of perception on two distinct levels.

Ehrenfels recognizes not only *complexes* of elementary perceptual data but also special *qualities* of such complexes, and the formations we perceive are such as to involve both. Just as for Mach, if two figures are similar, then this is because of an identity in the appurtenant nerve-processes or feeling-sensations, so also for Ehrenfels, if two figures are similar, then this is because of an identity in their associated Gestalten.¹³

Ehrenfels is explicit that this identity is to be explained by appeal to unitary presentational elements: when we hear a melody consisting of 8 notes, then there are (at least) nine presentations involved, 8 aural presentations of individual notes, and one unitary presentation of the associated Gestalt quality.¹⁴ Ehrenfels acknowledges that the notes constitute in and of themselves a certain complex whole, and that the

Gestalt quality is founded upon (is, precisely, a 'quality of') this complex whole. But the quality itself is not a whole embracing the individual sensational elements as parts: a view of this sort was developed only with the work of Wertheimer and the other members of the Berlin School. In this respect Ehrenfels, like Mach, can be said to have offered an elementarist solution to the problem of complex perception.

For Ehrenfels, as for Mach, no special intellectual effort, attention or attitude is needed to produce the awareness of a Gestalt quality: this awareness occurs as it were automatically. The problem of the 'universal givenness of Gestalt qualities with their foundations' is however a complex one. Ehrenfels asserts that

wherever a complex which can serve as the foundation for a Gestalt quality is present in consciousness, this quality is itself *eo ipso* and without any contribution on our part also given in consciousness (p. 111, above).

This remark relates only to the issue of the *genesis* of Gestalt qualities, to the question whether, on the basis of a given foundation, any activity or assistance is required on our part in order to bring a Gestalt quality to consciousness. Thus Ehrenfels points out that, at least in certain cases, 'the exertion we seem to require in order to grasp a shape or melody on the basis of a foundation already presented is much rather applied to the filling out of that foundation itself'. (p. 111, above)

He considers our perception of paintings, where sensation yields merely a starting point for further imaginative filling out:

A significant exercise of our capacities is required in order to utilize in our presentation the slight distinctions in light and colour and the foreshortenings in the perspective plane as associative tokens for the realization of the total luminosity and three-dimensionality of the painting. (pp. 111f., above)

But effort is needed, Ehrenfels argues, only in order to fix the indirectly seen parts of the whole. Someone who has developed in his consciousness the foundation for the Gestalt quality in the appropriate way will not find it necessary to generate this quality itself in a further act – and nor will he have any choice as to *which* quality will be generated: the quality is, as it were, given of itself. Ehrenfels' views on the genesis of Gestalt qualities are in this respect identical to those of Mach on the genesis of muscular innervations.

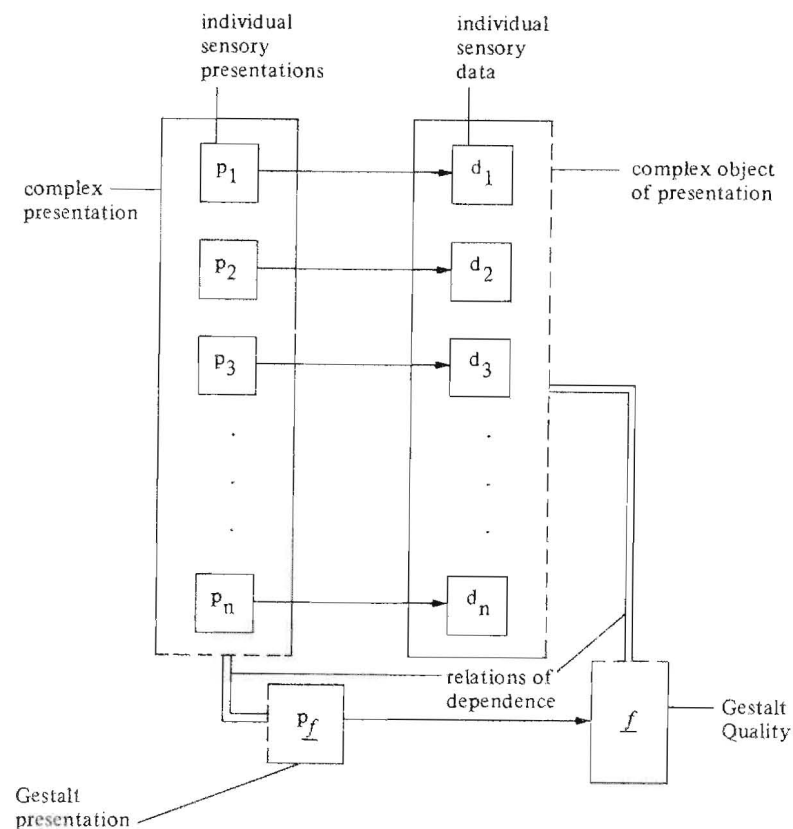
There is, however, in addition to the question of the genesis of Gestalt

qualities also another question, that of the ontological status of such qualities, and of their constitutive relations to the sensory data with which they are associated.¹⁵ Ehrenfels was perhaps the first to consider this problem in a serious way. He points out that if we assert a mutual dependence of Gestalt quality and foundation not merely in the genetic but also in this ontological sense, then this gives rise immediately to a problem of infinite multiplication. Mutual ontological foundation would signify first of all, harmlessly enough, that every Gestalt quality is necessarily such that it could not exist unless there exists also a corresponding complex of fundamenta. But it would signify also that every complex of fundamenta, too, is necessarily such that it could not exist unless an associated Gestalt quality existed also. *Every* arbitrary complex of given sensations, however delineated, would give rise to a Gestalt quality of its own. This would imply, however, that we would once more be in no position to explain that characteristic unity and integrity of perceptual complexes which is in fact experienced. Thus to hear a melody (e.g.) would be to hear also all constituent sub-melodies (and indeed, unless constraints on temporal and spatial proximity are introduced, all melodies built up on the basis of presently perceived tones together with tones previously heard). But further, since Gestalt qualities are themselves perfectly valid objects of presentation which may themselves serve as fundamenta of further Gestalt qualities, it would follow that, on hearing a sequence (s_1, s_2, \dots, s_n) of tones, we have not only the Gestalt quality, say f_1 which these immediately generate, but also the further Gestalt qualities f_2 – generated by the sequence $(s_1, s_2, \dots, s_n, f_1)$ – the quality f_3 – generated by the sequence $(s_1, s_2, \dots, s_n, f_1, f_2)$ – and so on. Now clearly, as Ehrenfels would say, there is nothing of all of this given in inner perception. And he concludes that, in the ontological sense, Gestalt qualities are merely *one-sidedly* dependent on their fundamenta.¹⁶ Mach seems not to have faced this problem, even though it arises in the self-same way within the framework of his own nervous *qualie* theory. He seems, rather, to have run together the genetic and the ontological dimensions and thereby to have been constrained to accept mutual dependence both in the genetic and in the ontological sense. As Smith points out in his essay above, the Meinongians accepted it in neither sphere, insisting on a one-sided dependence both genetically and ontologically. Thus they held first of all that Gestalt qualities (now called ‘founded contents’ and later ‘higher order objects’, or ‘objects of presentations of extra-sensory provenance’) are one-sidedly

ontologically dependent (‘founded’) on their fundamenta or ‘inferiora’. But they held also that such qualities are in need of being *produced* for presentation by a special exertion of consciousness, that the Gestalt quality must in a certain sense be teased out of the perceptual environment.¹⁷

We might display the essentials of Ehrenfels’ account in the form of a diagram, somewhat as follows:

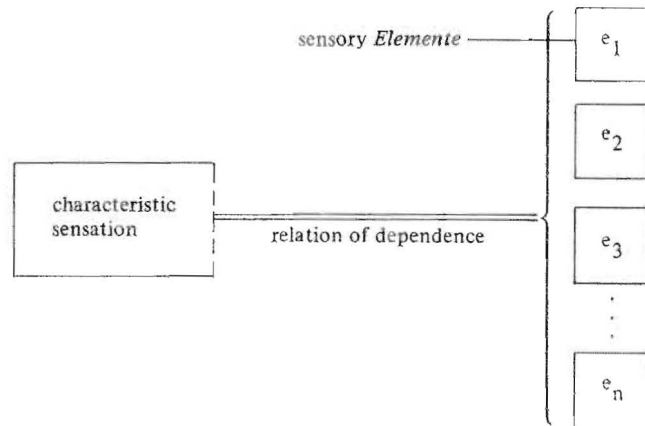
Diagram I.



Here the arrows represent relations of intentional directedness (between an act and its object), and the double lines represent relations of mutual dependence as in the diagrams on pp. 40, 47 etc. above.

Mach's theory, on the other hand, on the interpretation here advanced, might look like this:

Diagram 2.



It is of course the *differences* between these two figures that leap to the eye. The most important of these are: (1) Where act and object are distinguished by Ehrenfels and the other Brentanists, Mach embraces a conception of *Elemente* according to which sensory presentations and sensory data are not separate but are rather run together into a single unitary item. (2) Mach's atomism did not allow him to embrace either complex presentations or complex objects of presentation such as are to be found in the Ehrenfels theory.

In this paper however we shall be concentrating on what the two accounts have in common. For not only is it the case that Ehrenfelsian Gestalt qualities and Machian characteristic sensations perform the same job; both are also such as to stand to their respective underlying elementary data in the peculiar relation of non-causal dependence referred to above.

The investigation of this relation has a more than parochial interest. Notions of non-causal dependence form indispensable components not only of Mach's psychology and of the psychology of Ehrenfels, but also of the work of other thinkers in the Brentano tradition, particularly Stumpf, Meinong and Husserl, from where they exerted a wide influence, to a degree which has still hardly been appreciated.¹⁸ More important still, however, if our arguments are correct, are the implications of a

demonstration of the inadequacy of an account of dependence of the sort defended by Mach. For this account and its derivatives have been an unquestioned presupposition of almost all subsequent philosophy of science. To call it into question is to call into question a still powerful orthodoxy.

§5. Mach's Philosophy of Science

Mach is widely acknowledged as having been the first thinker to combine philosophical clarification, history of science and substantive scientific research in ways that are recognizable as philosophy of science as this is nowadays understood. He stands at the beginning of that strand in the history of Austrian philosophy which reaches its culmination (or its nadir) in the logical positivism of the Vienna circle.

But there is another, one might almost say phenomenological, aspect to his thinking. All Mach's arguments, however they are to be classified, are rigorously subordinated by him to a single goal: the goal of *increasing knowledge*.¹⁹ He is quite prepared to renounce any claim to the epithets 'physicist' or 'philosopher' if this contributes to the advancement of our understanding of the world ((1910), p. 11, Eng. p.38). He thereby stands in marked contrast to those philosophers and scientists who are all too ready to impose in advance requirements that enquiry has to satisfy if it is to be 'scientific', for example by foisting abstract 'criteria of rationality' on live traditions of research.

He shares with Husserl and others in the Brentano tradition the conviction that theoretical enquiry cannot afford to lose sight of the *origins* of our ideas (scientific and otherwise). Scientific ideas, as Mach conceives them, must have their origins in concepts – called by him 'inaugurating concepts' – derived directly from experience (and, like the phenomenologists, Mach was prepared to acknowledge the role played by introspection in the foundations of scientific enquiry). The science of heat, he argues, is derived from the concept of *felt warmth*, the science of light from the concept of *intensity of illumination*, the science of acoustics from the concept of *frequency*, and so on.²⁰

Mach shares with members of the phenomenological tradition a conception of the philosophy of science as something that must be tied to the actual practice of science. As Husserl puts it: 'A fruitful theory of

concept formation in the natural sciences can...only be a theory “from below”, a theory that has grown out of the work of the natural sciences themselves.’ The passage occurs in the context of a discussion by Husserl of a monograph by the Neo-Kantian Rickert in which a conception of the philosophy of science is manifested ‘which deals so much in general constructions, is so much a theory “from above”, that not a single example is to be found in the entire monograph and nor does this absence make itself felt’ (1979, p.147).

It is a recurring feature of Mach’s deservedly famous conceptual analyses of the ontological commitments of scientists e.g. to space and time, that he proceeds by gradually stripping away from these all purely conceptual baggage, all metaphysical free play not directly related to sense experience – and thereby arrives, step by step, at certain (as Mach conceives things) unambiguous and precise components, such as the inaugurating concepts mentioned above:

I see the expression of... economy clearly in the gradual reduction of the statical laws of machines to a *single one*, viz., the principle of virtual work: in the replacement of Kepler’s laws by Newton’s single law... and in the [subsequent] reduction, simplification and clarification of the laws of dynamics. I see clearly the biologic-economic adaptation of ideas, which takes place by the principles of continuity (permanence) and of adequate definition and splits the concept ‘heat’ into the two concepts of ‘temperature’ and ‘quantity of heat’; and I see how the concept ‘quantity of heat’ leads on to ‘latent heat’, and to the concepts of ‘energy’ and ‘entropy’. ((1910), p.6f., Eng. p.33)

He argues at length for a view of science as a continuous process of adaptation – the biological echo here is deliberate – of thoughts to facts and of thoughts to thoughts. And the aim of this adaptation (though not its biological explanation) is shared also by members of the Brentano tradition: it is precisely the realization of the fundamental requirement of univocity (*Eindeutigkeit*) of our ideas.²¹

And finally he shared a concern for the *presuppositionlessness* of description. Mach’s attitude here is neatly captured in Wittgenstein’s famous remark about psychology as consisting in ‘experimental methods, and conceptual confusions’.²² But the conceptual confusions which were the targets of Mach’s polemics were all, he thought, the result of employing concepts – of time and space, of causality, of the ‘inner’ and the ‘outer’ – without any basis in experience and experiment. One of the most striking examples here is Mach’s discussion of the ‘preconceived

opinions’ in the psychology of perception. These result, he claims, from a failure to examine perceiving itself, before transferring to the perceptual sphere, lock, stock and barrel, ideas derived from the sphere of physics (1903, ch. II).

Husserl got the main historical point exactly right in his comments on the use made of the ‘phenomenological method’ before the turn of the century by certain psychologists and natural scientists:

The sense of this method for men such as Mach and Hering lay in a reaction against the threat of groundlessness [*gegen die drohende Bodenlosigkeit*]; it was the reaction against a theorizing with the help of conceptual formations and mathematical speculation removed from intuition which brought no clarity into the correct sense and achievement of theories (1962, p. 302).

– and in this same passage Husserl stresses the similarity between the approaches of Mach and Hering on the one hand and that of Brentano on the other.²³

§6. Mach and the Brentano Tradition

The emphasis on description and sense experience in Mach corresponds in the work of the Brentanian psychologists to the emphasis on the need to create a scientific psychology on the basis of the unprejudiced description of inner experience.²⁴ The programme of descriptive – as opposed to genetic – psychology was common to all first-generation descendants of Brentano. Descriptive psychology deals with what we have called above ontological dependence relations and with associated structures in the sphere of conscious experiences. Genetic psychology deals rather with the coming and going of conscious experiences and with associated *causal* structures. The programme of descriptive psychology finds one of its most succinct formulations in Brentano’s *Meine letzten Wünsche für Österreich* ((1895), p.34), where Brentano describes the project of a ‘combinatoric’ of the basic psychic components which would yield psychic phenomena ‘as letters yield words’. The rigorous validity (necessity) of the laws of such a combinatoric would be contrasted with the empirical or inductive validity of the laws of genetic psychology, i.e. the laws of succession or of the coming and going of psychic phenomena.²⁵

Mach’s thought, and not least his theory of *Elemente*, might indeed be

described as a working out of a related programme. For this theory rests on a strikingly similar conception of the connections and combinations of *Elemente*:

The aim of all research is to ascertain the mode of connection of the elements... For us colours, sounds, spaces, times... are the ultimate elements, whose given connection it is our business to investigate. (1886, p.21; Eng. p.22.)

The antithesis of ego and world, sensation (phenomenon) and thing... vanishes, and we have simply to deal with the *connection* of the *elements*... of which this antithesis was only a partially appropriate and imperfect expression.... Science has simply to *accept* this connection, and to set itself aright (get its bearings) in the intellectual environment which is hereby furnished, without attempting to explain its existence. (*op.cit.*, p. 10, Eng. p.14.)

The great difference between the two programmes, on the other hand, is that, as already noted, the genetic and the ontological are simply run together in Mach, who knows nothing of the distinction between genetic and descriptive psychology of the Brentanists.

We have emphasized that the notion of *non-causal dependence* which lies at the root of Mach's theory is a notion which appears also as a fundamental component in the work of the Brentanians. And whilst the Machian and Brentanian formulations of this notion are not identical, the ways in which they are put to work are in many respects parallel.

Mach's views on how *Elemente* are related to each other have been adopted by subsequent philosophers in the positivist tradition (at least in part because, since they involve a denial of any *necessary* connection, they mesh well with the tenets of empiricism). They have indeed been absorbed to such an extent that they form an unquestioned and unanalysed component of present-day philosophy of science. Mach's critics and interpreters have concentrated in their writings much rather on the *Elemente* themselves, and the literature abounds with refutations of the 'phenomenalism' or 'neutral monism' which Mach is held to have propounded. This aspect of his thinking, too, exerted a powerful influence on the Vienna circle. But the question of the *relations between Elemente* is clearly no less important, despite the fact that it has received so little detailed consideration. It is important not only because Mach was almost certainly the first to have addressed the problem of providing such a theory without appeal to extraneous and ambiguous or unexplained notions like that of causality. It is important further because some of his most telling insights, not least those which are of relevance to the problem

of complex-perception, are directed precisely towards the project of a general theory of relations of the given sort.

§7. Mach on Variation

What, then, is Mach's theory of the relations between *Elemente*? To answer this question we must consider a further crucial notion underlying his approach, which also has its counterpart in the theories of the Brentano school: the notion of *variation*.

That science proceeds by identifying constancies and regularities in what is in flux in reality was a commonplace long before the writings of Mach. One thinks immediately of the writings on method of John Stuart Mill. But Mach gave this conception an important twist. The simple – and on reflection somewhat simplistic – opposition between what is constant and what is variable, is replaced in Mach's theory by the concept of an *all-pervading and continuous variation*. Thus the notion of scientific laws as simple generalizations has no place within his theory. The object of his researches is always the continuous transition from one mosaic of ordered connections to another. His strikingly elegant and original idea was that all connections between elements *and all constancy* can be understood entirely in terms of the idea of continuous transition or variation.

Science, according to Mach, takes as its starting point the orderings of phenomena given in experience and assigns appropriate numerical values to these phenomena in ways which reflect their dimensions of variability:

The method of change or variation presents us with like cases of facts containing components that are partly the same and partly different. It is only by comparing different cases of refracted light at changing angles of incidence that the common factor, the constancy of the refractive index, is disclosed. And only by comparing the refractions of light of different colours does the difference, the inequality of the indices of refraction, arrest the attention. Comparison based on change leads the mind simultaneously to the highest abstractions and to the finest distinctions. (1896, p.258 Eng. p.230f.)

Science, he argued, works by assigning quantitative values to the variables involved, so that scientific laws can be conceived as 'functional' or 'tabular' descriptions of such continuous transitions.

Mach's thesis concerning continuous variation can be understood on at least five distinct levels:

- It is first of all a thesis about the way the world (i.e. the totality of elements) is.

- It is secondly a thesis about how, within this totality, science actually proceeds or develops, a thesis about the 'economical' ordering activities of scientists.

- It is thirdly a thesis about the way science *ought* to proceed: a more adequate grasp of the notion of continuous variation would, Mach claims, make science more efficient (more economical).

- It is fourthly a thesis about the continuity of transitions between everyday experience as traditionally and habitually understood and the constructions of scientific theories.

- And finally it is a thesis about the interplay between *sense* experience – which is, in a certain sense, the only true reality – and those indirect, accessory adjuncts to this experience which are scientific theories.

Now there is one aspect of Mach's thinking here to which considerable attention has been paid in subsequent literature in the philosophy of science. Mach's functional descriptions – which almost always take the form of differential equations – involve no reference to extrinsic notions such as causality, space and time. The scientist rather *implicitly defines* the objects of his research in the very formulation of his equations, and particularly in his choice of variables. In this respect Mach can properly be said to have anticipated certain aspects of the conventionalist and operationalist accounts of the nature of science. But Mach was not simply a conventionalist. For the ordering activities of scientists, their drive to produce economical orderings of functional descriptions, has as its indispensable correlate in the Machian framework the ordered transitions and relations exhibited by the phenomena themselves.

§8. Mach on Dependence

A first provisional formulation of Mach's account of the relation of dependence might run as follows: two variables (continuously variable quantities) are dependent if and only if the variation in one is reflected in a simultaneous variation in the other. One phenomenon is dependent on another precisely when there is a regular covariation of the two.

Independence, on the other hand, is signalled by the absence of any regular covariation. Where tabular descriptions reflect constant covariation, there we have dependence amongst the phenomena represented, and thus the proper expression of relations of dependence is in functional equations.²⁶

It hardly needs pointing out that the notion of necessity, including the spurious necessity involved in so-called relations of causality, is entirely excluded from this framework. The very opposition between what is necessary and what is contingent dissolves in the face of Mach's commitment to an all-pervading and continuous variation.

Mach's notion of dependence is related in the first place to continuous *qualitative* covariation, but it is *quantitative* variation whose ordering and presentation is the primary function of science. Science must be quantitative, Mach holds, if it is to be useful (adaptive) at all. Only through numerical equations can we make predictions which take us beyond the merely qualitative (i.e. beyond that which, according to Mach, we know already).

Quantitative dependence is a particular, more simple case of qualitative dependence... In the case of quantitative dependence what we find is a surveyable, intuitive continuum of cases, while in the case of qualitative dependence it is always only necessary to consider a number of individual cases by themselves. ((1917), p.204, Eng. p.150, quoted by Schulzki, p.159.)

Even when we have to do with qualities (colours, tones) quantitative features of these are available. Classification here is so simple a task that it barely makes itself noticeable and even in the case of infinitely fine gradations, of a continuum of facts, the number system already lies ready to follow as far as is necessary. ((1896), p.438f., Schulzki, p.161)

Mach stresses further that dependence – or 'constancy [*Beständigkeit*] of covariation' – is always relative to the perspective adopted by the investigator or theorist.²⁷ Not *all* of what is continuously in flux can of course be grasped in any one functional description or equation. The scientist rather *selects* what is to be represented from this or that point of view. Scientific theories, the constantly adaptive products of the ordering activities of scientists, set out the connections between those functional descriptions which are revealed by such a process of selection. The latter picks out, for reasons of his own and appealing to convenience, analogy, habit, and so on, certain specific relata, and sets other relata out of account by restricting the range of variation which he will allow for consideration. Thus the gas equation, $pV/T = \text{constant}$, holds 'only for a

gaseous body of invariable mass for which pressure, volume and temperature have the same values in all its parts and provided the conditions are distant enough from liquefaction.' ((1917), p.445, Eng. p.353f.) The law of refraction $\sin\alpha/\sin\beta$ 'is narrowed by being related to a definite pair of homogeneous substances at a definite temperature and pressure, as well as to the absence of internal differences of electrical or magnetic potential.' (*loc.cit.*)²⁸

It is the principal thesis of this paper that the theory of dependence in terms of constant covariation is inadequate, a thesis we shall attempt to demonstrate in relation to the specific problems associated with our perception of what is complex. First, however, we must return to the treatment of dependence by Brentano's successors.

§9. Variation and Dependence in the Brentano Tradition

The writings of Brentano's pupils on variation and dependence are concerned primarily not, as in Mach's case, with quantitative and continuous variation; their employment of the notion is to a much lesser extent concentrated around phenomena which fall within the province of numerical science.²⁹ That there are, nonetheless, parallels with Mach's treatment, both of variation and of dependence, becomes clear when we look at the first important published treatment of dependence in the Brentano tradition – Stumpf's *Über den psychologischen Ursprung der Raumvorstellung* – which deals centrally, like Mach's paper of 1865, with problems associated with the structures of visual perception.³⁰

All presentations of colour in our experience, all 'colour-contents', to use Stumpf's term, are bound up with presentations of visual extent (with what we might call 'extension-contents').³¹ What is the nature of the relation between colour-contents and extension-contents? This relation cannot, Stumpf argues, be merely one of regular but contingent association – like, say, the regular association of 'Goethe' and 'Schiller' in the minds of German schoolboys. For however we attempt to vary colour- and extension-contents in imagination, in memory or in present experience, along all conceivable dimensions, we discover that it is impossible to separate the two. Systematic variation, Stumpf argues, reveals that the connection of contents of the two given types is a necessary connection – of precisely the kind to which appeal was made by

Mach, *en passant*, in his paper of 1865. Colour-contents and extension-contents are such that, as a matter of necessity, they cannot occur in isolation from each other. Within the quantitative, functional framework adopted by Mach in his later writings all such necessary connection is in effect eradicated (or perhaps we should say that its necessity is simply ignored). It would seem that its recognition is made possible only on the basis precisely of *qualitative* investigations of the type undertaken by the Brentanists, investigations in which, further, the ontological and the genetic dimensions are kept clearly separate.³²

The implications of this theory of necessary connection are manifold. As Stumpf points out, from the necessity of the connection between colour- and extension-contents it follows that it is misleading to conceive these as separate contents at all: each is, rather, something that is in itself intrinsically partial or incomplete, is what Stumpf calls a *Teilinhalt*. Each such partial content can exist only to the extent that it is supplemented, in the context of a larger whole, by one or more further partial contents of a complementary sort.

Teilinhalte – which play a role similar to that of distinctive features in phonology – are, we might say, sub-atomic units of experience. Their recognition thereby signifies a break with atomistic psychology that is no less radical than is the recognition of *sui generis* psychological complexes – for it implies that the simplistic notion of atomicity, derived as it was from the corpuscular theories of the Newtonian era, cannot serve within psychology as an adequate basis even for the treatment of *simple* sensations.

The two-sided relation of necessary connection between colour-content and extension-content is called by Stumpf a relation of mutual dependence, and we note that dependence relations between *Teilinhalte* of the given sorts have been isolated by Stumpf precisely by a method which involves appeal to a notion of variation related to qualitative orderings manifested in experience. The same 'method of variation' is used by Stumpf also in relation to other kinds of psychic contents to reveal whole families of species of *Teilinhalte* and two- or n-sided relations of mutual dependence between them.

It is at this point that we see the connection between the two key notions of dependence and variation as these are conceived within the Brentano tradition. The work of Husserl directly continues that of Stumpf, elaborating Stumpf's method of systematic variation in such a way that it could be applied, in principle, beyond the purely psychological

sphere. Husserl and his immediate followers extended the method still further, to reveal hierarchies of dependence relations not merely in relation to perceptual phenomena but also in other, highly disparate dimensions of experienced reality.³³

§10. On the Concept of Substance

Perhaps the most interesting parallels between the respective treatments of dependence and variation of Mach and of the Brentanists are revealed in their analyses of the traditional concept of substance.

For Mach, as we have seen, there is 'but one sort of constancy, which embraces all forms, namely *constancy of connection*'.³⁴ This applies particularly to the concept of substance. Substances (bodies) are not that which is identical through change, they are not that which endures. They are, rather,

no more than bundles of reactions connected in a law-governed fashion. The same is true of processes of every sort... waves and water which we follow with the eye and with the sense of touch..., shock-waves in the air which we hear and can only make visible by artificial means..., electric currents which can be followed in artificially produced reactions. What is constant is always and only the law-governed connection between reactions. *This is the critically purified concept of substance which science puts in the place of the vulgar concept.* (Mach, *Notizbuch*, p.188, as quoted by Schulzki (1980), p.88, our emphasis; cf. Dingler (1924), p.106.)

Thus it is constancy of connection which is at the heart of the Machian concept of substance: 'we term substance what is conditionally constant' (1903, p.256, Eng. p.328), and the 'constant connection between reactions expounded in the propositions of physics represents the highest degree of substantiality that enquiry has thus far been able to reveal.' ((1917), p.134, Eng., p.99)³⁵

Mach's views thereby signify also a rejection of the traditional conception of substance as a substrate of properties or bearer of accidents. Now this conception is still very much defended by Brentano,³⁶ but Meinong, Husserl and Stumpf each puts forward views in opposition to that of Brentano which constitute a rejection of the traditional notion exactly parallel to that of Mach. A substance is, they argue, just a whole

consisting of parts standing in relations of dependence, and manifesting constant and variable dimensions. Thus as Meinong once put it:

The nature of substance is to be sought in the fact that it is a complex of, so to speak, mutually dependent [*aufeinander angewiesenen*] properties. (Meinong 1906, p.27)

And as Stumpf – who had earlier been a colleague of Mach's in Prague – writes in the *Erkenntnislehre* (sec. 3.7): substance is a unity of interdependent parts each of which has its own dimension of variation.³⁷ Or, as he formulates the matter in his autobiography:

In the relation between colour and extent I thought I could see (and still think so) a striking example of or analogy with the relation which is taken to obtain between the properties of substance in metaphysics. (Stumpf 1924, p.8)

Kreibig, a follower of Meinong, even goes so far as to identify the *thing* as a specific sort of Gestalt quality: 'A thing is given in perception as the Gestalt quality of a sum of perceived characters' (1909, p.115). The perception of such a quality becomes associated with an existential judgment which ascribes external reality to that which is perceived. 'All other definitions of the thing are purely metaphysical in nature and alien to an empirical treatment of the problem.'³⁸ Stumpf's student Kurt Lewin takes this idea one step further and sees the mind or ego as a mere complex of interdependent parts, of 'strong' and 'weak' Gestalten, which are in part in communication with each other, in part such as to disclose no genuine unity at all.³⁹

§11. On the Nature of Dependence

What, then, is dependence? For the Brentanists the relation of dependence is a relation of real necessity, a reflection of structural laws concerning the necessary co-existence of objects. The necessity involved is *sui generis*; it is neither physical (causal) nor logical (conceptual). It is a necessity of a type which is illustrated not merely by the relation between colour and extension or between the distinctive features of a phoneme, but also, for example, by the relation between a promise, on the one hand, and a mutually correlated claim and obligation on the other (the

former cannot, as a matter of necessity, exist without the latter). In fact the concept of necessary dependence is a formal concept, a concept which is like the concepts of logic in that it can be applied in principle to all matters, whatever their qualitative determinations. It differs from the concepts of formal logic, however, in being *ontological*; it is a concept of formal ontology or, as Meinong would put it, of the formal 'theory of objects'.

In regard to the Machian theory of necessity we can note first of all that Mach typically opposes logical to physical necessity and seeks to reduce the latter to the former. Closer inspection reveals, however, that by 'logical necessity' he means only psychological necessity, a notion he explicates in terms of always defeasible expectations:

There is only logical necessity: if certain properties hold of a fact [*Zukommen*]... then I cannot simultaneously ignore this. That they hold is simply an experiential fact. There is no such thing as physical necessity. ((1896), p.437; cf. Musil, p.81f., Eng. p.58f.)

The agreement of concepts with one another is a logically necessary requirement, and this logical necessity is also the only necessity of which we have knowledge. The belief in a necessity in nature arises only where our concepts are closely enough adapted to nature to ensure a correspondence between logical inference and fact. But the assumption of an adequate adaptation of our ideas can be refuted at any moment by experience. ((1904), p.280; Eng. p.318)

In late editions of the *Mechanics*, Mach replies to Husserl's criticism that the principle of the economy of thought is unable adequately to comprehend the nature of logical necessity. The account of the economy of thought has to be supplemented, Husserl had argued, by an account of the role of formal concepts. Mach replies as follows:

As a natural scientist I am accustomed to investigating individual questions... and to move from these towards more general questions. I adhered to this custom in investigating the genesis of physical knowledge. I was obliged to proceed in this way because a general theory of theories was a task which was beyond me... I therefore concentrated on individual phenomena: the adaptation of thoughts to facts and to one another, thought economy, comparison, thought experiments, constancy and continuity of thought, and so on. I found it both profitable and sobering to consider ordinary thought and all science as a biological and organic phenomenon with logical thought as an ideal limit case.

But he goes on:

I would not want to doubt for a minute that investigation can begin at either end. And, as this makes clear, I am perfectly capable of distinguishing between logical and psychological questions, a distinction I think everyone is capable of making who is interested in the light psychology amongst other things can throw on logical processes. Someone who has once looked carefully at the logical analysis of what Newton says in my *Mechanics* will find it difficult to reproach me with the attempt to run together blind, natural thought and logical thought. Even if we have the complete logical analysis of all sciences before us, the biological and psychological investigation of their genesis... would still be needed; although this would not exclude submitting the latter in its turn to logical analysis. ((1904), p.537; Eng. p.592ff.)

Thus Mach is apparently prepared to concede that the two approaches – the logical and the biological/psychological – are complementary and do not at all contradict one another. If, however, we look at Mach's deservedly famous 'logical' analyses of Newton, then what we find is in fact *conceptual criticism* – albeit of the highest order⁴⁰ – not any recognition of the role of formal concepts, whether logical or ontological.

A letter from Husserl to Mach on receipt of his reply puts the main point clearly: the different formal concepts – proposition, implication, some, all, cardinal number, etc. – cannot be taken to be 'expressions of empirical generalities', they cannot be explained by the genetic psychology of judging, cognizing, etc., nor by reference to the economy of thought, for any such attempted explanation would be circular.⁴¹

There is in fact a fundamental unclarity in the concept of necessity that is employed by Mach, and thus we can anticipate a corresponding unclarity about what precisely dependence is, an unclarity which emerges most pointedly in Mach's two papers – replies to Planck and Stumpf – of (1910). Dependences are, he says, 'real', 'given'; physical dependences differ from psychological dependences in being more 'intrinsic' [*innig*], thereby yielding us our concepts of matter. All well and good, as intuitions go. But Mach was unable to produce a *theory* of the different types of dependence which could do justice to intuitions of this sort. Overimpressed by the relativity of a restricted range of examples of dependence conceived as more or less constant covariation, Mach came to see the latter as an exhaustive category whose inner structure is not capable of being further penetrated by science.

We have mentioned already that Husserl generalized Stumpf's theory of covariation beyond the sphere of psychic contents. Husserl went beyond Stumpf first of all in recognizing relations of one-sided in addition to those of mutual dependence. In this he was embracing an idea already

developed by Brentano in his theory of the types of psychic phenomena in the *Deskriptive Psychologie* and before him by Aristotle in the theory of individual accidents. Brentano's own examples of one-sided dependence are couched in the terminology of one-sided separability: a judgment cannot exist in separation from an associated presentation; a phenomenon of preference or aversion cannot exist in separation from an associated judgment, and so on. Other sorts of examples of one-sided dependence might be: the dependence of current or charge upon a conductor; of magnetic attraction on magnetized body; of action upon agent; of a depression over the Atlantic upon molecules of air; and so on. But all of these examples – and certainly all the examples treated by Brentano (and by Stumpf) – concern objects existing *simultaneously*. Brentano's theory is in this sense too narrow.

Husserl went further than both Brentano and Stumpf, secondly, in admitting trans-temporal dependence relations.⁴² Now, as we have seen, it was Ehrenfels in "Über 'Gestaltqualitäten'" who first took the notion of dependence as this was to be found in Brentano and Stumpf and applied it to examples of objects of sense that are spread out in time and to objects of sense that do not exist simultaneously or at an instant. In this way he was able to produce the first truly general theory of the perception of complexes, embracing both visual and (for example) aural complexes, both static and dynamic complexes, and also hybrid complexes of various kinds.⁴³

It was in the end however Husserl, in the 3rd *Logical Investigation*,⁴⁴ who succeeded in bringing together all of these strands – one-sided and mutual dependence and independence – within the framework of a single theory. Moreover, it was Husserl who managed to free the theory of dependence relations from the limitation to psychological examples (and to psychologically motivated criteria of dependence) and to develop the theory as a formal ontology applicable to all material varieties of objects, existing both simultaneously and across time. Husserl did not, however, ignore the question of the relation between this formal ontology and the field of psychological examples in which it has its roots. Indeed his *Logical Investigations* can be said to show the true indispensability of both mutual and one-sided dependence to the adequate understanding of the structures of mental phenomena, as also of the phenomena of language.

But how does this leave Mach? Given his notion of dependence as 'logically necessary' constant covariation, Mach, it is clear, cannot accept

even the possibility of one-sided dependence. Two or more variables can either vary simultaneously together, in which case, according to Mach, we have *mutual* dependence. Or they can fail to vary together, in which case there is *no dependence at all*. A third alternative simply fails to present itself within the tabular or functional conception of scientific laws defended by Mach – and, we might add, by almost all subsequent philosophers of science. All purported examples of one-sided dependence must therefore be rejected by these philosophers as spurious, to be explained away by a sufficiently deep analysis or reduction of the phenomena in question.

And while the recognition of a relation of necessary connection between characteristic sensation and foundation was, as we have seen, clearly expressed in Mach's 1865 paper, even at that stage, that is to say before the fully worked-out theory of *Elemente*, it is clear that Mach was unaware of the peculiarity of relations of one-sided dependence. Within the terms of Mach's official theory of dependence relations the insight into this peculiarity simply cannot find expression. Misled by the fact that his view of dependence as constant covariation is plausible for the bulk of the examples he treats (e.g. the gas laws⁴⁵), Mach adopts a theoretical framework which cannot permit the proper formulation of other sorts of examples, and he thereby misses distinctions which even he would otherwise have to admit as being crucial.

Perhaps the most important of these – to which we draw attention only in passing – was dealt with most succinctly by Kurt Lewin. It is the distinction between what might be called *successive* and *longitudinal* causality. Thus consider a sentence such as 'if the temperature of a gas is raised, then it will expand or its pressure will increase':

The essential meaning of such an assertion is this: events *a* and *b* are necessarily *dependent moments* of a single unified occurrence. The mathematical formula states the quantitative relations involved in the occurrence. Already in such cases the dependent moments of the occurrence are moments that obtain temporally *side by side*.

The part-processes in question, then, are to be understood as being related not by temporal succession as 'cause' and 'effect', but rather in such a way that they are 'brought into reciprocal functional dependence throughout the longitudinal section of the occurrence in question'. (Lewin 1927, p. 305) Two quite different sorts of dimension in nature are

involved in these two different forms of causality: Mach is able to give a clear account of neither.

§12. Epilogue

The implications of Mach's commitment to a universal mutual dependence are far-reaching. In relation to the concept of time, for example, it leads to a position that is difficult to distinguish from a Spinozistic pantheism, a view of the world which would make everything dependent on everything else (the night, in which all cows are black). As Musil writes, expounding Mach's theory:

space and time are themselves concepts for certain connections between phenomena: the oscillations of a pendulum, for example, take place in time only if its excursion depends on the position of the earth and so here the measurement of time amounts to measurement of angles or lengths of arcs. If we imagine the natural course of different events represented by equations involving time, then time may be eliminated from these equations (for example, an excess of temperature may be determined by space traversed by the falling body); the phenomena then appear simply as dependent on one another. It is therefore superfluous to emphasize time and space, since temporal and spatial relations merely reduce to dependences between the phenomena.

Thus the equations of physics refer to a very general connection. For to be a function of time now means to be dependent on certain spatial positions; and that all spatial positions are functions of time means that from the point of view of the cosmos all spatial positions depend on one another; but since spatial positions can only be recognized by reference to states we can also say that all states depend on one another. In our ideas of time, then, the profoundest and most universal connection of things finds expression. The same is true of our ideas of space, for every motion of a body K is a motion towards other bodies A, B, C..., and even if one says that a body preserves unchanged its direction and velocity in space this contains a reference to the need to take into account the whole world. (Musil 1908, p.72, Eng. p.52)

We have quoted Musil at such length, first of all in order to draw attention to the fact that our criticisms of Mach, here, are very much Musilian in spirit. But also because of the candour with which Musil expresses the implications of Mach's views. The theory of time presented in this passage carries the implication that Mach could not introduce a notion of one-sided dependence into his system by the back door, by

appealing to *trans-temporal* variation, such that a later variation would be non-reciprocally dependent upon an earlier. In fact, Mach identifies all attempts to state a dependence relation across time with attempts to save the banished notion of causality. But this signifies that the three dimensions of the temporal and the atemporal, of the possible and the necessary, and of the causal and the non-causal are, in effect, confounded within Mach's functional framework, where the more careful approach of Ehrenfels and of the other Brentanians had made it possible to keep them apart.

Only at one point does Mach recognize, in passing, that the commitment to universal mutual dependence does not exhaust all purely analytic possibilities. 'But we do not', he says, 'need to see any metaphysical problem in this' ((1904), p.548; Eng. p.351). Here as elsewhere his faith lies in the possibility that when all intervening variables are spelled out – e.g. between friction and heat – we shall be left with a system expressible entirely in terms of functional equations. But he is here directly contradicting his own principle that what is given in experience should be taken at face value. As Musil points out (*op.cit.*, p.77, Eng. p.55), the direct generation of heat through friction does not correspond to any *direct* generation in the opposite direction. The directionality or irreversibility of certain relations of dependence is given in experience. It is only in virtue of an impoverished theory of dependence that Mach can overlook this.

Notes

¹ Revised and expanded English version of "Mach und Ehrenfels: Über Gestaltqualitäten und das Problem der Abhängigkeit", in R. Fabian, ed., *Christian von Ehrenfels. Leben und Werk*, Amsterdam: Rodopi, 1985, 85-111. We have provided only relatively brief indications of the relevant Gestaltist literature here: the reader is invited to supplement the references provided by turning to the Bibliography at the end of this volume. References to items in this bibliography are given by author and year *without* parentheses; references in which the year is surrounded immediately by parentheses – 'Brentano (1895)', etc. – designate items in the list on pp. 155f. below.

² Note that, precisely speaking, Brentano and his students make up not a school but a loose association, a fact marked in what follows by our talking of 'the

Brentano tradition', 'Brentano and his heirs', etc. On the influence of this tradition: see Smith, ed. 1982. On its unifying philosophical features, see Mulligan (1980) and Mulligan (1986).

³ Note, however, Stumpf's remark (1939/40, I, p.243f.) to the effect that the discoveries of the Gestalt psychologists have led to false and exaggerated accusations that 19th century psychology was purely summative or atomistic. The assumption holds good, he points out, only in certain cases: e.g. Taine and the English associationist psychologists. It is not true of e.g. Lotze and James.

⁴ 1886, p.18, Eng. trans., p.20.

⁵ As Gustav Bergmann points out, Mach belongs with Meinong – he could have mentioned all Brentano's heirs – to the first group of philosophers who took seriously 'the introspective irreducibility' of certain 'relational characters' ((1950), p.7)

⁶ Cf. Meinong (1965), p.74. Mach does not mention the 1865 paper by name.

⁷ As Spinoza (*Ethics*, III, Proposition 2, Scholium) puts it: 'No one has yet determined what the body is capable of... For no one has yet come to know so exactly the structure [*fabrica*] of the body that he could explain all its functions.'

⁸ The passage in question is discussed in Becher 1911, pp.238ff., who points to the importance of Mach's 'hypothesis of identical accompanying phenomena' for the treatment of the problem of mind and body. Becher points out also however that this hypothesis goes beyond what is given in experience. See also Keiler 1982a, p.255, who sees in the hypothesis an anticipation of Köhler's isomorphism theory.

⁹ This is in contrast to Husserl in the *Philosophie der Arithmetik* of 1891, whose views in this respect are too often overhastily identified with those of Ehrenfels. See the discussion in §3 of the essay by Smith, above.

¹⁰ On the importance of the peculiarities of temporal Gestalten for the early work of the Berlin school see Ash 1982, pp.296f.

¹¹ Eng. p.287 of 1959 ed. Consider also the following passage, which illustrates clearly the connection between nervous *quale* and bodily movement:

To the three optical space-coordinates, viz., to the sensations of height, breadth, and depth, corresponds... simply a three-fold innervation, which turns the eyes to the right or to the left, raises or lowers them, and causes them to converge, according to the respective needs of the case... Whether we regard the innervation itself as the space-sensation, or whether we conceive the space-sensation as ulterior to the innervation [is] a question neither easy nor necessary to decide. (1886, p.77f., Eng. p.169f.)

¹² In his two papers of (1910), particularly where he is replying to criticisms of e.g. Stumpf, we do encounter references to an '*innigste Zusammenhang*', a notion which may be descended from the earlier notion of a 'necessary connection', but these references play no effective role within Mach's later theory. In particular, Mach makes it clear in these papers that such connections are merely pervasive and very frequent, and that they are 'necessary' exclusively in this sense (i.e. not necessary at all).

¹³ The Munich psychologist Cornelius, in his own paper "Über 'Gestaltqualitäten'" of 1900, criticizes both Ehrenfels and Mach for having drawn the wrong inferences from the existence of perceived similarity.

Ehrenfels was wrong, he held, for having conceived the Gestalt quality as a 'positive content of presentation' superadded to our perception of what is given on the level of sensation. And Mach was wrong for having missed the fact that feelings, too, whether muscular or non-muscular, are themselves varieties of Gestalt qualities. For Cornelius, talk of Gestalt qualities is a mere roundabout way of referring to similarity of complexes of sensations, which should simply be accepted as a primitive phenomenon.

¹⁴ Of course more presentations will be involved also in virtue of the workings of *memory*, which are required if the Gestalt-presentation is to be constituted at all; but we shall leave this matter aside in what follows since it bears no relation to our principal concerns.

¹⁵ That there are two distinct dimensions here is seen if we consider, for example, the relation between a child and his mother (or between God and His Creation). The child is genetically dependent upon its mother, could not have *begun to exist* unless the mother existed. But the child is clearly not dependent for its continuing to exist upon the continued existence of its mother. See Ingarden (1964/65) for the definitive philosophical treatment of this distinction.

¹⁶ Roughly: *a* is one-sidedly dependent on *b* if and only if *a* is such that, as a matter of necessity, it cannot exist unless *b* exists but not conversely. *a* is two-sidedly (mutually) dependent on *b* if and only if *a* and *b* are necessarily such that neither can exist without the other. Clearly mutual dependence can hold also in relation to any plurality of objects, however large. See Smith, ed. 1982 for further details. In the German version of this paper, where we concentrated rather on the genetic question, it was suggested erroneously that Ehrenfels did not use the notion of one-sided dependence. See, however, p. 88 above.

¹⁷ See §5 of the paper by Smith, above. Interestingly Ehrenfels, in his paper on Gestalt qualities of 1932 translated above, allies his own earlier work with that of the production theorists, though this retrospective interpretation seems not to be supported by the text of the paper of 1890.

¹⁸ Cf., again, Smith, ed. 1982, esp. the diagram on p.482.

¹⁹ See e.g. Feyerabend (1980), pp.262-68.

²⁰ Note that many, if not all, of Mach's inaugurating concepts are ordinal in nature: that is, they have to do with *intensive magnitudes*. See Bradley (1971), ch. II, on "Metrical Concepts".

²¹ See e.g. Mach (1896), p.452f., (1917), pp.446, 449f., Eng. pp.348, 351f. Compare Brentano (1968), p.58, and also the following passage from Husserl:

Depth [*Tiefsinn*] is a mark of chaos which genuine science aims to transform into a cosmos, into a simple, completely clear, analysed order. Genuine science knows no depth as far as its actual theory extends. Every piece of accomplished science is a whole made up of steps of thought each of which is immediately evident – and hence not at all 'deep'. Depth is a matter of wisdom, conceptual univocity and clarity – a matter of rigorous theory. ((1911), p.144 of the translation)

²² Wittgenstein (1953), II, xiv.

²³ Compare also the following discussion by Köhler of the 'puzzle' of external perception:

Although allegedly founded on processes in my interior, such percepts as tree, house, cloud, moon and thousands of others are clearly localized outside of me... Only a few authors, mostly men of great phenomenological power, have been able to recognize the apparent puzzle as what it really is: a most unfortunate pseudoproblem produced by inconsistent thinking. Such men were E. Hering, the physiologist, and E. Mach, the physicist and philosopher. (Köhler 1938, pp. 126f.)

Hering's important role in the early development of Gestalt psychology, above all in the matter of experimental approach, has been stressed above all by Ash (1982, pp.87-108). See especially Hering's *Outlines of a Theory of the Light Sense* of (1905). Hering's work contains considerations of the relationship of psychology and physiology and of the physiological correlates of perception related in important ways to those of Köhler 1920.

- ²⁴ See H. Lübke's "Positivismus und Phänomenologie" of 1972, an excellent account of the phenomenology of the *Analysis of Sensations*. See now also Sommer (1985).
- ²⁵ Cf. Kraus' remarks in Brentano (1924/25), I, p. xvii, and, for a fuller treatment, Brentano's *Deskriptive Psychologie* (1982). On the parallels between the Brentanian opposition between descriptive and genetic psychology and the synchronic/diachronic opposition of de Saussure see S. Raynaud (1982).
- ²⁶ Compare the papers of Grelling and Oppenheim and the discussion by Simons below.
- ²⁷ See 1903, p. 256, Eng., p. 328; and compare Musil, pp. 70f., Eng. pp. 51f.
- ²⁸ On the ideal gas example and other examples of 2-, 3- and n-dimensional manifolds see Weyl (1918), p. 75, and on this Stumpf 1939/40, sec. 26.3, pp. 649f. It seems indeed that the object investigated and the investigating observer will in a certain sense *interpenetrate*:

An element such as the warmth of a body A hangs not merely together with other elements whose aggregate we designate e. g. as a flame B; it hangs also together with the totality of the elements of our body e. g. of a nerve N. (*Mechanik*, 6th ed., p. 554, 9th ed., p. 484, Eng. p. 612)

- ²⁹ Exceptions would be the experimental work of Meinong, Stumpf and their pupils, above all Stumpf's quantitative investigations in acoustics and Benussi's work on Gestalt perception.
- ³⁰ Stumpf's ideas on dependence in this work almost certainly derive from ideas presented by Brentano in the already-mentioned lectures on descriptive psychology. Mulligan and Smith (1985) is an account of this latter work that is complementary to the present essay.
- ³¹ Compare e. g. 1886, p. 41, Eng. p. 103, and also James (1879) and (a corrective to James) Rubin (1977).
- ³² See, again, Smith, ed. 1982, esp. pp. 25-35 and Mulligan (1986).
- ³³ Indeed the project of Husserlian phenomenology can itself be described as being that of uncovering – albeit from a particular epistemological point of view – the various families of dependence structures involving consciousness.
- ³⁴ 1886, p. 157, Eng. p. 331, our emphasis; cf. also 1903, p. 258, Eng. p. 331.
- ³⁵ Cf. also the following passage from *Erkenntnis und Irrtum*:

When an equation is satisfied, then there is involved therein a widened and generalized concept of substance... In general it matters little whether we regard the equations of physics as expressions concerning substances (laws or forces), for at all events they express functional dependences. ((1917), p. 277, Eng. p. 207f.)

- ³⁶ Cf. Brentano (1933), pp. 140, 274, Eng. pp. 108, 194 (criticisms of Mill and Herbart). See also Chisholm (1978), Mulligan (1985), and Smith (1987).
- ³⁷ It is interesting at this point to compare Musil's sketch of this Stumpfian conception in his critique of Mach of 1908, pp. 54f., Eng. pp. 42f.
- ³⁸ See also pp. 118ff. for Kreibitz's criticisms of the traditional concept of substance.
- ³⁹ Lewin 1926, pp. 32f. of *Separatum*.
- ⁴⁰ Thus for example Mach's criticism of Newton's definition of mass in terms of density and volume is that it is circular ((1904), ch. 2.III, §5, Eng., p. 237). To appreciate the importance of such criticisms we need only think of their influence on Einstein.
- ⁴¹ In the same letter Husserl emphasizes the *one-sidedness* both of Mach's approach and of his own. This idea seems to have impressed Mach, and he returns to it in later work. See (1917), p. 282, Eng. p. 212. On 'logic' in Mach see further Musil, pp. 92f., Eng., p. 64f.
- ⁴² See Mulligan and Smith (1986), for further details.
- ⁴³ Ehrenfels saw for example that there exist hybrid Gestalten embracing both physical and psychical components – corresponding to verbs such as 'murder', 'promise', 'threaten', 'suffer', etc. describing complex actions.
- ⁴⁴ A start was made already in Husserl's paper of 1894.
- ⁴⁵ Compare again the passages from Weyl and Stumpf mentioned in n. 28 above.

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