



A choice-functional characterization of welfarism [☆]

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A B S T R A C T

Welfarism is the view that individual welfare is the only thing that matters. One important contribution of social choice theory has been to provide a precise formulation and axiomatic characterization of welfarism using Amartya Sen's framework of social welfare functionals. This paper is motivated by the observation that the standard formalization of welfarism is too restrictive, since a welfarist social planner need not be committed to maximizing a preference ordering or any other binary relation over alternatives. We therefore provide a characterization of welfarism in a more general choice-functional setting and show that welfarism, so understood, carries no commitment to rationalizability. This characterization is compatible with welfare values having any structure whatsoever. It also sheds light on different formulations of anonymity, revealing only some of these to be fundamental requirements of impartiality.

1. Introduction

Welfarism is, very roughly, the view that individual welfare is the only thing that matters (Sumner 1999, 184; Moore and Crisp 1996, 598; Kagan 1998, 48; Shaver 2004, 237). In informational terms, a welfarist social planner only needs to know how well off each individual would be in the available alternatives in order to decide what to do. One important contribution of social choice theory has been to provide a more precise formulation and axiomatic characterization of this doctrine using Sen's (1970a) framework of social welfare functionals.

A social welfare functional is a mapping which assigns a social preference ordering of alternatives to each profile of real-valued utility functions in its domain. A social welfare functional is welfarist if and only if the ordering it assigns to any profile is determined by a single ordering of utility vectors (Gevers, 1979). This means that the social welfare functional ignores all non-welfare features of the alternatives as well as the particularities of each profile. A foundational result in social choice theory is the *welfarism theorem*, which, given an unrestricted domain of utility profiles, characterizes welfarism in terms of a Pareto Indifference axiom and a utility-theoretic version of the Independence of Irrelevant Alternatives (Bossert and Weymark 2004, Theorem 2.2; see also d'Aspremont and Gevers 1977; Hammond 1979; Weymark 1998). Importantly, even for a single profile, this equivalence relies on the transitivity of social preference (Weymark 2017; the need for transitivity is also emphasized by Fleurbaey et al. 2003).

The standard characterization does capture one important way of being a welfarist. But it also seems possible for a social planner to be welfarist, in the rough sense stated above, even if her choices are not rationalizable as maximizing a single binary social preference relation—let alone an ordering—and thus not accurately modeled by the social welfare functional framework. For a simple example, consider a welfarist majoritarian view, on which it is permissible to choose x in a choice between x and y if and only if x is better than y for at least as many people as it is worse; from any larger menu of alternatives, it is permissible to choose any alternative in (say) the uncovered set with respect to this relation (Fishburn 1977; Miller 1977). This is a crude interpretation of promoting “the well-being of the greatest number” (de Tocqueville [1835] 2002, vol. I, part 2, ch. 6). This rule is not rationalizable as maximizing

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any binary relation (assuming the domain contains some Condorcet cycles), but it seems perfectly consistent with welfarism, in the intuitive sense that what it is permissible to choose from any menu of alternatives depends only on how well off each individual would be in those alternatives.

A perhaps more plausible example involves “partial aggregation” of harms. Some believe that in a choice between (a) saving one person from a severe impairment and (b) saving a much larger number of people from a moderate impairment, we ought to choose b; and in a choice between b and (c) saving some even larger number of people from a slight impairment, we ought to choose c; but in a choice between a and c, we ought to choose a, no matter how many people would slightly better off in c (Kamm 2007, 485; for a critical survey, see Horton 2021). Such a pattern of choices is not rationalizable as maximizing any binary relation. But it nonetheless seems compatible with (though of course not committed to) welfarism, in the intuitive sense stated above (see, e.g., Brown 2020; Voorhoeve 2014).

Other examples of nonrationalizable but arguably welfarist principles can be found in bargaining theory (Gaertner and Klemisch-Ahlert 1991; Gauthier 1985; Imai 1983; Kalai and Smorodinsky 1975), the theory of distributive justice (Fleurbaey et al. 2009; Tungodden and Vallentyne 2005), variable-population ethics (Boonin-Vail 1996; Kolodny 2022; McDermott 2019; Otsuka 2018; Temkin 2012), and ethical applications of tournament theory (Podgorski 2022, 2023).

Unfortunately, the extant literature does not seem to contain a characterization of welfarism in this more general sense. This paper addresses this gap, by defining welfarism in a choice-functional framework and providing an axiomatic characterization of welfarism so defined, which does not require social choice functions to be rationalizable.

We begin, in section 2, by laying out a generalization of Sen (1977)’s framework of functional collective choice rules. We then provide a simple characterization of profile-dependent welfarism in this framework (section 3), in terms of a choice-functional analogue of Pareto Indifference and novel restrictions of Sen’s properties α and β concerning contractions and expansions of menus of alternatives. We then extend this characterization to profile-independent welfarism via an Independence of Irrelevant Alternatives condition (section 4). Finally, we distinguish between two kinds of anonymity principles which might be imposed on welfarist choice rules, and provide a choice-functional characterization of anonymous welfarism (section 5).

The project of this paper is related to the classic literature on choice-functional analogues of Arrow’s impossibility theorem and other foundational results in social choice theory (for a survey, see Sen 1986). Sen (1999) offers reformulations of Arrow’s conditions in choice-functional terms which are jointly inconsistent even in the absence of any general conditions of rationality or consistency between menus. (Arrhenius 2004 pursues an analogous strategy in variable-population ethics.) Our results, by contrast, show the welfarist to be committed to certain principles of “internal consistency,” though these principles fall far short of rationalizability.

Some authors have formulated certain kinds of welfarism in choice-functional settings. Pattanaik and Suzumura (1994, 436) briefly consider a property of “Indistinguishability of Indifferent States,” which captures the same idea as our analogue of Pareto Indifference, and which they claim “rules out the possible use of any nonwelfare information for the purpose of social choice.” As we shall see, this claim is not quite correct, in the absence of our consistency axioms. Bossert (1998) considers a “weak welfarism” property for allocation mechanisms, which carries no commitment to rationalizability. He does not offer more basic axioms to characterize this property, however; his focus is on rationalizability by a social welfare ordering. Roemer (1988) formulates a notion of welfarism for bargaining theory on economic environments, which does not require rationalizability. Roemer shows this notion of welfarism to be equivalent, on a certain class of domains, to a requirement of consistency with respect to reductions in the dimension of a commodity space (for related work, see de Clippel 2015; Donaldson and Roemer 1987; Ginés and Marhuenda 2000; Kıbrıs and Sertel 2007; Martinet et al. 2024; Valenciano and Zarzuelo 1997). The present paper is concerned with more abstract alternatives, with minimal assumptions about the structure of individual well-being. It is interesting, however, that consistency axioms of some kind or other play such an essential role in characterizing welfarism across these different frameworks.

2. Framework

Let X be a nonempty set of alternatives and N a nonempty set of individuals. (We do not require the typical assumptions that $|X| \geq 3$ or $\infty > |N| \geq 2$.) For each individual $i \in N$, there is a nonempty set \mathbb{W}_i of possible “welfare values” for i . These welfare values can be any objects whatsoever. They could be real numbers, as in the standard framework of Sen (1970a). But they could also be vectors of numbers (as in Chipman 1960; List 2004; Sen 1980), non-numerical “grades” (as in Balinski and Laraki 2010; Morreau and Weymark 2016), “dimensioned quantities” of well-being (as in Nebel 2021, 2022a), or objects of any other kind. We also leave open, until section 5, whether different individuals have the same possible welfare values.¹

A *welfare profile* is a mapping $W : X \rightarrow \prod_{i \in N} \mathbb{W}_i$ which assigns a *welfare distribution* $W(x)$ to each alternative $x \in X$. (We do not call $W(x)$ a “vector” since these objects need not live in a vector space.) We are interested in some nonempty domain $\mathcal{D} \subseteq (\prod_{i \in N} \mathbb{W}_i)^X$ of possible welfare profiles.

¹ The welfarism theorem in the social welfare functional framework similarly does not depend on that framework’s presupposed real-valued representation of well-being (Nebel 2024). We simply make this generality explicit, for three reasons. First, it clarifies that unorthodox views about the structure of welfare—e.g., non-Archimedean views on which some quantity of a “higher” good is better than any quantity of “lower” goods (Arrhenius and Rabinowicz 2005; Carlson 2022; Nebel 2022b; Pivato and Tchouante 2024; Thomas 2018)—are fully compatible with welfarism. Second, it allows our framework to be applied to variable-population cases, by simply taking nonexistence to be an honorary “welfare value,” which may or may not be comparable to others. Third, as further discussed in section 6, it means that our results can be utilized even by many theorists who reject welfarism, by reinterpreting the sets \mathbb{W}_i to include nonwelfare characteristics of some relevant kind—e.g., which of i ’s rights are respected, or how deserving or responsible i is.

For any set S , let $\mathcal{F}(S)$ denote the set of all finite, nonempty subsets of S —i.e., *menus* of elements of S . Each $A \in \mathcal{F}(X)$ is a menu of alternatives. A *social choice function* $C : \mathcal{F}(X) \rightarrow \mathcal{F}(X)$ takes each menu A of alternatives and returns a nonempty subset $C(A) \subseteq A$ of acceptable choices. (For most of our results, it would suffice for the set of menus to be closed under finite union.)

It will be useful to relate some of our axioms and results to well-known properties of choice functions and conditions for rationalizability. We say that $C : \mathcal{F}(X) \rightarrow \mathcal{F}(X)$ is *rationalized* by a binary relation \succsim on X if and only if, for all $A \in \mathcal{F}(X)$, $C(A) = \{x \in A \mid x \succsim y \text{ for all } y \in A\}$. Rationalizability in this sense is equivalent to the conjunction of properties α and γ (Sen 1971):

Property α If $A \subseteq B$ then $C(B) \cap A \subseteq C(A)$.

Property γ $C(A) \cap C(B) \subseteq C(A \cup B)$.

We call a choice function *fully rationalizable* if and only if it is rationalized by an ordering. This status is equivalent to the conjunction of properties α and β :

Property β If $A \subseteq B$ then either $C(A) \cap C(B) = \emptyset$ or $C(A) \subseteq C(B)$.

Let \mathcal{C} denote the set of all choice functions on $\mathcal{F}(X)$. Adapting the terminology of Sen (1976, 1977, 1993), a *functional collective choice rule* is a mapping $\phi : D \rightarrow \mathcal{C}$ which assigns a social choice function to each welfare profile in its domain. (This simply replaces Sen’s domain of *preference profiles*— n -tuples of orderings on X —with one of welfare profiles.) For any profile W , we write C_W for $\phi(W)$.

We can distinguish two levels at which welfarism might be applied (Blackorby et al. 1990). It might first be applied only *within* each profile, to the social choice function assigned to that profile: that is, for any profile W , the choice function C_W ’s selection from any menu of alternatives might depend only on the welfare distributions assigned to those alternatives by W . This is *profile-dependent* welfarism. A stronger property applies *across* profiles. It says that there is a single choice function on the set of all menus of welfare distributions which determines the choice function C_W assigned to any profile W . This is *profile-independent* welfarism. We characterize these two ideas in turn.

3. Profile-dependent welfarism

For any profile W and subset of alternatives $S \subseteq X$, let

$$\mathbf{W}(S) := \{w \in \prod_{i \in N} \mathbb{W}_i \mid w = W(x) \text{ for some } x \in S\}$$

denote the set of welfare distributions attainable by alternatives in S in W . According to

Profile-Dependent Welfarism For any profile $W \in D$, there is a unique choice function $C_W^* : \mathcal{F}(\mathbf{W}(X)) \rightarrow \mathcal{F}(\mathbf{W}(X))$ such that, for all $A \in \mathcal{F}(X)$ and $x \in A$, $x \in C_W(A)$ if and only if $W(x) \in C_W^*(\mathbf{W}(A))$.

We call C_W^* the *distributive choice function* associated with C_W . Profile-Dependent Welfarism captures the idea that, within any given profile, the social choice from any menu of alternatives should be fully determined by their welfare distributions in that profile; non-welfare features of the alternatives can be ignored. However, it allows the distributive choice function to vary between profiles.

We first show Profile-Dependent Welfarism to be equivalent to the following condition:

Intraprofile Neutrality For any $W \in D$, $A, B \in \mathcal{F}(X)$, $x \in A$, and $y \in B$, if $\mathbf{W}(A) = \mathbf{W}(B)$ and $W(x) = W(y)$, then $x \in C_W(A)$ if and only if $y \in C_W(B)$.

Lemma 1. A functional collective choice rule ϕ satisfies Profile-Dependent Welfarism if and only if it satisfies Intraprofile Neutrality.

Proof. Suppose that ϕ satisfies Intraprofile Neutrality. For each $W \in D$, define C_W^* as follows: for any menu of welfare distributions $A^* \in \mathcal{F}(\mathbf{W}(X))$ and $w \in A^*$, $w \in C_W^*(A^*)$ if and only if there is some menu of alternatives $A \in \mathcal{F}(X)$ and $x \in A$ such that $\mathbf{W}(A) = A^*$, $W(x) = w$, and $x \in C_W(A)$. For any $A^* \in \mathcal{F}(\mathbf{W}(X))$, there must be some $A \in \mathcal{F}(X)$ such that $\mathbf{W}(A) = A^*$, and there must be some $x \in A$ such that $x \in C_W(A)$, in which case $W(x) \in C_W^*(A^*)$. Thus, C_W^* always returns a nonempty choice set, so it is a choice function.

Take any $A \in \mathcal{F}(X)$ and $a \in A$. Clearly $a \in C_W(A)$ implies $W(a) \in C_W^*(\mathbf{W}(A))$. For the converse implication, suppose $W(a) \in C_W^*(\mathbf{W}(A))$. Then there must be some $B \in \mathcal{F}(X)$ and $b \in B$ such that $\mathbf{W}(B) = \mathbf{W}(A)$, $W(b) = W(a)$, and $b \in C_W(B)$. By Intraprofile Neutrality, $a \in C_W(A)$ if and only if $b \in C_W(B)$. Thus, for all $A \in \mathcal{F}(X)$ and $x \in A$, $x \in C_W(A)$ if and only if $W(x) \in C_W^*(\mathbf{W}(A))$.

To see that C_W^* must be unique, take any other choice function C_W^{**} on $\mathcal{F}(\mathbf{W}(X))$ such that $x \in C_W(A)$ if and only if $W(x) \in C_W^{**}(\mathbf{W}(A))$ for all $A \in \mathcal{F}(X)$ and $x \in A$. These choice functions are distinct only if, for some $A \in \mathcal{F}(X)$, $C_W^{**}(\mathbf{W}(A)) \neq C_W^*(\mathbf{W}(A))$. This is impossible given our result that, for all $A \in \mathcal{F}(X)$ and $x \in A$, $x \in C_W(A)$ if and only if $W(x) \in C_W^*(\mathbf{W}(A))$.

Suppose next that ϕ satisfies Profile-Dependent Welfarism: there is a choice function $C_W^* : \mathcal{F}(\mathbf{W}(X)) \rightarrow \mathcal{F}(\mathbf{W}(X))$ such that, for all $A \in \mathcal{F}(X)$ and $x \in A$, $x \in C_W(A)$ if and only if $W(x) \in C_W^*(\mathbf{W}(A))$. Take any $A, B \in \mathcal{F}(X)$ and $W \in D$ such that $\mathbf{W}(A) = \mathbf{W}(B)$.

Table 1
Counterexamples to each of Pareto Indiscriminability, Redundant Contraction, Redundant Expansion, where $W(a) = W(b) \neq W(c)$.

Menu	Pareto Indiscriminability	Redundant Contraction	Redundant Expansion
{ a, b }	{ a, b }	{ a, b }	{ a, b }
{ b, c }	{ b, c }	{ b, c }	{ b, c }
{ a, c }	{ a }	{ a }	{ a }
{ a, b, c }	{ a }	{ a, b, c }	{ a, b }

By Profile-Dependent Welfarism, $x \in C_W(A)$ if and only if $W(x) \in C_W^*(\mathbf{W}(A))$, and $y \in C_W(B)$ if and only if $W(y) \in C_W^*(\mathbf{W}(B))$. Thus, if $W(x) = W(y)$, then $x \in C_W(A)$ if and only if $y \in C_W(B)$, so Intraprofile Neutrality is satisfied. \square

Intraprofile Neutrality is, in turn, equivalent to the conjunction of three independent conditions. The first is a choice-functional variation on Pareto Indifference. It says that the social choice function assigned to any given profile cannot discriminate between two alternatives which have the same welfare distribution, in the sense that either both or neither are acceptable choices from any menu to which they both belong:

Pareto Indiscriminability For any $W \in D$, $A \in \mathcal{F}(X)$ and $x, y \in A$, if $W(x) = W(y)$, then $x \in C_W(A)$ if and only if $y \in C_W(A)$.²

The second condition is a restriction of property α to menus which have the same attainable welfare distributions. To motivate this idea, say that an alternative is *redundant* on a menu if there is some other alternative on that menu with the same welfare distribution (as in Dhillon and Mertens 1999). Redundant Contraction captures the idea that removing redundant alternatives cannot make an initially acceptable choice suddenly unacceptable:

Redundant Contraction For any $W \in D$ and $A, B \in \mathcal{F}(X)$, if $\mathbf{W}(A) = \mathbf{W}(B)$ and $A \subseteq B$, then $C_W(B) \cap A \subseteq C_W(A)$.

The third condition is the analogous restriction of property β :

Redundant Expansion For any $W \in D$ and $A, B \in \mathcal{F}(X)$, if $\mathbf{W}(A) = \mathbf{W}(B)$ and $A \subseteq B$, then either $C_W(A) \cap C_W(B) = \emptyset$ or $C_W(A) \subseteq C_W(B)$.

The main result of this section is that these three conditions are jointly equivalent to Intraprofile Neutrality, and thus to Profile-Dependent Welfarism:

Theorem 2. A functional collective choice rule ϕ satisfies Profile-Dependent Welfarism if and only if it satisfies Pareto Indiscriminability, Redundant Contraction, and Redundant Expansion.

Proof. Suppose first that ϕ satisfies Pareto Indiscriminability, Redundant Expansion, and Redundant Contraction. By Lemma 1, it suffices to show that ϕ satisfies Intraprofile Neutrality.

Take any $W \in D$, $A, B \in \mathcal{F}(X)$, $x \in A$, and $y \in B$ such that $\mathbf{W}(A) = \mathbf{W}(B)$ and $W(x) = W(y)$. Suppose $x \in C_W(A)$. Then by Redundant Expansion, either $C_W(A) \cap C_W(A \cup B) = \emptyset$ or $x \in C_W(A \cup B)$. If $C_W(A) \cap C_W(A \cup B) = \emptyset$, then $A \cap C_W(A \cup B) = \emptyset$ by Redundant Contraction, and thus $B \cap C_W(A \cup B) = \emptyset$ by Pareto Indiscriminability, yielding $C_W(A \cup B) = \emptyset$, which is impossible. Thus, $x \in C_W(A \cup B)$. This implies $y \in C_W(A \cup B)$ by Pareto Indiscriminability, and thus $y \in C_W(B)$ by Redundant Contraction, as desired. By exactly parallel reasoning, $y \in C_W(B)$ implies $x \in C_W(A)$, so Intraprofile Neutrality is satisfied.

Suppose next that ϕ satisfies Profile-Dependent Welfarism and thus Intraprofile Neutrality. Pareto Indiscriminability is simply the restriction of Intraprofile Neutrality to cases where $A = B$. Redundant Contraction and Redundant Expansion follow immediately from the restriction of Intraprofile Neutrality to cases where $x = y$. \square

These three axioms—Pareto Indiscriminability, Redundant Expansion, and Redundant Contraction—are independent in the sense that no two of them entail the third, assuming that X contains at least three alternatives and D contains at least one profile W that is not constant on X (that is, $W(x) \neq W(y)$ for some $x, y \in X$). For example, let $X = \{ a, b, c \}$ and $W(a) = W(b) \neq W(c)$. Consider the choice functions depicted in Table 1. There are four non-singleton menus in $\mathcal{F}(X)$, one in each row. Each column depicts a choice function that violates the axiom listed while satisfying the other two. The set in each cell is the value of the corresponding choice function (column) in the corresponding menu (row).

² Pareto Indiscriminability on its own is equivalent to the existence of a unique function $\tilde{C}_W : \mathcal{F}(X) \rightarrow \mathcal{F}(\mathbf{W}(X))$, where $\tilde{C}_W(A)$ is a nonempty subset of $\mathbf{W}(A)$ for every $A \in \mathcal{F}(X)$. (Simply define $\tilde{C}_W(A) = \{ w \in \mathbf{W}(A) \mid w = W(x) \text{ for some } x \in C_W(A) \}$.) This condition could be considered a kind of “menu-dependent” welfarism, since it allows $\tilde{C}_W(A) \neq \tilde{C}_W(B)$ even when $\mathbf{W}(A) = \mathbf{W}(B)$. This is distinct from the notion of a menu-dependent choice function in Sen (1997, Theorem 3.2), which is equivalent to nonrationalizability. I thank an anonymous referee for suggesting the idea of menu-dependent welfarism.

Table 2
Nonrationalizability of Aggregate Relevant Harms.

	Person 1	2, 3	4, ..., 8
<i>a</i>	3	1	2
<i>b</i>	0	3	2
<i>c</i>	0	1	3

All three choice functions in Table 1 generate the same *base relation* \succsim_W , defined by $x \succsim_W y$ if and only if $x \in C_W(\{x, y\})$: namely, $a \succ_W c \sim_W b \sim_W a$. This relation only rationalizes the counterexample to Redundant Expansion, however. The counterexample to Pareto Indiscriminability is not rationalizable because it violates γ . The counterexample to Redundant Contraction is not rationalizable because any such counterexample violates α . Interestingly, even though this base relation satisfies Pareto Indifference (since $a \sim_W b$), there is no binary social welfare relation \succsim_W^* on $\mathbf{W}(X)$ with the feature that, for all $x, y \in X$, $x \succsim_W y$ if and only if $W(x) \succsim_W^* W(y)$. As Weymark (2017) observes, the welfarist significance of Pareto Indifference in the social welfare functional framework depends on the transitivity of social preference (see also Bosmans and Öztürk 2022; Fleurbaey et al. 2003).³

An obvious corollary of Theorem 2 is that, when C_W is fully rationalizable for every profile $W \in D$, Profile-Dependent Welfarism is equivalent to Pareto Indiscriminability; and, when C_W is rationalizable (though not necessarily by an ordering) for every $W \in D$, Profile-Dependent Welfarism is equivalent to the conjunction of Pareto Indiscriminability and Redundant Expansion. However, while our axioms are compatible with rationalizability, they do not require it. For example, Voorhoeve (2014) suggests that if we can save one person from death, a hundred thousand people from a moderate impairment, or a billion from a very slight impairment, we ought to save the hundred thousand; but, if we lack the option to save the one person from death, we ought to save the billion from very slight impairment. On the face of it, this pattern violates property α , but it is perfectly consistent with Pareto Indiscriminability, Redundant Contraction, and Redundant Expansion, and thus with Profile-Dependent Welfarism.⁴

It will be useful to consider a more precise formulation of Voorhoeve’s view, due to Brown (2020). Assume $\mathbb{W}_i = \mathbb{R}$ for all $i \in N$. For any profile $W \in D$, menu $A \in \mathcal{F}(X)$, alternative $x \in A$, and individual $i \in N$, let $H_i^A(W(x)) := \max_{y \in A} W_i(y) - W_i(x)$ denote the magnitude of i ’s “harm” in x relative to her best alternative in A , according to profile W . For each profile W , there is a ratio $\rho_W \in [0, 1]$ such that i ’s harm $H_i^A(W(x))$ in x counts as *relevant* (in menu A and profile W) if and only if $H_i^A(W(x)) \geq \rho_W H_j^A(W(z))$ for all $j \in N \setminus \{i\}$ and $z \in A \setminus \{x\}$. Let $R(x, A, W)$ denote the set of individuals whose harms in x are relevant in menu A and profile W . Voorhoeve’s view can be operationalized by the following functional collective choice rule:

Aggregate Relevant Harms For all $W \in D$, $A \in \mathcal{F}(X)$,

$$C_W(A) = \operatorname{argmin}_{x \in A} \sum_{i \in R(x, A, W)} H_i^A(W(x)).$$

To see how Aggregate Relevant Harms violates property α , consider the profile W depicted in Table 2. Let $\rho_W = 1/2$. There are eight individuals. The harms faced by person 1 in alternatives b and c are relevant in the menu $\{a, b, c\}$. The harms faced by persons 2 and 3 in a and c are also relevant in this menu. But the harms faced by the five remaining people in a and b are not relevant in this menu, because they are less than half of the greatest harm with which they compete (namely person 1’s). Thus, according to Aggregate Relevant Harms, $C_W(\{a, b, c\}) = \{b\}$, since b minimizes the sum of relevant harms. However, when a is no longer an option, the smaller harms to the five become relevant, so that $C_W(\{b, c\}) = \{c\}$. This violates property α . Indeed, the base relation generated by C_W is $a \succ_W c \succ_W b \succ_W a$, which cannot rationalize a choice function.

This violation of α , however, is perfectly consistent with Redundant Contraction, and it is not difficult to see that Aggregate Relevant Harms satisfies Profile-Dependent Welfarism. For each profile W , C_W can be associated with the following distributive choice function. For each $A^* \in \mathcal{F}(\mathbf{W}(X))$, $w \in A^*$, and $i \in N$, let $H_i^{A^*}(w) := \max_{v \in A^*} v_i - w_i$. The harm $H_i^{A^*}(w)$ counts as relevant (in A^* and W) if and only if $H_i^{A^*}(w) \geq \rho_W H_j^{A^*}(u)$ for all $j \in N \setminus \{i\}$ and $u \in A^* \setminus \{w\}$. Let $R(w, A^*, W)$ denote the set of individuals whose harms in w are relevant in menu A^* and profile W . Define $C_W^* : \mathcal{F}(\mathbf{W})(X) \rightarrow \mathcal{F}(\mathbf{W})(X)$ as follows: for all $A^* \in \mathcal{F}(\mathbf{W})(X)$,

$$C_W^*(A^*) = \operatorname{argmin}_{w \in A^*} \sum_{i \in R(w, A^*, W)} H_i^{A^*}(w).$$

This shows that Profile-Dependent Welfarism does not require rationalizability.

³ It is not enough for social indifference to be transitive. Where X and W are as in Table 1, Weymark’s example is the relation $a \succ_W c \succ_W b \sim_W a$, which satisfies the transitivity of indifference. Note that any choice function which generates this base relation satisfies Redundant Expansion.

⁴ Voorhoeve’s own view is that alternatives should be individuated in a fine-grained way that makes his view compatible with α (compare Broome 1991, 1993). For a critical discussion of this general strategy, see Baccelli and Mongin (2021).

4. Profile-independent welfarism

One feature of Aggregate Relevant Harms, as we have formulated it, is that the ratio ρ_W can vary between profiles. So a person's harm in some alternative might count as relevant in some menu in one profile, without counting as relevant in another profile, even when the welfare distributions of the alternatives on that menu are held fixed. This profile-dependence seems difficult to explain on a welfarist view. This section extends our choice-functional characterization of welfarism in a profile-independent way.⁵

Let $\Omega := \{ w \in \prod_{i \in N} \mathbb{W}_i \mid w = W(x) \text{ for some } W \in D, x \in X \}$ denote the set of all welfare distributions attainable across all profiles. Let

$$D^* := \{ A^* \in \mathcal{F}(\Omega) \mid A^* = \mathbf{W}(A) \text{ for some } W \in D, A \in \mathcal{F}(X) \}$$

denote the set of all menus of welfare distributions which are attainable by some menu of alternatives in some profile or other. According to

Profile-Independent Welfarism There is a unique choice function $C^* : D^* \rightarrow D^*$ such that, for all $W \in D, A \in \mathcal{F}(X)$, and $x \in A, x \in C_W(A)$ if and only if $W(x) \in C^*(\mathbf{W}(A))$.

We call C^* the *distributive choice function* associated with the functional collective choice rule ϕ .

Profile-Independent Welfarism is equivalent to the following strengthening of Intraprofile Neutrality:

Interprofile Neutrality For any $A, B \in \mathcal{F}(X), x \in A, y \in B$, and $W, W' \in D$, if $\mathbf{W}(A) = \mathbf{W}'(B)$ and $W(x) = W'(y)$, then $x \in C_W(A)$ if and only if $y \in C_{W'}(B)$.

Lemma 3. A functional collective choice rule ϕ satisfies Profile-Independent Welfarism if and only if it satisfies Interprofile Neutrality.

Proof. Suppose that ϕ satisfies Interprofile Neutrality. Then ϕ satisfies Intraprofile Neutrality (by letting $W = W'$) and thus, by Lemma 1, Profile-Dependent Welfarism. We can then define $C^* : D^* \rightarrow D^*$ as follows: for any $A^* \in D^*$ and $w \in A^*, w \in C^*(A^*)$ if and only if there is some $W \in D$ such that $w \in C_W^*(A^*)$. For every $A^* \in D^*$, there is some $w \in A^*$ and $W \in D$ such that $w \in C_W^*(A^*)$, so $C^*(A^*)$ is a nonempty subset of A^* for every $A^* \in D$. Thus, C^* is a choice function.

Take any $A \in \mathcal{F}(X), a \in A$, and $W \in D$. Clearly $a \in C_W(A)$ implies $W(a) \in C^*(\mathbf{W}(A))$, by Profile-Dependent Welfarism and the definition of C^* . For the converse implication, suppose $W(a) \in C^*(\mathbf{W}(A))$. Then there must be some $B \in \mathcal{F}(X), b \in B$, and $W' \in D$ such that $\mathbf{W}'(B) = \mathbf{W}(A), W'(b) = W(a)$, and $b \in C_{W'}(B)$, which implies $a \in C_W(A)$ by Interprofile Neutrality. So, for all $A \in \mathcal{F}(X)$ and $x \in A, x \in C_W(A)$ if and only if $W(x) \in C^*(\mathbf{W}(A))$.

The demonstrations that C^* is unique and that Profile-Independent Welfarism entails Interprofile Neutrality are analogous to the corresponding parts of the proof of Lemma 1. \square

Obviously, if D contains only a single profile, then Profile-Independent Welfarism and Interprofile Neutrality are equivalent to Profile-Dependent Welfarism and Intraprofile Neutrality. In this sense, Theorem 2 already establishes Profile-Independent Welfarism when there is just a single profile. However, if D contains multiple profiles, then we need additional assumptions.

Let us assume that D is unrestricted in the following sense:

Unrestricted Domain For any $A \in \mathcal{F}(X), A^* \in D^*$, and $g : A \rightarrow A^*$, there is a $W \in D$ such that $W(x) = g(x)$ for all $x \in A$.

Even when $\mathbb{W}_i = \mathbb{R}$ for all $i \in N$, Unrestricted Domain is considerably weaker than the usual axiom of that name. It is compatible, for example, with certain people's utilities always being of the opposite sign, or always being integers.⁶

Given Unrestricted Domain and a further assumption stated below, Interprofile Neutrality is, in turn, equivalent to the conjunction of Intraprofile Neutrality and

Independence of Irrelevant Alternatives For all $W, W' \in D$ and $A \in \mathcal{F}(X)$, if $W(x) = W'(x)$ for all $x \in A$, then $C_W(A) = C_{W'}(A)$.

Our further assumption has to do with the number of alternatives. It is simplest to assume that X is infinite. We make this assumption in Theorem 4 below. However, the result would still hold if X were finite, so long as there are more alternatives than welfare distributions in Ω . This more complicated version (Theorem 8) is proved, and the independence of the axioms is demonstrated, in the appendix.

⁵ On the distinction between profile-dependent and -independent welfarism in the social welfare functional framework, see Blackorby et al. (1990); d'Aspremont and Gevers (2002). Adler (2022); Fleurbaey et al. (2003); Weymark (2016) take welfarism to require profile-independence; see also the distinction between welfarism and "quasi-welfarism" in Fleurbaey (2003).

⁶ Unrestricted Domain (along with other conditions that require collective choice rules to be defined on a domain of multiple profiles) has been challenged on the grounds that the welfare distributions of alternatives, when fully described, are necessarily fixed (Blackorby et al. 2006; Hurley 1985; Morreau 2015). See Hedden and Nebel (2024); Nebel (2024) for discussion.

Theorem 4. *If a functional collective choice rule ϕ satisfies Unrestricted Domain and $|X| = \infty$, then ϕ satisfies Profile-Independent Welfarism if and only if ϕ satisfies Pareto Indiscriminability, Redundant Contraction, Redundant Expansion, and Independence of Irrelevant Alternatives.*

Proof. Assume that ϕ satisfies Unrestricted Domain, Pareto Indiscriminability, Redundant Contraction, Redundant Expansion, and Independence of Irrelevant Alternatives. By Theorem 2, ϕ satisfies Intraprofile Neutrality. Take any $A, B \in \mathcal{F}(X)$ and $W, W' \in \mathcal{D}$ for which $W(A) = W'(B)$. Since $|X| = \infty$ and A and B are finite, we can find some $A' \in \mathcal{F}(X)$ which is disjoint from A and B and some bijection $f : A \rightarrow A'$ (thus $|A| = |A'|$). By Unrestricted Domain, there are profiles V and V' such that:

- For all $x \in A$, $V'(f(x)) = V(f(x)) = V(x) = W(x)$, and
- For all $y \in B$, $V'(y) = W'(y)$.

Take any $x \in A$ and $y \in B$ such that $W(x) = W'(y)$. Independence of Irrelevant Alternatives and Intraprofile Neutrality imply (in alternating order) that $x \in C_W(A)$ if and only if $x \in C_V(x)$ if and only if $f(x) \in C_{V'}(A')$ if and only if $f(x) \in C_{V'}(A')$ if and only if $y \in C_{V'}(B)$ if and only if $y \in C_{W'}(B)$. Thus, we have $x \in C_W(A)$ if and only if $f(x) \in C_{W'}(B)$ for all $x \in A$, so Interprofile Neutrality—and thus Profile-Independent Welfarism, by Lemma 3—is satisfied.

It is easy to see that Interprofile Neutrality (and thus Profile-Independent Welfarism) implies Independence of Irrelevant Alternatives and Intraprofile Neutrality and, therefore, Pareto Indiscriminability, Redundant Contraction, and Redundant Expansion. \square

The full strength of Unrestricted Domain is not necessary for the equivalence in Theorem 4. For example, suppose there is a nonempty $S \subseteq X$ such that $W(x) = W'(x)$ for all $x \in S$ and $W, W' \in \mathcal{D}$. This violates Unrestricted Domain as long as there are at least two attainable welfare distributions. But clearly the restriction of Interprofile Neutrality to A or B in $\mathcal{F}(S)$ would hold as long as Intraprofile Neutrality is satisfied. We could of course weaken Unrestricted Domain to accommodate this sort of possibility, as long as $X \setminus S$ is either infinite, bigger than Ω , or empty. One question for further research is how much further Unrestricted Domain can be weakened while maintaining the necessary equivalence of Interprofile Neutrality to the conjunction of Intraprofile Neutrality and Independence of Irrelevant Alternatives, given a suitable number of alternatives.

To see the need for our assumption about the number of alternatives, suppose that X was finite but no larger than Ω . Then consider any distinct profiles $W, W' \in \mathcal{D}$ in which X contains no redundant alternatives—i.e., $W(x) \neq W(y)$ for all distinct $x, y \in X$, and likewise for W' . Independence of Irrelevant Alternatives and Intraprofile Neutrality impose no constraint whatsoever on the choice from menu X in these profiles, since $W(x) \neq W'(x)$ for some $x \in X$ and there are no redundant alternatives. In the absence of any general consistency conditions, Independence of Irrelevant Alternatives is not sufficient to rule out profile-dependence, unless X is infinite or larger than Ω .

5. Anonymous welfarism

A welfarist believes that welfare is the only thing that matters. This is precisified by Theorems 2 and 4. But we might also believe that it should not matter *who* has what welfare. This does not follow from Profile-Independent Welfarism as we have defined it. Many welfarists will want to impose some further constraint to capture a requirement of impartiality between individuals.

Anonymity principles are meant to reflect this idea of impartiality.⁷ There are two ways in which a distributive choice function C^* might be anonymous. The first is that it may be unable to discriminate between welfare distributions related by a permutation of individuals. For any distribution $w \in \Omega$ and permutation $\sigma : N \rightarrow N$, let σw denote the distribution defined by $(\sigma w)_i = w_{\sigma(i)}$ for all $i \in N$.

Anonymous Indiscriminability For all $A^* \in \mathcal{D}^*$ and $w, v \in A^*$, if there is a permutation $\sigma : N \rightarrow N$ such that $v = \sigma w$, then $w \in C^*(A^*)$ if and only if $v \in C^*(A^*)$.

Many social choice principles which are naturally modeled in the functional collective choice rule framework, however, are incompatible with Anonymous Indiscriminability. For example, as mentioned in section 1, many people think we ought to save a single person from severe harm (such as torture or death) rather than any number of people from a slight impairment (such as a headache). Consider the distributions in Table 3, where welfare values are represented by real numbers. Suppose that losing 99 units of welfare corresponds to a severe harm whereas losing 1 unit corresponds to a merely slight harm. Then, on views of this kind—e.g., Aggregate

Table 3
Violation of Anonymous Indiscriminability.

	Person 1	Person 2	...	Person 100
w	100	1	...	99
v	1	2	...	100

⁷ For prior characterizations of anonymous welfarism in single- and multi-profile frameworks, see especially Blackorby, Bossert, and Donaldson (2006, 2005a), respectively.

Table 4
Violation of Anonymous Invariance.

	Person 1	Person 2	...	Person 100
w'	1	100	...	99
v'	2	1	...	100

Relevant Harms with a single profile-independent relevance ratio $\rho > 1/100$ —we ought to choose w rather than v , in violation of Anonymous Indiscriminability (Brown 2020; Parfit 2003; Voorhoeve 2014).

This violation of Anonymous Indiscriminability, however objectionable it may be, does not seem to involve any failure of impartiality. A social planner who chooses w rather than v need not care more about *person 1* than anyone else; rather, they may simply care more about preventing severe harms, whomever might befall them, than preventing any number of minor ones. In particular, C^* may satisfy the following condition, which really does seem a requirement of impartiality:

Anonymous Invariance For all $A^*, B^* \in \mathcal{D}^*$, if there is a permutation $\sigma : N \rightarrow N$ and a bijection $f : A^* \rightarrow B^*$ such that $f(w) = \sigma w$ for all $w \in A^*$, then $w \in C^*(A^*)$ if and only if $f(w) \in C^*(B^*)$ for all $w \in A^*$.

For example, if we chose w rather than v from Table 3 but v' rather than w' from Table 4, that would violate Anonymous Invariance.

In order to derive Anonymous Invariance, we need another assumption about the domain of our functional collective choice rule:

Interpersonal Richness For any profile $W \in \mathcal{D}$ and permutation $\sigma : N \rightarrow N$, there is a profile $W' \in \mathcal{D}$ such that $W_i = W'_{\sigma(i)}$ for every $i \in N$.

This is not already implied by Unrestricted Domain, which is compatible with different individuals having no possible welfare values in common; Interpersonal Richness rules this out. (Nor is Unrestricted Domain implied by Interpersonal Richness, which is compatible with there being no constant profiles.)

Given Profile-Independent Welfarism and Interpersonal Richness, Anonymous Invariance is equivalent to imposing the following anonymity condition on our functional collective choice rule:

Interprofile Anonymity For all $W, W' \in \mathcal{D}$, if there is a permutation $\sigma : N \rightarrow N$ such that $W_i = W'_\sigma(i)$ for all $i \in N$, then $C_W = C_{W'}$.

Proposition 5. *If a functional collective choice rule ϕ satisfies Profile-Independent Welfarism and Interpersonal Richness, then ϕ satisfies Interprofile Anonymity if and only if its distributive choice function C^* satisfies Anonymous Invariance.*

Proof. Assume Interprofile Anonymity, Profile-Independent Welfarism, and Interpersonal Richness. Take any $A^*, B^* \in \mathcal{D}^*$ for which there is a permutation $\sigma : N \rightarrow N$ and a bijection $f : A^* \rightarrow B^*$ such that $f(w) = \sigma w$ for all $w \in A^*$. Since $A^* \in \mathcal{D}^*$, there must be some $A \in \mathcal{F}(X)$ and $W \in \mathcal{D}$ such that $\mathbf{W}(A) = A^*$. By Interpersonal Richness, there is also a profile $W' \in \mathcal{D}$ such that $W_i = W'_{\sigma(i)}$ for all $i \in N$.

Take any $w \in A^*$. There must be some $a \in A$ such that $W(a) = w$. Profile-Independent Welfarism implies that $w \in C^*(A^*)$ if and only if $a \in C_W(A)$. Interprofile Anonymity then implies that $a \in C_{W'}(A)$ if and only if $a \in C_W(A)$. Since $W'(a) = f(w)$, Profile-Independent Welfarism then implies that $a \in C_{W'}(A)$ if and only if $f(w) \in C^*(B^*)$. Thus, Anonymous Invariance is satisfied.

It is easy to see that Anonymous Invariance and Profile-Independent Welfarism imply Interprofile Anonymity. \square

In light of Proposition 5, I call a functional collective choice rule *anonymously welfarist* if and only if it satisfies Profile-Independent Welfarism and Interprofile Anonymity. For example, Aggregate Relevant Harms with profile-independent ρ is anonymously welfarist in this sense, even though it violates Anonymous Indiscriminability. Its violation of Anonymous Indiscriminability is related to its nonrationalizability. For example, consider the distributions in Table 5, and suppose $\rho > 1/2$. Then, where C^* is the distributive choice function associated with Aggregate Relevant Harms, we have $C^*({u, v, w}) = {u, v, w}$, but $C^*({u, v}) = {u}$, $C^*({v, w}) = {v}$, and $C^*({w, u}) = {w}$, in violation of both α and Anonymous Indiscriminability.

Table 5
Anonymous Indiscriminability and Nonrationalizability.

	Person 1	Person 2	Person 3
u	1	2	3
v	2	3	1
w	3	1	2

Table 6
Anonymous Indiscriminability without Anonymous Invariance.

Menu	β	α	γ, β
$\{u, v\}$	$\{u, v\}$	$\{u, v\}$	$\{u, v\}$
$\{v, w\}$	$\{v\}$	$\{w\}$	$\{v, w\}$
$\{u, w\}$	$\{u, w\}$	$\{u\}$	$\{w\}$
$\{u, v, w\}$	$\{u, v\}$	$\{u, v, w\}$	$\{w\}$

The relationship between Anonymous Invariance, Anonymous Indiscriminability, and rationalizability is summarized by the following result:

Proposition 6. *Assume Interpersonal Richness and that $C^* : D^* \rightarrow D^*$ is fully rationalizable. Then, if C^* satisfies Anonymous Indiscriminability, it also satisfies Anonymous Invariance. If C^* satisfies Anonymous Invariance and N is finite, then C^* also satisfies Anonymous Indiscriminability.*

Proof. Assume Interpersonal Richness and that C^* is fully rationalizable. C^* therefore satisfies properties α and β .

First assume Anonymous Indiscriminability. Take any $A^*, B^* \in D^*$ such that, for some permutation $\sigma : N \rightarrow N$ and bijection $f : A^* \rightarrow B^*$, $f(w) = \sigma w$ for all $w \in A^*$. Anonymous Indiscriminability implies that $C^*(A^* \cup B^*) \cap A^* = \emptyset$ if and only if $C^*(A^* \cup B^*) \cap B^* = \emptyset$. But at least one of these sets must be nonempty, so both of them are. It follows, by property α , that $C^*(A^* \cup B^*) \cap C(A^*) \neq \emptyset$ and $C^*(A^* \cup B^*) \cap C(B^*) \neq \emptyset$. So, by β , $C^*(A^*), C^*(B^*) \subseteq C^*(A^* \cup B^*)$, and therefore $C^*(A^*) = C^*(A^* \cup B^*) \cap A^*$ and $C^*(B^*) = C^*(A^* \cup B^*) \cap B^*$ by α . Thus, for any $w \in A^*$, $w \in C^*(A^*)$ if and only if $w \in C^*(A^* \cup B^*)$, and $f(w) \in C^*(A^* \cup B^*)$ if and only if $f(w) \in C^*(B^*)$. By Anonymous Indiscriminability, $w \in C^*(A^* \cup B^*)$ if and only if $f(w) \in C^*(A^* \cup B^*)$. So $w \in C^*(A^*)$ if and only if $f(w) \in C^*(B^*)$, as Anonymous Invariance requires.

Next assume Anonymous Invariance and that N is finite. Take any permutation $\sigma : N \rightarrow N$. Since N is finite, σ is the product of finitely many transpositions τ_1, \dots, τ_m . (A transposition is a permutation that swaps exactly two elements.) Take any $w^0 \in D^*$ and let $w^j = \tau_j w^{j-1}$ for all $j \in \{1, \dots, m\}$, so that $w^m = \sigma w^0$. All of these distributions are in D^* by Interpersonal Richness. We show that whenever $w^0, w^m \in B^*$ for any $B^* \in D^*$, $w^0 \in C^*(B^*)$ if and only if $w^m \in C^*(B^*)$. Anonymous Invariance implies that $C^*({w^{j-1}, w^j}) = {w^{j-1}, w^j}$ for all $j \in \{1, \dots, m\}$. Property β implies that $C^*({w^0, w^1, \dots, w^m}) = {w^0, w^1, \dots, w^m}$. Property α implies that $C^*({w^0, w^m}) = {w^0, w^m}$. β then implies that whenever $w^0, w^m \in B^*$ for any $B^* \in D^*$, $w^0 \in C^*(B^*)$ if and only if $w^m \in C^*(B^*)$. Therefore, Anonymous Indiscriminability is satisfied. \square

We have already seen how a distributive choice function can satisfy Anonymous Invariance while violating Anonymous Indiscriminability. Interestingly, it is also possible, in the absence of rationalizability, to satisfy Anonymous Indiscriminability while violating Anonymous Invariance. Suppose for example that $N = \{1, 2\}$ and $\mathbb{W}_1 = \mathbb{W}_2 = \{0, 1, 2\}$. Let $u = (2, 0)$, $v = (0, 2)$, and $w = (1, 1)$. The choice functions in Table 6 all satisfy Anonymous Indiscriminability but violate Anonymous Invariance. The choice function in the leftmost column violates property β , the middle one violates α , and the one on the right violates γ and β . Unlike the violations of Anonymous Indiscriminability witnessed above, these violations of Anonymous Invariance seem inexplicable from an impartial perspective. This confirms our suspicion that Anonymous Indiscriminability does not, on its own, capture a fundamental commitment to impartiality; it seems better regarded as a *consequence* of impartiality on the assumption of full rationalizability.

Propositions 5 and 6 also shed some light on requirements of impartiality in more standard, “relational” frameworks for social welfare evaluation. For example, Blackorby et al. (2005b, ch. 7) explore a framework of *social decision functionals*, which assign a (possibly incomplete) quasiordering to each profile of real-valued utility functions in some domain. They require the functional to be welfarist in the sense that the comparison of two alternatives in any profile is determined by a single quasiordering of utility vectors. Our results suggest that, in such a framework, the analogue of Interprofile Anonymity will not force all permutations of a utility vector to be equally good; it will only require the quasiordering of utility vectors to be invariant to common permutations, so that for any vectors u and v and permutation of individuals σ , u is at least as good as v if and only if $\sigma(u)$ is at least as good as $\sigma(v)$. An example of a social decision functional which satisfies only the latter condition is the strong Pareto rule axiomatized, in an Arrovian setting, by Weymark (1984).

The difference between these anonymity conditions bears on other foundational issues in welfarist ethics. According to the “person-affecting restriction,” one alternative can be better than another only if there is someone for whom it is better. This principle faces well-known challenges in variable-population cases (Parfit 1984) but is widely thought to be a plausible welfarist principle in fixed-population cases (Arrhenius and Rabinowicz 2012; Blackorby et al. 2006; Goodin 1991). However, when welfare levels are only partially ordered, the person-affecting restriction is in tension with anonymity-as-indifference, given the weak Pareto principle, even in fixed-population cases (Nebel 2020). It is perfectly consistent, however, with anonymity-as-invariance. The difference is also important in settings with infinite populations, where, given a suitable set of utility vectors, only the anonymity-as-invariance condition is compatible with the strong (or even weak) Pareto principle (Asheim et al. 2010; Askill 2018).

6. Conclusion

The standard characterization of welfarism in the social welfare functional framework appeals crucially to the transitivity of social preference. We have seen that an analogous characterization survives in a choice-functional framework even when social choice functions are not rationalizable by any binary relation, let alone an ordering. This vindicates our initial suspicion that collective choice rules can be, in a natural sense, welfarist—indeed, anonymously welfarist—even if their prescriptions are not rationalizable.

In fact, we have characterized a much more general class of ethical principles, since (as mentioned in Note 1) nothing in the formalism requires us to interpret the elements of \mathbb{W}_i as *welfare* values, as opposed to other attributes of individuals.⁸ A theorist who accepts our choice-functional “welfarism” axioms when the welfare values are replaced by other such properties would be committed to choosing between alternatives on the basis of individuals’ characteristics in those alternatives alone, but not necessarily just their welfare characteristics. This doctrine, which might be called *individualism*, would seem acceptable to many critics of welfarism (such as Scanlon 1998; Sen 1970b), though not all of them (such as Moore 1903). We leave a more thorough exploration of this view, and of how to distinguish welfarism from the more general class of individualistic principles, for another occasion (for important work in this direction, see Blackorby et al. 2005a).

CRedit authorship contribution statement

Jacob M. Nebel: Conceptualization, Formal analysis, Writing – original draft, Writing – review & editing.

Declaration of competing interest

None.

Data availability

No data was used for the research described in the article.

Appendix A. Variation on Theorem 4 with finitely many alternatives

We first prove the following:

Lemma 7. *If a functional collective choice rule ϕ satisfies Unrestricted Domain, Intraprofile Neutrality, and Independence of Irrelevant Alternatives, and $\infty > |X| > |\Omega|$, then for any $W, W' \in \mathcal{D}$, if there is a transposition $\tau : X \rightarrow X$ such that $W(x) = W'(\tau(x))$ for all $x \in X$, then for all $x \in X$, $x \in C_W(X)$ if and only if $\tau(x) \in C_{W'}(X)$.*

Proof. Assume Unrestricted Domain, Intraprofile Neutrality, and Independence of Irrelevant Alternatives, and take any $W, W' \in \mathcal{D}$ for which there is a transposition $\tau : X \rightarrow X$ such that $W(x) = W'(\tau(x))$ for all $x \in X$.

If $|X| \leq 2$, then $|\Omega| = 1$ since $|X| > |\Omega|$, in which case the conclusion follows trivially from Intraprofile Neutrality. So suppose $|X| > 2$. Without loss of generality let the support of τ (the set of elements moved by τ) be $\text{supp}(\tau) = \{a, b\}$. If $W(a) = W(b)$ then $W = W'$ so $C_W = C_{W'}$. Thus, suppose $W(a) \neq W(b)$. Since $|X| > |\Omega|$, there must be some $c \in X \setminus \{a, b\}$ such that $W(c) = W(x)$ for some $x \in X \setminus \{c\}$. Note also that $W(c) = W'(c)$, since $\tau(c) = c$, and that $W'(c) = W'(x)$ for some $x \in X \setminus \{c\}$.

We now use Unrestricted Domain to construct three profiles $W^1, W^2, W^3 \in \mathcal{D}$:

- $W^1(x) = W(x)$ for all $x \in X \setminus \{c\}$, and $W^1(c) = W(a)$;
- $W^2(x) = W^1(x)$ for all $x \in X \setminus \{a\}$, and $W^2(a) = W(b)$;
- $W^3(x) = W^2(x)$ for all $x \in X \setminus \{b\}$, and $W^3(b) = W(a)$.

Intraprofile Neutrality and Independence of Irrelevant Alternatives imply (in alternating order) that $a \in C_W(X)$ if and only if $a \in C_W(X \setminus \{c\})$ if and only if $a \in C_{W^1}(X \setminus \{c\})$ if and only if $c \in C_{W^1}(X \setminus \{a\})$ if and only if $c \in C_{W^2}(X \setminus \{a\})$ if and only if $c \in C_{W^2}(X \setminus \{b\})$ if and only if $c \in C_{W^3}(X \setminus \{b\})$ if and only if $b \in C_{W^3}(X \setminus \{c\})$ if and only if $b \in C_{W'}(X \setminus \{c\})$ if and only if $b \in C_{W'}(X)$.

Similarly, they imply (again, in alternating order) that $b \in C_W(X)$ if and only if $b \in C_W(X \setminus \{c\})$ if and only if $b \in C_{W^1}(X \setminus \{c\})$ if and only if $b \in C_{W^1}(X \setminus \{a\})$ if and only if $b \in C_{W^2}(X \setminus \{a\})$ if and only if $a \in C_{W^2}(X \setminus \{b\})$ if and only if $a \in C_{W^3}(X \setminus \{b\})$ if and only if $a \in C_{W^3}(X \setminus \{c\})$ if and only if $a \in C_{W'}(X \setminus \{c\})$ if and only if $a \in C_{W'}(X)$.

For any $x \in X \setminus \{a, b, c\}$ (if there is one), we have $x \in C_W(X)$ if and only if $x \in C_W(X \setminus \{c\})$ if and only if $x \in C_{W^1}(X \setminus \{c\})$ if and only if $x \in C_{W^1}(X \setminus \{a\})$ if and only if $x \in C_{W^2}(X \setminus \{a\})$ if and only if $x \in C_{W^2}(X \setminus \{b\})$ if and only if $x \in C_{W^3}(X \setminus \{b\})$ if and only if $x \in C_{W^3}(X \setminus \{c\})$ if and only if $x \in C_{W'}(X \setminus \{c\})$ if and only if $x \in C_{W'}(X)$.

⁸ Analogous observations regarding the standard characterization of welfarism have been made by Kelsey (1987), Mongin (1998), and Bossert and Weymark (2004).

Table 7
Counterexamples to each of Pareto Indiscriminability, Redundant Contraction, Redundant Expansion, where $W(a) = W(b) = w$ and $W(c) = v$.

Menu	Pareto Indiscriminability	Redundant Contraction	Redundant Expansion
$\{a, b\}$	$\{a\}$	$\{a, b\}$	$\{a, b\}$
$\{b, c\}$	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$
$\{a, c\}$	$\{a\}$	$\{a\}$	$\{a, c\}$
$\{a, b, c\}$	$\{a\}$	$\{a, b, c\}$	$\{a, b\}$

Thus, for any $x \in X \setminus \{c\}$, we have $x \in C_W(X)$ if and only if $\tau(x) \in C_{W'}(X)$. Since $W(c) = W(x)$ for some $x \in X \setminus \{c\}$, Intraprofile Neutrality implies that $c \in C_W(X)$ if and only if $x \in C_W(X)$ for some such x . We then have $\tau(x) \in C_{W'}(X)$, and since $W'(\tau(x)) = W(x) = W(c) = W'(\tau(c))$, $\tau(c) \in C_{W'}(X)$ as well. \square

Theorem 8. *If a functional collective choice rule ϕ satisfies Unrestricted Domain and $\infty > |X| > |\Omega|$, then ϕ satisfies Profile-Independent Welfarism if and only if ϕ satisfies Pareto Indiscriminability, Redundant Contraction, Redundant Expansion, and Independence of Irrelevant Alternatives.*

Proof. As in the proof of Theorem 4, we prove only the right-to-left direction of the biconditional. Assume that ϕ satisfies Unrestricted Domain, Independence of Irrelevant Alternatives, Pareto Indiscriminability, Redundant Contraction, and Redundant Expansion (and thus Intraprofile Neutrality), and that $\infty > |X| > |\Omega|$. Take any $A, B \in \mathcal{F}(X)$ and $W, W' \in \mathcal{D}$ such that $W(A) = W'(B)$. Take any $w \in \Omega$, $a \in A$, and $b \in B$ such that $W(a) = W'(b) = w$. There must be some $A' \subseteq A$ which contains a and some $B' \subseteq B$ which contains b , neither of which contains any redundant alternatives in W or W' respectively—that is, $W(x) \neq W(y)$ for all distinct $x, y \in A'$, and similarly for B' . We use Unrestricted Domain to construct profiles V and V' as follows:

- $V(x) = W(x)$ for all $x \in A'$; $V(y) = w$ for all $y \in X \setminus A'$.
- $V'(x) = W'(x)$ for all $x \in B'$; $V'(y) = w$ for all $y \in X \setminus B'$.

Intraprofile Neutrality and Independence of Irrelevant Alternatives imply (in alternating order) that $a \in C_W(A)$ if and only if $a \in C_W(A')$ if and only if $a \in C_V(A')$ if and only if $a \in C_V(X)$. They also imply (in the same order) that $b \in C_{W'}(B)$ if and only if $b \in C_{W'}(B')$ if and only if $b \in C_{V'}(B')$ if and only if $b \in C_{V'}(X)$. We therefore need only to show that $a \in C_V(X)$ if and only if $b \in C_{V'}(X)$, in order to establish Interprofile Neutrality and thus (by Lemma 3) Profile-Independent Welfarism.

There is a permutation $\pi : X \rightarrow X$ such that $V(x) = V'(\pi(x))$ for all $x \in X$, with $V(x) \in B'$ for all $x \in A'$, so in particular $\pi(a) = b$. Since X is finite, π is the product of some transpositions τ_1, \dots, τ_m on X . We can then use Unrestricted Domain to construct profiles V^1, \dots, V^{m-1} as follows: $V^1(x) = V(\tau_1(x))$ for all $x \in X$; for each $k \in \{2, \dots, m-1\}$, $V^k(x) = V^{k-1}(\tau_k(x))$. By Lemma 7, we have $a \in C_V(X)$ if and only if $\tau_1(a) \in C_{V^1}(X)$ if and only if \dots if and only if $\tau_{m-1}(\dots(\tau_1(a))\dots) \in C_{V^{m-1}}(X)$ if and only if $\tau_m(\dots(\tau_1(a))\dots) = \pi(a) = b \in C_{V'}(X)$. \square

Appendix B. Independence of the axioms in Theorems 4 and 8

The axioms in Theorems 4 and 8 are independent so long as \mathcal{D} contains at least one profile W that is not constant on X , where $|X| \geq 3$. For each axiom, we state (without proof) an example of a functional collective choice rule which violates only that axiom.

Unrestricted Domain When \mathcal{D} contains just a single, nonconstant profile $W \in (\mathbb{R}^N)^X$, Aggregate Relevant Harms satisfies all of the axioms except for Unrestricted Domain.

Independence of Irrelevant Alternatives Let $\mathcal{D} = (\mathbb{R}^N)^X$. Consider a version of Aggregate Relevant Harms where $\rho_W \neq \rho_{W'}$ for some $W, W' \in \mathcal{D}$. This rule satisfies all of the axioms except for Independence of Irrelevant Alternatives.

Pareto Indiscriminability Let $X = \{a, b, c, \dots\}$, $\Omega = \{w, v\}$ and $\mathcal{D} = \Omega^X$. For every profile $W \in \mathcal{D}$ where $W(a) = w$, let $C_W(A) = \{a\}$ if $a \in A$, otherwise $C_W(A) = A$. For every $W \in \mathcal{D}$ where $W(a) \neq w$, let $C_W(A) = \{x \in A \mid W(x) = w\}$ if $W(x) = w$ for some $x \in A$, otherwise $C_W(A) = A$. A violation of Pareto Indiscriminability is illustrated in Table 7, where $W(a) = W(b) = w$ and $W(c) = v$.

Redundant Contraction Let X , Ω , and \mathcal{D} be as in the previous example. For every $W \in \mathcal{D}$ and $A \in \mathcal{F}(X)$, let $C_W(A) = \{x \in A \mid W(x) = w\}$ whenever there is exactly one $x \in A$ such that $W(x) = w$; otherwise, $C_W(A) = A$. See Table 7.

Redundant Expansion Let X , Ω , and \mathcal{D} be as in the previous example. For every $W \in \mathcal{D}$ and $A \in \mathcal{F}(X)$, let $C_W(A) = \{x \in A \mid W(x) = w\}$ whenever there is more than one $x \in A$ such that $W(x) = w$; otherwise, let $C_W(A) = A$. See again Table 7.

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