

# Inferentialism without Verificationism: Reply to Prawitz\*

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## Abstract

I discuss Prawitz's claim that a non-reliabilist answer to the question "What is a proof?" compels us to reject the standard Bolzano-Tarski account of validity, and to account for the meaning of a sentence in broadly verificationist terms. I sketch what I take to be a possible way of resisting Prawitz's claim—one that concedes the anti-reliabilist assumption from which Prawitz's argument proceeds.

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What is a proof? Professor Prawitz's paper has two aims: to show that this is a hard and important question, and to suggest a possible answer—one, he argues, that requires us to reject the standard Bolzano-Tarski account of validity, and to account for the meaning of a sentence in terms of the grounds for asserting it. Section 1-3 attempt to resist Prawitz's attack to the standard conceptions of meaning and validity. Section 4 briefly raises three potential worries about Prawitz's preferred answer to our initial question. Section 5 offers some concluding remarks.

## 1 Legitimate inferences and knowledge of validity

The following definition of a proof may seem initially promising: a proof is *a chain of inferences that is seen to be valid*. The question arises, though, as to which inferences can legitimately figure in a proof. Prawitz suggests that only *legitimate inferences* can, where an inference *I* is legitimate just if by making *I* a subject who has a justification for the premises gets a justification for the conclusion.

To be sure, not all inferences are legitimate in this sense: only the ones that are known to be valid—Prawitz assumes—are. However, he argues, on pain of starting an infinite regress, knowledge of legitimate inferences must be ultimately non-inferential,

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i.e. it “must either be self-evident somehow, presumably in virtue of the meaning of the sentences involved, or be implicit in some sense that would have to be explained” (p. 6).<sup>1,2</sup> And, Prawitz continues, whether we can make sense of either of these two options depends on what one means by ‘valid’. More precisely, if validity is to be known non-inferentially, the validity of an inference may not be explicated, as the Bolzano-Tarski tradition would have it, as truth-preservation for all variations of the meaning of the non-logical vocabulary. Prawitz concedes that, on the Bolzano-Tarski account, a rule like  $\wedge$ -I may be non-inferentially or implicitly known to be valid, provided that we know  $\wedge$ ’s meaning, viz. that, for every valuation  $v$ ,  $A \wedge B$  is true at  $v$  just if  $A$  is true at  $v$  and  $B$  is true at  $v$ , and that we know that an inference is valid just if it preserves truth at every valuation. But, he argues, for more complicated cases “we find that we cannot go from the meaning of the sentences given in terms of truth-conditions to the validity of the inferences without a considerable amount of reasoning on the meta-level” (p. 7). If validity is to be accounted for the standard way, Prawitz concludes, “we cannot require that a subject should know the validity of an inference before she can use it in a proof (*Ibid.*)”.

## 2 Moderate inferentialism

This conclusion, it seems to me, can be resisted. Ordinary speakers are typically unaware of the logician’s explications of the informal notion of validity. Hence, it does not seem plausible in the first place to require that they know—whether self-evidently or implicitly—facts about validity (see also Pagin, *ibid.*, pp. 18-9). A more plausible option would be to say that an inference  $I$  is legitimate if being disposed to infer according to  $I$  is constitutive of our understanding of some of the expressions figuring in  $I$ . Following Boghossian (2003), let us call this the *meaning-entitlement* view:

(ME) By making  $I$  a subject  $S$  who has a justification for the premises gets a justification for the conclusion if (roughly)  $S$  knows the meanings of the expressions in virtue of which  $I$  is valid, i.e.  $S$  is disposed to infer according to  $I$ .

Two points are worth mentioning. First, this view does not require speakers to know that inferences are valid. All they need to know is the meaning of (some of) the expressions occurring in them—that is, if  $I$  is constitutive of  $E$ ’s meaning, speakers must be willing to infer according to  $I$ , in order for  $I$  to be legitimate. Second, ME is compatible with the Bolzano-Tarski account of validity.

Let *moderate inferentialism* be the view that introduction and elimination rules (thereafter, I- and E-rules respectively) determine the truth-conditions of the logical operators, on the assumption that these rules are truth-preserving, and that to

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<sup>1</sup>Page references are to Prawitz (2010), except where otherwise stated.

<sup>2</sup>One might reject this disjunction, and attempt to block the regress along broadly *reliabilist* lines (see e.g. Rumfitt, 2008, pp. 62-3). Prawitz is aware of this option, but declares that he will not take it into consideration. I shall too set it aside, for the sake of argument.

understand a logical operator  $\$$  is to be willing to infer according to its I- and E-rules.<sup>3</sup> Following Hodes (2004) and MacFarlane (2005), moderate inferentialists may distinguish between the *sense* of a logical constant, whose grasp is constituted by a willingness to infer according to its basic introduction and elimination rules, and its *referent*, e.g. the truth-function it denotes. Indeed, they may have independent reasons for doing so. Classically, for instance,  $A \vee B$  (“*A* or *B*”) and  $A \nmid B$  (“not both *A* and *B*”) have the same truth-conditions. Yet, we may want to say, ‘ $\vee$ ’ and ‘ $\nmid$ ’ differ in some aspect of meaning (MacFarlane, 2005, § 6.2). Moderate inferentialists have the resources to account for such a difference: ‘ $\vee$ ’ and ‘ $\nmid$ ’ have different senses, which are specified by their different I- and E-rules.

Closer to our present concerns, moderate inferentialists can take the truth-conditions of the logical operators to select the class of *admissible models*—models in terms of which validity can be defined in model-theoretic terms. Thus Vann McGee: “The rules of inference determine truth-conditions. The truth-conditions together [...] determine the logical consequence relation” (McGee, 2000, p. 72).

### 3 Prawitz’s objections to inferentialism

Prawitz considers a response along similar lines, viz. that “the meaning of expressions to be determined by inference rules that govern their use” (p. 10). He examines two versions of the view, and finds them both wanting.

On the first version, propounded by Gentzen and, indeed, Prawitz himself during the 70’s and the 80’s, only I-rules are taken to be meaning constitutive—more precisely, I-rules whose conclusion is logically *more complex* than any of the premises or of the discharged hypotheses. Prawitz objects that, first, this approach delivers a “notion of validity of an inference” which “gives no information concerning the question when an inference is legitimate” (p. 10), and that, second, if we take only *some* inferences to be meaning-constitutive, “it is unclear what makes the remaining ones legitimate” (p. 13).

Concerning the first objection, though, we have already seen that it is a mistake to identify the legitimate inferences with the ones that are known to be *valid*, irrespective of whether validity is accounted for in model- or proof-theoretic terms. As for the second, the objection assumes that there are legitimate inferences that are legitimate or entitling, but not meaning-constitutive. In some sense of the term ‘proof’, this may well be. What counts as a proof for an exceedingly bright mathematician may not be a proof for a high-school student. But, I take it, we are here trying to define an idealized notion of a proof: one that could *in principle* be recognized by a competent speaker. On *this* understanding of ‘proof’, I submit, the class of legitimate inferences may well coincide with the meaning-constitutive ones.<sup>4</sup>

On the second approach, all “the immediate inferences accepted in a language”

<sup>3</sup>See e.g. Smiley (1996) and Garson (2010).

<sup>4</sup>To be sure, the question remains as to how, in our linguistic practice, speakers are entitled to make inferences that are not meaning-constitutive. I do not have space to address this problem here. It is worth noting, however, that it equally affects Prawitz’s view—see Pagin (ming, p. 19).

are taken as meaning-constitutive, and “a person who knows the meanings of all the expressions occurring in a [basic] inference also knows immediately that the inference is valid, because to know these meanings is among other things to know that the inference is valid in the language in question; no reasoning is required to establish the validity” (p. 10). Prawitz’s main objection to this brand of inferentialism is essentially Arthur Prior’s point that not all combinations of I- and E-rules can define a logical expression, since some combinations give rise to inconsistency (see Prior, 1960). There have been attempts to “find a restriction (other than the condition in terms of complexity present in Gentzen’s approach), which imposed on the inference rules that can be taken as meaning constitutive, would guarantee consistency”, but, Prawitz writes, “these attempts have been fruitless” (pp. 12-3). They appeal to some notion of *harmony* between I- and E-rules, where—roughly—a pair of I- and E-rules for an operator \$ is harmonious just if \$-E allows us to infer from \$-statements precisely what follows from the grounds for introducing \$-statements, as specified by \$-I. However, Prawitz observes, examples of harmonious and yet inconsistent rules have long been known. For instance, consider the following pair of rules (first given by Stephen Read, 2000):

$$\begin{array}{c}
 [\bullet]^i \\
 \vdots \\
 \bullet\text{-I, } i \frac{\perp}{\bullet}
 \end{array}
 \quad
 \begin{array}{c}
 \bullet\text{-E} \frac{\bullet \quad \bullet}{\perp} .
 \end{array}$$

These rules are as harmonious as one may wish them to be. Yet, it is easy to show that, given the standard structural rules, they yield inconsistency.<sup>5,6</sup>

Prawitz’s dismissal of the inferentialist’s criteria for meaning-constitution seems too quick. To begin with, it is difficult to see why a combination of restrictions on admissible introduction rules, on the hand, and admissible combinations of I- and E-rules, on the other, may not allow inferentialists to characterize a class of meaning-constitutive rules, at least insofar as these rules are intended to define *logical* expressions. Indeed, Gentzen’s restriction on admissible I-rules precisely points in that direction. Secondly, inferentialists may follow Belnap (1962) and require that admissible rules yield a conservative extension of the structural fragment of our system.<sup>7</sup> Trivializing operators, from Prior’s tonk to Read’s  $\bullet$ , typically fail to meet this requirement, pending some drastic revision of the structural rules.

<sup>5</sup>Prawitz (1965, p. 94) provides another example of harmonious and yet inconsistent rules.

<sup>6</sup>Prawitz concedes that, on this second approach, one might alternatively follow Cozzo (1994) and take inconsistent (and indeed trivializing) rules to be meaning-constitutive. He persuasively argues, though, that doing so would oblige us to treat as primitive rules that can be meaning-theoretically justified.

<sup>7</sup>Not, *nota bene*, of the system as a whole. On the standard inferentialist accounts of higher-order quantification, this would immediately rule out higher-order logics of level 3 and above (see Wright, 2007).

## 4 Grounds

Now to Prawitz's own proposal. Let a *ground* for a sentence  $A$  be whatever justifies an assertion of  $A$ . Then, Prawitz tells us,  $A$ 's meaning is given by what counts as a ground for  $A$ . If  $A$  is an atomic observational sentence, a ground for  $A$  is a set of "relevant observations". If  $A$  is a compound sentence, a ground for  $A$  is defined in terms of the grounds for asserting  $A$ 's sub-sentences. For instance, if  $\alpha$  and  $\beta$  are grounds for, respectively,  $A$  and  $B$ , one can join these two grounds into a compound ground for  $A \wedge B$ . Prawitz calls this operation *conjunction introduction*:

( $\wedge$ -I)  $\gamma$  is a ground for  $A \wedge B$  just if  $\gamma = \wedge i(\alpha, \beta)$ , where  $\alpha$  is a ground for  $A$  and  $\beta$  is a ground for  $B$ .

Grounds for assertions made under assumptions can be likewise defined by reference to *unsaturated* grounds (p. 14). In short: grounds are operations that transform the grounds for the premises of an inference into grounds for its conclusion. The upshot is a somewhat richer notion of an inference rule:

An inference should be taken as being determined or individuated not only by its premisses and conclusion [...], but also by an operation applicable to the given grounds for the premisses. (p. 17)

Indeed, Prawitz argues, an inference  $I$  is *valid* just when the operation it is associated with yields a ground for the conclusion whenever we are given grounds for the premises and we infer according to  $I$ . As a result, whenever we make a valid inference, we apply an operation to the premises which yields a ground for that inference's conclusion. Our initial question is finally answered: legitimate inferences are valid inferences, and *vice versa*. As for *proofs*, they are spatio-temporally located "means for finding a ground". Two different proofs may deliver the same grounds. Grounds, we must infer, are Platonic entities, outside of space and time.

I will limit myself to making three observations. First, if grounds are Platonic objects, and if the grounds for observational sentences are "observations", one wonders what these observations may be. They cannot be observations made by some particular agent at some particular time, at least if different proofs, or justifications, can sometimes deliver the same ground. They must therefore be rather akin to what Austin called *sensibilia*—disembodied *sense data*.<sup>8</sup> Now, Prawitz's main reason for (re)introducing *sensibilia*, is twofold: first, that only on the assumption that proofs are delivered by objective grounds can proofs be independent of what *we think* is a proof; and second, that only if grounds are atemporal can *truth* also be atemporal. Here I limit myself to noticing that Prawitz's second reason essentially depends on his verificationism, viz. that a sentence  $A$  is true just if there is—in an atemporal sense of 'is'—a ground for  $A$ .

Second, the operations on grounds Prawitz considers all validate I-rules satisfying Gentzen's complexity requirement. However, consider the following operation on grounds:

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<sup>8</sup>Thanks to Tim Williamson here.

(•-I)  $\gamma$  is a ground for  $\bullet$  just if  $\gamma = (\bullet i \xi) \beta(\xi)$ , where  $\beta(\xi)$  is some unsaturated ground for asserting  $\perp$  under the assumption  $\bullet$ .

This operation, if it exists, validates the infamous •-I. Hence, just as inferentialists are faced with the problem of defining admissible I-rules, Prawitz may need to tell us which operations on grounds are admissible, and why.

Finally, in order to avoid a *holistic* view of meaning, verificationists need to specify what counts as a *canonical* ground for a sentence  $A$ , on the assumption that either the non-canonical grounds for  $A$  can be reduced to its canonical grounds, or a grasp of what counts as a non-canonical ground is not part of one's understanding of  $A$ , irrespective of whether non-canonical grounds can be reduced to the canonical ones. Elsewhere, Prawitz states a clear preference for the first option. It is unclear, however, whether this option—Dummett's so-called *Fundamental Assumption*—can be plausibly maintained for *atomic* sentences. Yet, as Prawitz himself acknowledges,

it is the whole verificationist project that is in danger when the fundamental assumption cannot be upheld. (Prawitz, 2006, p. 523)

By contrast, the problem need not affect moderate inferentialists, at least insofar as they do not identify the sense of a sentence with what counts as a (canonical) ground for asserting it.

## 5 Conclusions

In Prawitz's view, proofs are chains of *legitimate inferences*—inferences that, when made, yield a justification for the conclusion if we have a justification for the premises. However, Prawitz argues, making sense of legitimate inferences has drastic consequences in the philosophy of logic, viz. it forces us to reject the Bolzano-Tarski account of validity. *Moderate inferentialists*, I have suggested, can resist this conclusion, *pace* Prawitz's general reservations about inferentialism. As for Prawitz's suggestion that rules are associated with validity-making operations on Platonic grounds, it seems to me that it faces the same problems besetting the inferentialist approach, with some additional worries concerning the metaphysics of grounds.

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