MUST GOOD REASONING SATISFY CUMULATIVE TRANSITIVITY?

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0. Introduction

One way that we revise our attitudes is by reasoning from other attitudes that we have. And to first approximation, a piece of reasoning is good when the attitudes that results from it will be rational if attitudes that it began with are rational. Good reasoning so understood has been studied from a variety of different perspectives by philosophers, logicians, and computer scientists.

Interestingly, despite their different perspectives (and except for certain important exceptions that we will discuss below), these theorists have converged on a particular idea about the structure of good reasoning. Very roughly, the idea is that if two pieces of reasoning are good on their own, then a longer piece of reasoning that consists of performing these two pieces of reasoning back-to-back must also be good. We will see later that idea more precisely and accurately is that good reasoning must have a structural property known as *cumulative transitivity* or for short, *CUT*. But this preliminary gloss will be enough for now to appreciate the role that this convergence plays in theorizing.[1]

Consider the shape that puzzles about reasoning typically take.[2] Generally, puzzles about reasoning isolate a number of *prima facie* plausible pieces of reasoning and show that when combined in a certain way, these pieces of reasoning lead to unacceptable results. This is then taken to be a *reductio* of at least one of the *prima facie* plausible pieces of reasoning. This illustrates how theorizing about good reasoning is standardly constrained by the idea that I identified: it is because of this convergence that these theorists approach puzzles about reasoning as though the reasoning to the problematic conclusion has to be good if each step in that reasoning is individually good.

In this paper, I wish to argue against this orthodoxy. In doing so, I will not give a counterexample to this idea or argue for a solution to some puzzle that relies on giving it up. To do that, I would have to look at each puzzle in detail and argue that the best solution to that puzzle would involve a failure of good reasoning

[1] This convergence was perhaps first noticed in formal work (Gabbay 1985; Kraus, Lehmann, and Magidor 1990; and Makinson 1994 are the seminal discussions). But as I explain in the next paragraph, it has been influential even when it has gone unnoticed.

[2] A sampling of puzzles about reasoning of the kind that I have in mind may include the bootstrapping problem (see Cohen 2002), the lottery paradox (see Douven and Williamson 2006), the surprise test paradox (see Kripke 2011), and the problem of conflicting obligation in ethics (see Goble 2009).

to satisfy CUT over the puzzling inferences. While I (as well as others) have done this elsewhere, I wish to adopt a more general perspective in this paper that does not focus on any particular puzzle.[3]

I adopt this general perspective because to fully evaluate whether a proposed solution to a puzzle is satisfying, we must not only focus on whether the proposal adequately captures our considered judgments about the particular pieces of reasoning involved in the puzzle or adequately captures domain specific principles related to the puzzle. We must also look at how it fits with our commitments on the nature and structure of good reasoning in general. For this reason, I will focus on high-level issues that suggest that good reasoning must satisfy CUT.

Now as often happens with orthodox views, it is hard to find explicit arguments in favor of the idea that good reasoning must satisfy CUT. Instead, the idea operates as an implicit commitment that is revealed by how theorists approach puzzles and as an emergent pattern among formal theories of reasoning. Nonetheless, we can extract two kinds of considerations that support thinking good reasoning must satisfy CUT.

First, theorists have articulated certain powerful intuitive thoughts behind that idea that good reasoning must satisfy CUT. I will explain how these thoughts suggest three compelling arguments for the conclusion that good reasoning must satisfy CUT and then explain why these arguments are unsound (§2-4).

Second, the idea that good reasoning must satisfy CUT can look attractive simply because it is hard to see how good reasoning might fail to satisfy CUT. To respond to this worry, I develop an informal model of reasoning that is informed by work in traditional epistemology and show that it gives us a simple picture of how good reasoning might not satisfy CUT (§5). But before I dive into the details of my arguments, I will more carefully introduce our topic and bring out its importance (§1).

1. Our Question

Let's begin then by clarifying what good reasoning is and how to think about its structure.

1.1 Good Reasoning from a Pre-theoretical Perspective

We have a pre-theoretical grip on what it is for a piece of reasoning to be good. We can see this by considering ordinary belief formation and certain puzzles.

For example: Suppose that you believe that normally, birds fly and you believe that Tweety is a bird. And suppose that based on these two beliefs, you form the new belief that Tweety flies. [4] This transition from the two beliefs to the formation of the new belief that Tweety flies is good reasoning in at least one reasonable and pre-theoretical sense of 'good reasoning'.

We can also illustrate this pre-theoretical sense of 'good reasoning' by considering certain puzzles such as the so-called *bootstrapping* problem in epistemology.[6] The problem arises in different ways for different

[3] See, e.g., Horty 1993 and 1997 and Nair 2014
[4] See, e.g., Reiter 1980.
[6] See Cohen 2002.

theories but for simplicity consider a theory that accepts the following two inferences: the inference from '*o* looks red' to '*o* is red' (similarly for other colors) and the inference from a whole track of claims of the form '*o* looks red and is red', '*o** looks blue and is blue', etc. to the claim 'my color vision is reliable'.

The puzzle has us imagine an agent who does not start out believing that her color vision is reliable. This agent has her friend set up a slide show and the friend does not tell her what color the slides are. She sits in front of the screen and the slide show begins. Based on her visual experience she comes to believe the slide looks red. By the first inference rule, she may conclude that the slide is red. The next slide comes up and she comes to believe that the slide looks blue. Based on the first inference rule she may conclude the slide is blue. The slide show goes on and by repeating this process, she comes to believe a whole track record of claims of the form 'the slide looks red and is red', 'the slide looks and is blue'. So using the second inference rule, she may conclude that her color vision is reliable. But it is not good reasoning to come to believe that your color vision is reliable in this way.

Standard responses to this puzzle involve either accepting that the reasoning is good or rejecting one of the inferences that the theory is committed to. This can look like an unpalatable choice. If, however, good reasoning might not satisfy CUT, we have another option. We could consider the idea that each of these inferences is good on its own but that they cannot be performed back-to-back.

This is of course not say that this is the correct solution to the bootstrapping problem or any other puzzle. But the task of the present paper is only to argue against the idea that good reasoning *must* satisfy CUT and thereby open the door for rejecting CUT as a *possible* solution to these puzzles about reasoning.

1.2 An Unofficial Gloss on Good Reasoning

The focus of this paper is this notion of good reasoning that we have a pre-theoretical grasp of. Though it is an interesting question what the best theory of good reasoning is, this is not a question that I will answer in this paper.

That said, having at least an unofficial gloss on what good reasoning is may help us to better fix ideas. As I am thinking of it, we reason from some attitudes to the formation of new attitudes or revision or reaffirmation of old attitudes. So I take it that some (perhaps improper) subset of your attitudes constitute the starting points of your reasoning and reasoning is a mental process that returns attitudes as outputs when given these starting points as input. In the remainder of this paper, we will primarily focus on reasoning with beliefs.[8]

One way to elaborate on why the reasoning that we looked at earlier is good is as follow: The beliefs that we said that the agent has before the reasoning constitute all and only her (relevant) starting points. The belief that results after the reasoning is the output of a process that takes all and only these starting points as input. And this is good reasoning in the sense that the output of the process is a *rationally permissible* attitude given that all of the inputs are rationally permissible.

[8] The only exception to this will be our discussion of how reasoning with suppositions can lead to the formations of new beliefs.

This characterization does not say that good reasoning always involves the formation of rational beliefs. One can reason well to the formation of irrational beliefs (e.g., by coming to irrationally believe that *p* or *q* based on the irrational belief that *p*). What our unofficial gloss says is that a piece of reasoning to irrational beliefs is good just in case the mental process that it is an instance of is guaranteed to return rationally permissible beliefs when given a collection of beliefs such that (a) each member of the collection is permissible and (b) the collection consist of all and only the beliefs that constitute your starting point.[9, 10] Just as valid arguments need not have a true conclusion, pieces of good reasoning need not output rational beliefs.

So this is our gloss on what good reasoning is and it is only intended as an unofficial heuristic that may aid in thinking about our topic.

1.3 An Official Assumption about Good Reasoning

That said, I will insist on one official theoretical assumption. The assumption is that we are to think of belief in purely qualitative terms. This means we are to think of belief as distinct from credences or partial beliefs. We are not to think of belief in terms of probabilities. This assumption is important because the theories that I wish to discuss—the theories that converge on the idea that good reasoning satisfies CUT—only concern beliefs understood in this purely qualitative way. So in order to address this convergence directly I will accept this picture of the nature of belief.

What's more, the issues that we are discussing look very different if we reject this assumption because theories that reject the assumption tend to also reject the idea that good reasoning satisfies CUT.[13] In other work, I compare how CUT fails according to these probabilistic approaches to how CUT fails on the picture developed here. But I do not have the space here to discuss the issues other than to note that the picture

[9] The inclusion of (b) ensures our gloss is neutral on whether good reasoning satisfies CUT. So one of the questions that we will explore in detail is whether (b) is a vacuous or otherwise unimportant condition on what it takes for a process of reasoning to be good. Cf. Makinson and van der Torre 2000.

[10] What are our starting points in reasoning? Below we will see that good reasoning fails to satisfy MONOTONICITY and this suggests our starting points must be our total belief set (or as I will claim §5, our total foundational belief set). If this is so, one may reason well to irrational belief by either starting with beliefs that are irrational or starting with less than the total set of beliefs. And indeed, generally if one starts with irrational beliefs, one will reason to forming an irrational belief in the conclusion. But sometimes one may reason well from a subset of one's total beliefs to the formation of a *rational belief*. When is this so? All theories of reasoning have trouble answering this question and the official definition in the text is silent on this. But one natural answer says that the belief one forms from reasoning with a subset of one's total beliefs is rational only in cases where the reasoning from the subset to the conclusion is good *and* reasoning from one's total belief set would be good. This is only a rough answer because there remain complications that I cannot fully discuss here that require a subtler definition still.

[13] Recent explicit arguments for this assumption include Buchak 2014 and Ross and Schroeder 2014. Those who reject this assumption may be divided into at least five camps. There is the so-called classic Lockean approach to reducing beliefs to credences (Foley 1993) according to which CUT may fail (though it is controversial whether these are really theories of *reasoning*, see, e.g., Staffel 2013 and Hlobil 2016a for discussion). There are approaches to reducing beliefs to credences that are not classic Lockean approach such as Leitgeb 2013, Lin 2013, van Fraassen 1995, Weatherson 2005, and Wedgwood 2012. It turns out to be a complicated question whether these views can take advantage of the distinctive picture developed in this paper. I hope to return to considering this issue in future work. There are eliminativists about belief such as Jeffrey 1970 whose work has no obvious interaction with the ideas considered here. And there are those who reduce credence to belief such as Easwaran 2016. These theorists may accept the arguments of this paper but it is an interesting question that deserves further exploration whether this will have any consequences for the nature of credences. Finally, there are those such as Harman 1986 who are eliminativists about credence who may accept the arguments of this paper.

developed here works very differently from the probabilistic picture.[14, 15]

1.4 How to Frame Questions about Structure

Now that we know what good reasoning is, the next thing to do is introduce a more precise way of thinking about its structure. As it turns out, there are a variety of formal theories of reasoning that model belief in different ways. This makes these theories difficult to compare. That said, logicians and computer scientists have developed at least one way to compare these theories that has proven fruitful even though it abstracts from many of the details of each view. What I wish to do now is introduce this way of comparing theories as a way of making our question more precise.

To start, let's step back from reasoning and consider classical logic. Classical logic is intended to characterize *valid* arguments where a valid argument is one in which the truth of the premises *guarantee* the truth of the conclusion. Classical logical consequence is a relation that holds between a set of premises and a conclusion just in case the argument consisting of those premises and that conclusion is valid.

In the twentieth century, logicians discovered a number of interesting structural properties of the logical consequence relation. If we use uppercase Greek letters (A, B, X etc.) for sets of sentences, lowercase Greek letters (α, β, χ etc.) for sentences, and $\vdash$ for the classical logical consequence relation, we can write $A \vdash \chi$ for 'χ is a logical consequence of A'. And we can now succinctly state three structural properties that will be important for our purposes:

$\vdash$ satisfies *REFLEXIVITY* just in case $A \vdash \alpha$ for all $\alpha \in A$
$\vdash$ satisfies *MONOTONICITY* just in case if $A \vdash \chi$ and $A \subseteq B$, then $B \vdash \chi$
$\vdash$ satisfies *CUT* just in case if $A \vdash \beta$ for all $\beta \in B$ and $A \cup B \vdash \chi$, then $A \vdash \chi$

Classical logical consequence has these structural properties because of the way classical logic attempts to capture when the truth of the premises guarantees the truth of the conclusion.[20]

[14] So probabilistic theories are the first notable exception to the consensus that good reasoning satisfies CUT. The other notable exception is the system of inheritance reasoning developed in Horty, Thomason, and Touretzky 1990 (see especially their §5.3). This theory bears some similarities to the one that I develop, but because these theories model belief very differently than I do, I am unable at this time to make any precise comparisons. I hope in future work to be able to more adequately discuss this system of inheritance reasoning.

[15] In this connection, it is also worth taking a moment to mention the topic of so-called "transmission failure" that bears some some similarity to our topic. The center mass of literature has focused on (a) whether Moorean argument as a case of transmission failure, cases where warrant or justification fails to transmit across deductive inference (Pryor 2012 and Wright 2004) and (b) whether a systemic model of transmission failure can be provided that delivers this result (Piazza 2013 and Okasha 2004). Though there are interesting connection between this topic an ours. Five observations are worth making: First the issues in (a) concern whether particular cases exhibit transmission failure while we are not focusing on particular cases in this paper. Second as far as I know the only models that have been proposed to understand transmission failure are probabilistic. Third, this debate generally assumes that if you are permitted to believe something, you are permitted to believe anything in the deductive closure of that belief. What is up for debate is whether the original belief can provide evidence for the belief in its closure. But our question only concerns what it is permissible to believe. Fourth, this debate focuses on cases in which the reasoning corresponds to a deductively valid inference. Our discussion does not assume this. Fifth, this debate does not distinguish between CUT and transitivity. §2 shows that this distinction is important. Similar observations apply to so-called "chain reaction" arguments (see, e.g., Goldman 2004 and Williamson 2000: ch. 10)

[20] There are non-classical logics according to which logical consequence fails to satisfy CUT. For example, Williams 2008 and 2011 defends a view about vagueness that entails that that logical consequence fails to satisfy CUT and Ripley 2013 defends a view of truth that entails that logical consequence fails to satisfy CUT. If we were to adopt such a non-classical logic and claim that one of the

Now one question that we can ask whether good reasoning shares these structural properties. In asking this question, we must keep in mind that while logical consequence is normally thought of as a relation among sentences, good reasoning is a relation that holds between mental states.

But it has proven fruitful in making comparisons between different formal theories to define, for each theory, what we might call a "a good reasoning consequence relation" on analog to a logical consequence relation. So let us write $A \Vdash_{s,\,t} \chi$ to represent the claim that for an agent *s* at a time *t*, the transition from belief in each proposition expressed by the sentences in A to the belief in the proposition expressed by χ is a piece of good reasoning. For simplicity, let us leave implicit reference to agents and times and just write $A \Vdash \chi$. And let us for simplicity, succinctly put this as the claim that it is good reasoning to conclude χ from A. Finally, let us call what is to the left of $\Vdash$ the premises and what is to the right the conclusion.[21]

Using this technique, logicians and computer scientists have been able to draw a number of interesting comparisons between formal theories. And one result of this comparative work is that despite being designed to model very different kinds of reasoning (e.g., generic reasoning, legal reasoning, causal reasoning) all these formal theories (save the exceptions discussed above) converge on the result that good reasoning relation satisfies CUT where we say:

$\Vdash$ satisfies CUT just in case if $A \Vdash \beta$ for all $\beta \in B$ and $A \cup B \Vdash \chi$, then $A \Vdash \chi$

The precise statement of our question, then, is whether good reasoning must satisfy CUT and the answer that I

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counterexamples to logical consequence satisfying CUT that these logic generates is also a counterexample to good reasoning satisfying CUT, this would give us a quick argument for the conclusion that good reasoning might not satisfy CUT. I do not however wish to rest my arguments in this paper on these controversial premises.

There may however be a less controversial way of making use of the resources of non-classical logic to argue that good reasoning might not satisfy CUT. Consider reasoning with logically inconsistent premises. It is natural to think it is bad reasoning to reason explosively from inconsistent premises (i.e., reason to any conclusion you like). We may then think that some form of non-classical logic is the correct theory of reasoning with inconsistent premises even if it is not the correct logic (see Burgess 2009: ch. 5 for an introduction to these logics). And certain non-classical logics designed to deal with inconsistent premise sets claim that logical consequence fails to satisfy CUT so this would give us a route to the conclusion that good reasoning might not satisfy CUT.

I do not pursue this more restrictive strategy here for three reasons. First even if some form of non-classical logic is the correct theory of reasoning with inconsistent premise sets, it is controversial which form of non-classical logic is correct. Indeed, the dominant non-classical logic for dealing with inconsistent premise sets is relevant logic and this logic entails logical consequence does satisfy CUT (see Anderson and Belnap 1975 for a classic discussion of relevant logic).

Second, this strategy, even if successful, tells us little about the issues that motivate interest in the question discussed in this paper. Those issues were our approach to puzzles about reasoning and the convergence among formal theories of reasoning. The logics that we are presently considering would help with puzzles only on the assumption that the starting points in the puzzles are genuinely logically inconsistent. This is, to my mind, not a genuine solution to these puzzles; it is triage. And the approach would tell us nothing about the convergence among formal theories of reasoning because these theories are designed to capture ampliative reasoning from consistent premise sets.

Third, this strategy even if successful would not tell us how to respond to the intuitive appeal of CUT or the arguments for it developed below (§2-4). And it would not give us a model of the sort developed below that tell us how and why good reasoning might fail to satisfy CUT (§5). Thus, this restricted strategy also relies controversial premises and even if it is successful, the alternative approach developed in this paper is still of interest because it promises to illuminate issues that the restricted strategy does not speak to.

[21] Indeed, I will abusively use 'premises' at different times to refer to the set of sentences on the left, the set of propositions expressed by those sentences, and the set of beliefs in those propositions. I trust context will make clear my meaning. Similar comments apply to my use of 'conclusion'.

will be pursuing is that good reasoning might not satisfy CUT.[22] And another way of thinking of this question is as asking whether this convergence is just an accident resulting from the range of cases considered by theorists or if it reflects a deeper truth about the nature of reasoning.[23]

1.5 How to Argue about Structure

Now it may be hard to see how we could argue about the structure of reasoning at the level of abstraction that I am interested in. Below, I will look at some unsuccessful argument of this sort that concern CUT. But it helps to start by seeing how a successful high-level argument about structure works. This argument shows that good reasoning does not satisfy MONOTONICITY.

Good reasoning does not satisfy MONOTONICITY because some good reasoning is *ampliative* in the sense that it can be good reasoning to draw conclusions that are not guaranteed to be true by our premises but are only likely to be true or in some other way reasonable given our premises.[24] To see this, take a piece of good ampliative reasoning from A to χ, since A does not guarantee the truth of χ it is possible to (consistently) add new information to A that makes χ false or unreasonable. But it would not be sensible to say that reasoning from A together with this new information to χ is good reasoning because χ is not likely to be true or otherwise reasonable given this large set of premises.[25]

The Tweety example can illustrate this: Where A={'Tweety is a bird', 'Normally, birds fly'} and χ be 'Tweety flies', $A \Vdash \chi$. But it would not be good reasoning to conclude that Tweety flies based on the larger belief set consisting of the belief that Tweety is a bird, the belief that normally birds fly, the belief that Tweety is a penguin, and the belief that normally, penguins don't fly. So where B represents this larger set, $B \nVdash \chi$.

For this reason, good reasoning is does not satisfy MONOTONICITY. This shows that even at this level of abstraction, there are certain high level features of reasoning that allow us to argue about structure. The question that we will pursue in this paper is whether we can isolate some high-level property of good reasoning like the property of ampliativity that entails that good reasoning must satisfy CUT.

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[22] Here is a sampling of the diverse array of formal result concerning this convergence. There are results concerning the relationships between properties of proofs and structural properties of good reasoning (Gabbay 1985). There are results that categorize different theories according to which structural properties they say good reasoning has (Makinson 1987 and 1994). There are results concerning whether and how different theories of good reasoning can be represented in a certain kind of preferential semantics (see Kraus, Lehmann, and Magidor 1990; Makinson 1987: 8-10 and 1994: §3.4; and Shoham 1987 for the seminal discussions and for more recent work, see Arieli and Avron 2000, Bezzazi, Makinson, and Pérez. 1997, and Schelechta 2007). And there are results concerning a correspondence between structural properties of good reasoning and different ways of choosing models where *choice* is analyzed with tools from social choice theory (see Rott 2001: ch. 6-8).

[23] Goble 2013: §6.3 explicitly raises the worry that it reflects a deep truth and criticizes Horty 1993 and 1997 and Nair 2014 on these grounds.

[24] Two clarifications are in order. First in saying this, I should not be taken as implicitly assuming that good reasoning is *supraclassical* in the sense that if $A \vdash \beta$, then $A \Vdash \beta$. I am officially neutral about this question (see Harman 1986 for arguments that good reasoning is not supraclassical). Second good reasoning being ampliative is compatible with there being certain domains (e.g. mathematics) in which every piece of good reasoning corresponds to a valid inference.

[25] I am not making a formal point but a philosophical one: *good* reasoning that is ampliative does not satisfy MONOTONICITY. There are formal systems whose consequence relations are ampliative and satisfy MONOTONICITY (see Makinson 2005's paraclassical logics).

2. The Argument from Brute Intuition

The first argument in favor of thinking that good reasoning must satisfy CUT seeks to settle the issue on brute intuition alone. The idea is that it is simply intuitively obvious that good reasoning must satisfy CUT.

While this argument is a particularly blunt one, it is a reaction that I often get when I first tell people about the question that I am interested in so it is worth discussing. And there is some merit to this reaction. After all, we do seemingly unproblematically engage in back-to-back reasoning and even I find the idea that good reasoning satisfies CUT intuitively plausible when I reflect on it.

What's more, this idea receives further support from the fact that we seemingly unproblematically assert conclusions of our reasoning (without asserting the premises) and thereby entitle others to believe them and reason with them without regard to how we came to believe them. But if good reasoning fails to satisfy CUT, then when we assert β, it may matter whether we have β as a conclusion from A or β from the start. For if CUT is not satisfied, one may not be able to conclude χ in settings whether β is a conclusion from A. Thus, if CUT fails, those who form beliefs on the basis of our assertion may be in certain kind of danger.And so our practices of assertion may not be in good order.[26] The first argument, then, is that our practices of assertion make it intuitively obvious that good reasoning must satisfy CUT.

My response is that we should not rest an argument about the high-level properties of good reasoning on brute intuition or observations about assertion like this. This response does not come from general skepticism about resting arguments on brute intuition or data about assertion. Rather I will show that in the case of high-level structural properties of good reasoning in particular our intuitions and this data lead us astray.

To see this, let me begin by reporting that many people who find CUT intuitively obvious also find REFLEXIVITY and a close cousin of CUT, TRANSITIVITY, intuitively obvious:

$\Vdash$ satisfies *REFLEXIVITY* in the sense that $A \Vdash \alpha$ for all $\alpha \in A$
$\Vdash$ satisfies *TRANSITIVITY* just in case if $A \Vdash \beta$ for all $\beta \in B$ and $B \Vdash \chi$, then $A \Vdash \chi$

And indeed these claims are also supported by similar observations about assertion. The trouble is that good reasoning cannot satisfy both REFLEXIVITY and TRANSITIVITY because it is known thesthey entail MONOTONICITY and we know good reasoning does not satisfy MONOTONICITY.[27]

To see the entailment begin by assuming the antecedent of MONOTONICITY:

(1) $A \Vdash \chi$
(2) $A \subseteq A^{+}$

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[26] The objection can be strengthened: according to the so-called argumentative theory of reasoning (Mercier and Sperber 2011), this social role of reasoning is essential to what reasoning is. So facts about assertion are not just probative but essential to the nature of reasoning. I unfortunately do not have the space her to discuss this theory and how it interacts with the ideas of this paper, but the response given in the text cautions against any easy argument from this theory to the idea that good reasoning must satisfy CUT.

[27] See, e,g,, Gabbay 1985.

By REFLEXIVITY and (2), we know:

(3) $A^+ \Vdash \alpha$ for all $\alpha \in A$

Then by TRANSITIVITY, (3), and (1), we know:

(4) $A^+ \Vdash \chi$

and (4) is just the consequent of MONOTONICITY.

Thus, the fact that our intuitions and certain observations about assertion favor both REFLEXIVITY and TRANSITIVITY even though they cannot both be true illustrates that our intuitions and these observations about high-level features of reasoning are unreliable. Indeed as I show in a note, this same point can be made even without the assuming REFLEXIVITY holds.[28] For this reason, we should not rely on such intuitions or observations in the case of CUT either.

3. The Argument from Suppositional Reasoning

The second and to my mind more compelling argument seeks to show that that CUT must hold because if good reasoning were to involve a failure of CUT, we would no longer be able to engage in the epistemically important practice of suppositional reasoning.

3.1 The Fruitfulness of Suppositional Reasoning

Lou Goble, in correspondence, nicely illustrates the epistemologically important practice of suppositional reasoning:

> In mathematics there may be interest in whether a certain proposition, φ, is true, or follows from presumptively true axioms, e.g. ZFC. Maybe it's a very hard question, many people work on it. Someone finally proves, not the result, but that φ follows from the axioms, e.g., ZFC, + [...]ψ, where ψ seems credible itself. That doesn't answer the original question, but it may well be regarded as significant progress. Now someone else, after much hard work, proves that ψ is not independent of ZFC, but is entailed by it. That is hailed as a significant result. Moreover, the original question concerning φ now seems answered, which was, of course, the original point of interest.[30]

This case illustrates how we often reason with a supposition to some conclusion, later show that the supposition

[28] Let us not suppose REFLEXIVITY always holds. Still we can argue that good reasoning fails to satisfy TRANSITIVITY. Here is an example: it is good reasoning to conclude 'Tweety is a bird and birds normally fly' from 'Tweety is a bird and birds normally fly and Tweety is a penguin and penguins normally don't fly'; it is, as we noted before, good reasoning to conclude 'Tweety flies' from 'Tweety is a bird and birds normally fly'; yet it is not good reasoning to conclude 'Tweety flies' from 'Tweety is a bird and birds normally fly and Tweety is a penguin and penguins normally don't fly'. In the lingo of contemporary epistemology the structure of the argument is this: *p* may defeasible justify *q* while *p* and *d* does not justify *q* where *d* is a defeater. Nonetheless, *p* and *d* may justify *p*.

[30] Cf. Burgess 2009: 105.

is true, and in light of this, accept the conclusions that we arrived at under the supposition. And it shows how this is an ordinary and important practice in theoretical inquiries such as mathematics. But Goble does not intend this example to be narrowly restricted to inquires like mathematics where it is perhaps true that all forms of good reasoning correspond to deductively valid inferences, inquires where ampliative inference is not good reasoning. Instead, it is one instance of a general practice that we engage in both in mathematics and in other inquires that allow for ampliative reasoning.[31]

Having established that suppositional reasoning plays this fruitful role, the argument is that good reasoning must satisfy CUT in order to allow for such reasoning.[32] In Goble's example, a mathematician reasons well from not just her set of premises, ZFC, but her set of premises together with ψ to the conclusion φ. In symbols, ZFC, $\psi \Vdash \varphi$. Then later we are told that a mathematician is able to reason well to the conclusion ψ from ZFC itself. So this means ZFC $\Vdash \psi$. In cases like this, we have now shown φ. That is, ZFC $\Vdash \varphi$. But in order for this to be so, it seems that we must think that ZFC $\Vdash \varphi$ follows from ZFC $\Vdash \psi$ and ZFC, $\psi \Vdash \varphi$. And that is CUT.[33]

The idea then is that the fruitful practice of suppositional reasoning shows that there is no difference in the inferential power of premises and conclusions and thereby shows that good reasoning must satisfy CUT.

3.2 Suppositional Reasoning without CUT

I should begin my response to this argument by conceding that suppositional reasoning is a fruitful practice and that if it were true that good reasoning must satisfy CUT in order to allow for this practice, I would agree that this a good argument that good reasoning must satisfy CUT. Luckily, however, we can allow for suppositional reasoning even if good reasoning does not satisfy CUT.

To see this, recall that CUT says that if $A \Vdash \beta$ and $A, \beta \Vdash \chi$, then $A \Vdash \chi$. So for CUT to fail is for there at least one claim β and one claim χ such that you can reason to χ when β is a premise but not when it is a conclusion. This does not entail that in reasoning with any claim we must be sensitive to whether it is a premise or a conclusion. And it does not mean even for those claims β which do require us to be sensitive to whether they are premises or conclusions that we cannot reason *at all* with them when they are conclusions. All it says

[31] Cf. Cohen 2010: 152-153

[32] This argument is inspired by discussion with Lou Goble as well as his case. It is also suggested in Kraus, Lehmann, and Magidor 1990: 177-178 and Makinson 1994: 43.

[33] David Makinson expresses a similar but more general thought about the importance of CUT (together with another property known as cautious monotony) for our inferential practices:

> Why should these conditions be seen as important? Because they correspond to certain very natural and useful ways of organizing our reasoning. They tells us that when we are reasoning, we may accumulate our conclusions into our premises without loss of inferential power (cautious monotony) or amplification of it (CUT). In this sense, the reasoning process is taken to be stable. (1994: 41)

Stalnaker 1994: 18 and Jeff Horty in conversation have expressed similar sentiments.

is that we cannot reason in *exactly the same way* whether or not β is a premise or a conclusion.

This opens the door for an account of suppositional reasoning. In general, the idea would be that even for those claims that are sensitive to whether they are premises or conclusions for certain kinds of reasoning, there are other kinds of reasoning that are indifferent to whether they are premises or conclusions. Since this idea is abstract, consider an example:

LOGICAL SUPPOSITIONAL REASONING*:* if $A \Vdash \beta$ and $A, \beta \vdash \chi$, then $A \Vdash \chi$

LOGICAL SUPPOSITIONAL REASONING says that if you can reason to β from A and if χ is a logical consequence of A and β together, then you can reason to χ from A. This tells us that insofar as we are performing logically valid inferences we do not have to keep track of whether β is a premise or a conclusion. This allows for suppositional reasoning but restricts its role. It says that if suppositional reasoning is to inform our belief formation, we must restrict ourselves to only logically valid inferences under suppositions.

Importantly, LOGICAL SUPPOSITIONAL REASONING is only an illustration of one way there could be suppositional reasoning even if good reasoning fails to satisfy CUT.[35] More generally, we can allow for suppositional reasoning by isolating a class of inferences that are good regardless of whether a claim is a premise or a conclusion. When I present a positive picture of how good reasoning might not satisfy CUT, we will see that this picture allows us to state in simple terms which kinds of inferences are good regardless of whether a claim is a premise or a conclusion (§5.5).

Of course, it is true that if good reasoning fails to satisfy CUT, we cannot reason indiscriminately under suppositions. But we have yet to see why this is a cost of the proposal. After all, examples like Goble's do not establish that our fruitful practice is indiscriminate in this way. All that they establish is that we often engage in reasoning under suppositions that informs our beliefs. The examples alone do not tell us anything about the precise scope of this practice. Investigating this question would require us to look at the details of particular domains of reasoning and puzzles about reasoning which is beyond the scope of this paper which focuses on abstract features of reasoning.

What's more, the strategy used to address this argument promises to generalize to deal with many (though perhaps not all) other arguments in favor of thinking that there is no difference in inferential power between premises and conclusions. After all, other arguments will tend to also use cases to show that there are certain general practices that require no inferential difference between premises and conclusions. And my response will be that a restricted set of inferences can in principle explain the practice. So considered at the level of generality that we are working with in this paper, the prospects of developing any argument directly for the conclusion that good reasoning must satisfy CUT—look dim.

[35] So though LOGICAL SUPPOSITIONAL REASONING assumes supraclassicality (see n. 20), my response to the objection does not.

4. The Argument from Reasoning with Conditionals

The final argument that I will consider is similar in some ways to the last argument. And my response will draw on themes from the responses to the previous two arguments.[38]

This argument focuses on the relationship between conditionals and good reasoning. In particular, the argument begins by claiming that there is a tight connection between good reasoning and conditionals that is captured by the following claims:

DEDUCTION THEOREM: $A, \beta \Vdash \chi$ iff $A \Vdash \beta \rightarrow \chi$

MODUS PONENS: if $A \Vdash \beta \rightarrow \chi$ and $A \Vdash \beta$, then $A \Vdash \chi$[39]

If these claims are both true, then good reasoning must satisfy CUT. To see this, assume the antecedent of CUT:

(i) $A \Vdash \beta$

(ii) $A \cup \{\beta\} \Vdash \chi$

By (ii) and the left-to-right direction of DEDUCTION THEOREM:

(iii) $A \Vdash \beta \rightarrow \chi$.

By (i), (iii), and MODUS PONENS, we have:

(iv)$A \Vdash \chi$.

And that is the consequent of CUT.

My first pass response to this argument is the same as my response to the argument of the last section: We can explain ordinary examples of reasoning that fit the pattern of DEDUCTION THEOREM and MODUS PONENS in terms of a more restricted set of rules of inference that are safe in the context of such reasoning without embracing this unrestricted version of them. And when I give my positive theory, I will provide a way of identify this set (§5.5).[40]

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[38] This argument was first suggested to me by Fabrizio Cariani and later Ralph Wedgwood.An anonymous referee at this journal also pressed this objection, helped me to see its force, and provided me with literature references that allowed me to develop the response given in this section..

[39] It is perhaps better to call this "meta modus ponens". For discussion of the distinction between this claim and the "object level" claim that $\{\beta \rightarrow \chi, \beta\} \Vdash \chi$, see Zardini 2013 and Fjellstadt 2016. The argument in the text requires "meta modus ponens".

[40] To preview, I restrict DEDUCTION THEOREM. I am not forced to this response. I could instead restrict MODUS PONENS. But the theory given §5 does not offer much by way of guidance on what the restriction would be. A supplementary theory of the behavior of the indicative conditional however could be added to the theory given here to restrict MODUS PONENS.

This first pass response will not satisfy everyone because some believe that DEDUCTION THEOREM and MODUS PONENS are the key to providing a *semantics* or *logic* of the indicative conditional and so must hold in every case. If this is right, restricted version of these principle would not suffice.

My response to this strengthened objection is that no one should accept both DEDUCTION THEOREM and MODUS PONENS.[41] This is because these claims, as we have already seen, entail CUT and it is known that CUT and DEDUCTION THEOREM together with an innocuous assumption entail MONOTONICITY.[42] Since good reasoning does not satisfy MONOTONICITY, something has got to go.[43]

Let me present this argument in greater detail. Begin by assuming the antecedent of MONOTONICITY:

(i) $A \Vdash \chi$

Now by REFLEXIVITY (i.e., for any set A, $A \Vdash \alpha$ for all $\alpha \in A$) we also have:

(ii) $A \cup \{\chi\} \cup \{\beta\} \Vdash \chi$

This is the innocuous assumption that I alluded to earlier. I will explain why it is innocuous below, but for now let us see where it leads. From (ii) and the left-to-right direction of DEDUCTION THEOREM, we have:

(iii) $A \cup \{\chi\} \Vdash \beta \rightarrow \chi$

Next by (i), (iii), and CUT, we have:

(iv) $A \Vdash \beta \rightarrow \chi$

Finally, by (iv) and the right-to-left direction of DEDUCTION THEOREM, we have the consequent of MONOTONICITY:

(v) $A \cup \{\beta\} \Vdash \chi$

Since good reasoning does not satisfy MONOTONICITY, everyone must reject at least one of REFLEXIVITY, DEDUCTION THEOREM, or CUT.[44] Let us consider each in turn.

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[41] To be clear, variants of these principles concerning logical consequence or semantic entailment may be true (indeed, restrictions such as these are in the spirit of my approach cf. LOGICAL SUPPOSITIONAL REASONING).

[42] This result is discussed in Hlobli 2016b and Morgan 2000 (cf. Shoham 1987: 734).

[43] Note that this response strategy will suffice even if we invent a connective that is defined so as to satisfy DEDUCTION THEOREM. No such connective allows us to remain consistent if it satisfies MODUS PONENS given then at good reasoning fails to satisfy MONOTONICITY.

[44] But cf. Morgan 2000 who argues for MONOTONICITY on something like these grounds.

REFLEXIVITY as it is used here is innocuous. While it may fail in certain contexts[45], it holds in many contexts in which MONOTONICITY fails. This is enough for the argument. The skeptical reader can check this for herself by plugging in her favorite example of a piece of reasoning that does not satisfy MONOTONICITY and note that it nonetheless satisfies (ii).[46]

One may instead reject DEDUCTION THEOREM. Now this may appear to undermine the argument for the conclusion that good reasoning must satisfy CUT. However, the proof that DEDUCTION THEOREM and MODUS PONENS entail CUT only relies on the left-to-right direction of DEDUCTION THEOREM. My response to that argument, on the other hand, relies on both directions of DEDUCTION THEOREM. So a stable way to argue for CUT without having to accept MONOTONICITY is to reject only the left-to-right direction of DEDUCTION THEOREM and maintain MODUS PONENS.

This argument however is less compelling than the original argument. The original argument told us to accept DEDUCTION THEOREM in every case because it is the key to providing an adequate semantics or logic for the indicative conditional. But on a natural understanding of this approach, both direction of DEDUCTION THEOREM are needed (because it is only when we have both directions that we have rules for "introducing" *and* "eliminating" the indicative). This new argument, then, can no longer be supported by the needs of an adequate semantics and logic for the indicative. At this stage, my initial response that proposed restricting these principles is sufficient. More conservatively still, my response in this section much like my response in §2 should significantly reduce our confidence in our judgements about what principles of reasoning are "key" to semantics or logic. I conclude that there is no way of rejecting DEDUCTION THEOREM that still allows one to make a strong case that good reasoning must satisfy CUT.

Finally, one could reject CUT. And indeed, some (perhaps, Hlobli 2016a) have taken this to be argument for just this conclusion. While endorsing this would help my case in this paper, it does require one to reject MODUS PONENS at least, if one wishes to preserve an unrestricted version of DEDUCTION THEOREM. Since I am open minded about which of these two principles require restriction, I will not claim that this is a decisive argument for CUT. Instead, all I claim is that this argument brings into sharp relief just how controversial the question of whether good reasoning satisfies CUT should be. [47, 48]

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45 Example: perhaps, one may start out permissibly believing some paradoxical proposition and end up coming to reject that proposition by reasoning.

46 Example: A is a set of sentences describing a large sample of white swans that have been observed to be white, χ is 'the next swan will be white', and β is 'the sample is biased'.

47 There are of course still other principles that we could consider (e.g., so-called *Right Weakening*, *Conjunction in the Conclusion*, *Disjunction the Premises*, *Rational Monotonicity*, *Disjunctive Rationality*, and others can in various combinations entail CUT). Though it would be interesting to consider each of these on their own, my response in outline is the same. I will reject at least one of the principles and offer a restriction. Exactly which principle and which restriction to offer, as I have been suggesting, is an issue that is to be decided by considering the particular case and particular domain of reasoning.

48 A more general worry is that the very idea that there is an epistemically important difference between premises and conclusions is dubious. In particular, one might think the distinction between premise and conclusion is a dialectical or pragmatic one and not an epistemic one. A supplement (Nair ms-b) develops this objection in detail and argues that almost any plausible theory of reasoning must make some epistemically important distinction between premises and conclusions. And the informal picture in the next section develops an interpretation of premises and conclusions on which the distinction is epistemically significant.

5. A Foundationalist Interpretation of the Structure of Reasoning

So far we have discussed arguments in favor of thinking that good reasoning must satisfy CUT and how they fail. In this discussion, I have taken an abstract perspective in order to remain neutral between different kinds of theories about the nature of reasoning. This has the virtue of making the premises of my arguments against the idea that good reasoning must satisfy CUT acceptable to many different audiences. But it has the vice of being so abstract that it hard to see exactly how it is that good reasoning might fail to satisfy CUT.

In the remainder of the paper, I propose to take on a more partisan perspective. In particular, I will be assuming a certain minimal foundationalist epistemology is correct. And I will use it to provide a more concrete interpretation of the main structural claims about good reasoning that we are discussing and I will show how within this picture, good reasoning might fail to satisfy CUT. Though I do not claim that this is the only approach that allows us to understand how good reasoning might fail to satisfy CUT, we will see that it is an approach that allows us to see relatively simply how good reasoning might fail to satisfy CUT.[50]

5.1 The Framework

The broadly foundationalist framework that I will be working with has two components; one more psychological, the other overtly epistemological. The psychological component is that an agent's belief state is structured so that some of her beliefs count as foundational beliefs and others count as non-foundational beliefs. The non-foundational beliefs in an agent's psychology are the ones that are based on her foundational beliefs where one way for non-foundational beliefs to be based foundational beleifs is for them to be products of reasoning from the foundational beliefs.[51]

In addition to this psychological component, there is an epistemological component of foundationalism that consists of two theses. First, the permissibility of non-foundational beliefs is determined by their relationship to foundational beliefs. If we use ⌈believe that α⌉ as shorthand for ⌈believe the proposition expressed by α⌉ and ⌈believe that A⌉ as shorthand for ⌈believe each of the propositions expressed by the sentences in A⌉[52], the idea more precisely is this: it is permissible for you to non-foundationally believe that β just in case (i) it is good reasoning to conclude β from A, (ii) you permissibly believe that A, and (iii) your beliefs that A are all and only your foundational beliefs. Second, the permissibility of foundational beliefs is not determined in this way. Different foundationalist views can give different accounts of how the permissibility of foundational belief is determined. For example, one account says a foundational belief is permissible in virtue of being caused in the right way by perception.

[50] My approach is similar to Rott 2001: ch. 5, but the discussion starting at §5.3 sheds new light on how good reasoning might not satisfy CUT.

[51] It is beyond the scope of this paper to discuss what "basing" is in general here. I allow that it may occur without reasoning, but focus only on cases involving reasoning in this paper.

[52] I adopt similar shorthand for grammatical variants from 'believe that' such as 'believes that', 'believe', and 'belief'.

This minimal form of foundationalism is a credible albeit controversial theory in epistemology. And it is worth emphasizing here that it is minimal in a way that makes it agnostic about many of the most controversial assumptions that traditional forms of foundationalism endorse. For example, it is compatible with but not committed to any of the typical internalist assumptions associated with foundationalism.[53]

5.2 The Interpretation

This minimal framework alone suffices for us to make an epistemically important distinction between premises and conclusions and to make better sense of (but not settle) the main questions about the structure of reasoning that we have discussed so far [54]

5.2.1 Premise/Conclusion Distinction

The first thing that I want to do is provide an interpretation of the distinctions between premises and conclusions. More precisely, I will provide an account of what an agent's premises and conclusions are at a given time. The account is that an agent's premises at a time are her foundational beliefs at that time and her conclusions at a time are her non-foundational beliefs at that time.[55]

The foundationalist framework ensures that there is a sharp epistemically important distinction between premises and conclusions so understood.

452.2 Learning and MONOTONICITY

The next thing that I want to do is provide an interpretation of the main structural claims that we have been discussing. In order to introduce this interpretation in an accessible way, it helps to focus on a different kind of structure that we have not yet discussed. It is what I call the *structure of standard cases of learning*. We can represent the structure as follows:

$$\begin{array}{ccccccc} \beta & & \chi & & \beta & \chi & \varepsilon \\ & \Uparrow & & \textit{Add}\ \delta & & \Uparrow & \\ & \mathrm{A} & & & & \mathrm{A}, \delta & \end{array}$$

Here is how read this figure. The left hand side of it depicts an agent who starts out with permissible foundational beliefs that A. And this agent permissibly non-foundationally believes that β and that χ based on

[53] See Fumerton 2009 for an introduction to more traditional and less minimal forms of foundationalism and Chisholm 1989 for a classic defense. I do not assume as Pollock and Cruz 1999 do that foundationalism makes the so-called doxastic assumption. Moreover, though my discussion is conducted in terms that are not congenial to Pollock and Cruz's direct realism, this is only to simplify the discussion.

[54] This interpretation is similar to thinking of reasoning that does not satisfy MONOTONICITY as a kind of belief revision (cf. Stalnaker 1994: §4 and Rott 2001). A standard formal theory of belief revision is the AGM theory (Alchourrón, Gärdenfors, and Makinson 1985) and the Makinson-Gärdenfors identity (Makinson and Gärdenfors 1991) provides a bridge between that theory and consequence relations that do not satisfy MONOTONICITY. However, that system of reasoning must (in the finite case at least) satisfy CUT (in the presence of supplementary postulate K*7, see Makinson and Gardenfors 1991: 197 for discussion). More generally, we break with AGM in representing a belief state as a structured entity rather than a flat classical theory (cf. Rott 2001: ch. 5).

[55] This is only one example of an epistemically important way of distinguishing premises and conclusions. Another example: premises are foundational beliefs; conclusions are belief whether they be foundational or non-foundational. This choice is not epistemically neutral. The choice in the text more easily allows us to make sense of reasoning that fails to satisfy REFLEXIVITY.

good reasoning from A. Then, perhaps due to perception, δ is added to the agent's foundations. So on the right hand side of the figure we see that the agent now also has a permissible foundational belief that δ. This results, we may assume, in the agent now permissibly non-foundationally believing that ε based on good reasoning from A and δ.

I call this 'the structure of standard cases of learning' because often when we learn something new, we are able to draw further conclusions because of this. And this is what is represented in this figure.

As we have seen, good reasoning fails to satisfy MONOTONICITY and this means not all cases of learning work like this. Instead sometimes we can learn something new and have to take back some of our conclusions. We can then have this structure:

$$\begin{array}{ccc} \beta & & \chi \\ & \Uparrow & \\ & A & \end{array} \qquad\qquad \textit{Add } \delta \qquad\qquad \begin{array}{c} \beta \\ \Uparrow \\ A, \delta \end{array}$$

Here learning δ made it so that we have to retract some of our old conclusions.

5.2.3 CUT

We can similarly interpret CUT. To see what the question of whether good reasoning satisfies CUT amounts to in this setting, it helps to compare it to the structure of standard learning. The natural interpretation of that structure is one on which the proposition expressed by δ is distinct from the proposition expressed by β, by χ, and each of the propositions expressed by the sentences in A.

The question of whether good reasoning satisfies CUT arises when the thing that we add to the foundations is the same as something that we already had as a conclusion. So for good reasoning to fail to satisfy CUT is for the following structure to be possible:

$$\begin{array}{ccc} \beta & & \chi \\ & \Uparrow & \\ & A & \end{array} \qquad\qquad \textit{Add } \beta \qquad\qquad \begin{array}{ccc} \beta & \chi & \varepsilon \\ & \Uparrow & \\ & A, \beta & \end{array}$$

Here β was a conclusion but adding it to the foundations leads to us being newly permitted in believing ε. If good reasoning fails to satisfy CUT, a structure like this is possible. If it satisfies CUT, a structure like this is not possible.

5.2.4 The Fruits of the Interpretation

This then gives us a simple interpretation of the main claims about structure that we have discussed. This interpretation allows us to isolate a question in foundationalist epistemology—can adding β to your foundational beliefs add to your permissible non-foundational beliefs if you were already permitted to non-foundationally believe that β?—and the issue of whether reasoning satisfies CUT is settled by how we answer this question. Though some epistemologists tacitly assume an answer to this question, as far as I know the question itself has gone unnoticed in foundationalist epistemology. It is nonetheless a substantive question. And as we have seen, the core elements of foundationalism do not entail an answer to it.

In fact, it is somewhat unsurprising that foundationalism is compatible with the idea that good reasoning fails to satisfy CUT. After all, foundationalism already gives foundational beliefs a distinctive epistemic role. In particular, the permissibility of non-foundational beliefs is explained by the relation that they stand to foundational beliefs. But the permissibility of foundational beliefs is not explained in this way. Thus, the foundationalist framework makes a sharp distinction between premises and conclusions and gives premises a distinctive epistemic role.

Seeing this makes it natural to wonder what further assumptions we would have to make in order to go from foundationalism being merely compatible with good reasoning failing to satisfy CUT to actually entailing that good reasoning fails to satisfy CUT. What I want to do in the remainder of this section is to explain what kind of assumption we would need to add to the foundationalist framework in order for it to entail that good reasoning fails to satisfy CUT.

5.3 How CUT Might Fail

To begin, we can informally thinking of NON-MONOTONICITY in terms of defeasible permissibility—the belief that Tweety is a bird and the belief that Tweety flies defeasibly permit you to believe Tweety flies. The permission is defeasible in the sense that you are permitted unless a certain condition, e.g., your believing that Tweety is a penguin, holds. Formal theories make this informal talk precise. But for the purposes of illustrating my main idea from the informal perspective that we have adopted in this paper, let's stick to this informal way of speaking.

In all of the examples that we have looked at, the 'unless'-clauses of these claims about defeasible permission simply mention other beliefs that you might have (e.g., the belief that Tweety is a penguin). But according to foundationalism, to fully characterize your belief state it is not enough just to say *what* beliefs you have; the *distinctive organizational structure* of your beliefs must also be mentioned. And this in turn leaves it open that these 'unless'-clauses may not just mention which other beliefs you have but also mention the distinctive organizational structure of your beliefs.

I will now provide two examples of what such 'unless'-clauses might look like. The first example will not involve a failure of good reasoning to satisfy CUT but it will illustrate how organization might matter. This will pave the way for a second example that will illustrate how good reasoning might fail to satisfy CUT.

5.3.1 A Preliminary Example

So consider the following case. Suppose that you foundationally believe of some object *o* that *o* seems to have a rough surface. And suppose this belief defeasibly permits you to believe that *o* has a rough surface. Finally suppose you come to believe that your sense of touch is malfunctioning. What may you conclude by reasoning now that you have learned this?

One answer is that you may no longer conclude that *o* has a rough surface. Another answer is that you may still conclude that *o* has a rough surface. And a third answer is that it depends. Though I do not know of any knock down argument against the first two answers, I wish to explore the third by way of illustrating how the *organization* of your beliefs may matter.

In particular, I wish to explore the idea that it depends on how your belief that *o* seems to have a rough surface came about: The idea is that your beliefs permit you to believe *o* has a rough surface unless you acquired your believe that *o* seems to have a rough surface through touching. So an agent who acquired the belief that *o* appears to have a rough surface by touching *o* would not be permitted to believe that *o* has a rough surface. But an agent who acquires belief that *o* appears to have a rough surface through visual perception would be permitted to believe *o* has a rough surface.

This illustrates that how your belief came to be foundational might affect what other beliefs are permissible. We can also look at this issue from a more abstract perspective: What we have observed so far is that sometimes the belief that β might make χ reasonable so long as that belief was arrived at in a certain way. Accordingly, claims about defeasible permissibility that make reference to how our beliefs were arrived at occupy a certain strategic position.

Without such 'unless'-clauses, we are faced with a choice. We can either say that the belief that β does not defeasibly permit believing that χ or we can say that it does, but make no reference to how the belief that β was arrived at. Both options seem unsatisfying. The first alternative may be too conservative because at least when the belief that β comes about in the right way, the belief that χ seems reasonable.[57] The second alternative may be too liberal because it claims the belief that χ is permissible in cases where it seems unreasonable.

So the example that I sketched earlier together with this more abstract consideration suggest that claims about defeasible permissibility might have 'unless'-clauses that mention the organizational structure of your beliefs. Now this example does not involve a failure of CUT. But it does give us a taste of why 'unless'-clauses might mention structure and this nicely paves the way for our second (abstract) example which will illustrate a failure of CUT.

5.3.2 A Failure of CUT

Here's the example: Suppose (1) the beliefs corresponding to A defeasibly permit the belief that β and (2) the beliefs corresponding to A together with the belief that β defeasibly permit the belief that χ. And suppose that if we unpack (2)'s 'unless'-clause it says the beliefs corresponding to A together with the belief

[57] One way of pressing this alternative is to say that the belief that β together with belief that this belief came about in the right way defeasibly permit the belief that χ. But given that the foundationalist psychology includes not just beliefs but also an account of what place within a larger structure these beliefs occupy and why they occupy this place, it is hard to see why we must insist that one have explicit beliefs about how a belief comes about for them to play the kind of role that I am suggesting that they play.

that β defeasibly permit the belief that χ unless the belief that β is solely based on the beliefs corresponding to A.

Let's look at what these claims predict. If we also assume the belief that δ defeasibly permits the belief that β, they predict that we can have a situation like this:

$$\begin{array}{c} \beta \\ \Uparrow \\ A \end{array} \qquad \qquad \textit{Add}\ \delta \qquad \qquad \begin{array}{ccc} \beta & & \chi \\ & \Uparrow & \\ & A, \delta & \end{array}$$

Though this case is not what I called a standard case of learning, it shares its structure. In standard cases of learning, the new information plays a direct role in permitting the new belief. But in this case, it is the beliefs corresponding to A and the belief that β permit the new belief. What the belief that δ does is make it so the belief that β is no longer solely based on the beliefs corresponding to A and this makes it so the 'unless'-clause of (2) is no longer true. In the lingo, the belief that δ is a defeater defeater.

So in that example the 'unless'-clause of (2) is no longer true because β comes to be partially based on the belief that δ. But another way the 'unless'-clause of (2) could no longer be true is by β becoming part of the foundations so not based on any other beliefs. That is, we could have a situation like this:

$$\begin{array}{c} \beta \\ \Uparrow \\ A \end{array} \qquad \qquad \textit{Add}\ \beta \qquad \qquad \begin{array}{ccc} \beta & & \chi \\ & \Uparrow & \\ & A, \beta & \end{array}$$

And now notice that this situation is one that demonstrates how good reasoning might fail to satisfy CUT.

Returning to the less rich notation that we used in the critical portion of this paper, the picture of what happens after adding β tells us that $A, \beta \Vdash \chi$. This is because if your foundational beliefs corresponding to A are permissible and your foundational belief that β is permissible, then you would be permitted to believe that χ. The picture before adding β shows us that $A \Vdash \beta$. And it shows us that $A \nVdash \chi$ because the claims about defeasible permissibility that we are considering say that the beliefs corresponding to A and the belief that β permit you to believe that χ unless the belief that β is based solely on the beliefs corresponding to A.

The fundamental idea is that good reasoning within a minimal foundationalist framework might fail to satisfy CUT by being sensitive not just to which beliefs you have but also to the *distinctive organizational structure* of those beliefs.

5.4 Applications

As I have said, here is not the place to develop and defend any application to a puzzle in detail. But considering how we might use this model to approach one puzzle will illustrate how the model may be helpful.

So return to the bootstrapping problem. The problem arose by accepting two pieces of reasoning. First is the inference from '*o* appears red' to '*o* is red' and similarly for other colors. Second is the inference from a whole track record of claims of the form '*o* appears red and is red', which we will call *Track Record*, to 'my color vision works'. The solution to this puzzle that is suggested by the present approach is to specify the 'unless'-clause of these claims about reasoning in a way that makes reference to the organizational structure of your beliefs.

In particular, suppose we say the belief that *Track Record* permits believing that your color vision works unless the belief that *o* appears red and is red is solely based on the belief that *o* appears red. This predicts that {'*o* appears red', '*o** appears blue', …} $\Vdash$ *Track Record* by the first inference rule. It also predicts that {'*o* appears red', '*o** appears blue', …}, *Track Record* $\Vdash$ 'my color vision works' by the second inference rule. But crucially {'*o* appears red', '*o** appears blue', …} $\nVdash$ 'my color vision works' because the 'unless'-clause of the second inference rule is triggered.

This shows how the model developed here may be applied to the bootstrapping problem. An interesting question for further exploration is whether this application is satisfactory for this puzzle or any other puzzle.[58] But since CUT is implicated in any puzzle that has multiple steps, there are many potential further applications of this idea and it remains to be seen which of these applications are fruitful.

5.5 Taking Stock

Let's take stock. We introduced a foundationalist framework that gave us an epistemically important distinction between premises and conclusions and allowed us to better understand the questions about the structure of reasoning that we have been focused on. We noted that within this framework it might be that good reasoning fails to satisfy CUT. We then showed how the framework together with claims about permissibility that mention the *distinctive organizational structure* of your beliefs actually entails that good reasoning fails to satisfy CUT. Finally, we put the model to work by applying it to the bootstrapping problem.

This model also allows us to shed new light on questions that we discussed earlier. For example, we considered the question of how to do suppositional reasoning and reasoning with conditionals if good reasoning fails to satisfy CUT. And we said that for it to be possible, there must be some class of inferences that is not sensitive to the premise/conclusion distinction. The present model allows us (informally at least) to isolate that class. The model says that inferences whose 'unless'-clauses *do not* mention the organizational structure of belief are the ones that will be guaranteed to be safe in the context of suppositional reasoning and reasoning with conditionals.

Thus, we have isolated a distinctive way in which reasoning with qualitative belief might not satisfy CUT.[59]

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[58] See Weisberg 2010 for further discussion. Nair ms-a evaluates and comparesthis approach to other approaches to the bootstrapping problem.

[59] Of course, I have not argued that good reasoning must fail to satisfy CUT. In fact, we could introduce a formal trick to ensure that

6. Conclusion

We have then two main results: First, there are no successful arguments for thinking good reasoning must satisfy CUT (§2-4). Second within our minimal foundationalist model, the question of whether good reasoning fails to satisfy CUT turns on the question of whether what we are permitted to believe depends not just on what beliefs we have but also on the *distinctive organizational structure* of those beliefs (§4).

We should then be open to exploring rejecting CUT as a possible solution to the problems in logic, computer science, ethics, and epistemology that involve multiple steps of reasoning. And more generally, I hope that by clearing certain conceptual barriers to understanding reasoning that does not satisfy CUT, this paper has brought to light fresh philosophical and formal questions about the nature and structure of good reasoning.[61]

CUT holds such as labelling sentences according to whether they are held non-inferentially or inferentially. But this redescription does not change the significance of our results. We still have seen how within a certain picture of qualitative belief the idea that premises play a special role in reasoning makes sense. This is the philosophical work that we need to do in order to see why we need to resort to redescriptions like this in order to save CUT.

That said, the redescription strategy does suggests that in developing a formal model, we should consider using more structured objects (e.g., ordered pairs of propositions and their derivational source) as the relata of our model's consequence relation and so this more complicated relation might satisfy CUT (cf. Brewka 1991).

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