Chancy Modus Ponens*

Sven Neth
nethsven@berkeley.edu

August 27, 2019

Abstract

Chancy *modus ponens* is the following inference scheme: '*probably* ϕ', '*if* ϕ*, then* ψ', *therefore,* '*probably* ψ' (Yalcin 2010; Moss 2015). I argue that Chancy *modus ponens* is invalid in general. I further argue that the invalidity of Chancy *modus ponens* sheds new light on the alleged counterexample to *modus ponens* presented by McGee (1985). I close by observing that, although Chancy *modus ponens* is invalid in general, we can recover a restricted sense in which this scheme of inference is valid.

1 Introduction

Chancy *modus ponens* is the following inference scheme (Yalcin 2010; Moss 2015):[1]

Probably, ϕ.

If ϕ, then ψ.

Therefore, probably ψ.

On the face of it, this looks like a valid scheme of inference. Chancy *modus ponens* seems to be a natural generalization of *modus ponens* for reasoning

*Forthcoming in *Analysis*. See https://doi.org/10.1093/analys/anz022 for final version.

[1]Note, however, that Yalcin (2010: 935) restricts the scheme to 'sentences which are themselves not probabilistic or epistemically modalized'. In contrast, Moss (2015: 57) accepts the scheme in full generality, as long as all modal operators are interpreted with respect to the same context. Moss could thus respond to the counterexamples below by arguing that different modal operators are interpreted with respect to different contexts.

under uncertainty, and many instances of Chancy *modus ponens* appear to be valid arguments. Consider the following example:[2]

> It's probably raining.
>
> If it's raining, the street is wet.
>
> Therefore, the street is probably wet.

2 Counterexamples to Chancy Modus Ponens

Despite its apparent plausibility, there are good reasons to think that Chancy *modus ponens* is invalid in general. Consider the following case, inspired by McGee (1985):

> **Horse Race.** There's a horse race with three horses A, B and C. Horse A and C belong to team red, horse B to team blue. Horse A will win with 55% probability, horse B with 30% probability and horse C with 15% probability.

In this case, the following are true:

(1) Probably, a team red horse wins.

(2) If a team red horse wins, then if it's not horse A who wins the race, it's horse C.

(1) is true because by stipulation, the probability that a team-red horse wins is 70%. (2) is true because horse A and horse C are the only members of team red. Therefore, if a team red horse wins and it's not horse A, it must be horse C. However, the following statements are false:

(3) Probably, if it's not horse A who wins the race, it's horse C.

(4) If it's not horse A who wins the race, it's probably horse C.

I take (3) and (4) to be equivalent.[3] (3) and (4) are false because if it's not horse A who wins the race, it is probably horse B, and not horse C, who wins the race. Now if Chancy *modus ponens* were valid, then (1) and (2) would

[2] A similar example is discussed by Forrest (1981: 39), who takes this to be a valid argument. However, Forrest does not take a stance on whether Chancy *modus ponens* is valid in general.

[3] As other authors have observed, the conditional constructions *probably, if* ϕ*,* ψ and *if* ϕ*, probably* ψ generally sound equivalent in English (Van Fraassen 1976: 272-273). Also see Adams (1965) and Stalnaker (1970). Note, however, that my argument doesn't require that these forms are *always* equivalent – only that they are equivalent in this particular case.

entail (3). But since (1) and (2) are true and (3) is false, Chancy *modus ponens* is invalid.
Here is another counterexample with the deontic modal 'should':

> **Stock market.** Stocks by company X are likely to increase in value.

In this case, the following is natural to say in giving advice:

(5) If company X's stocks will increase in value, you should invest all of your retirement funds in them.

This is because, if the stocks will double in value, you will double your retirement funds. Further,

(6) Probably, company X's stocks will increase in value.

Yet, since there is a risk of losing all your investments, the following also seems like good advice:

(7) You shouldn't invest all of your retirement funds in company X's stocks.

If Chancy *modus ponens* were valid, (5) and (6) would entail

(8) Probably, you should invest all of your retirement funds in company X's stocks.

and it seems like (7) and (8) are in tension with each other. In particular, conjoining (7) and (8) yields the following rather awkward construction:

(9) #You shouldn't invest all of your retirement funds in company X's stocks, but probably, you should.

Observe that (9) is of the form *not-ϕ, but probably ϕ*. Sentences of this form are known as *epistemic contradictions* and generally sound infelicitious (Yalcin 2007: 1015). This means that if Chancy *modus ponens* were valid, an unacceptable epistemic contradiction would follow from the true premisses (5), (6) and (7). This is another good reason to think that Chancy *modus ponens* is invalid.

3 Chancy *modus ponens* vs. *modus ponens*

As noted above, our first counterexample to Chancy *modus ponens* is similar to the (alleged) counterexample to *modus ponens* presented by McGee (1985). However, the failure of Chancy *modus ponens* is different from the failure of *modus ponens*. This is because we can validate *modus ponens* without validating Chancy *modus ponens*. An example of such a semantics is presented by Yalcin (2012).

In this semantics, we assign semantic values to sentences relative to points of evaluation, which are pairs of possible worlds and *information states*. We write $[\![\phi]\!]^{w,i}$ to denote the semantic value of ϕ relative to world w and information state i. For our purposes, information states i are probability spaces, consisting of a domain Ω_i of epistemically possible worlds, some algebra $\mathscr{F}_i$ of subsets of Ω_i and a probability measure $\mathbb{P}_i$ on $\mathscr{F}_i$. We say that an information state i *accepts* ϕ if ϕ is true at all worlds in Ω_i. We define the set of ϕ-worlds relative to information state i, written $[\phi]^i$, as follows:

$$[\phi]^i = \{w : [\![\phi]\!]^{w,i} = 1\}.$$

We define the notion of information state i *updated with* ϕ, written i^ϕ, as follows:

$$i^\phi = \langle \Omega_i \cap [\phi]^i, \{X \cap [\phi]^i : X \in \mathscr{F}_i\}, \mathbb{P}_i^\phi \rangle,$$

where $\mathbb{P}_i^\phi$ is $\mathbb{P}_i$ conditionalized on $[\phi]^i$.[4]

With this machinery in place, we can give the following semantic clauses for *probably* (Δ) and indicative conditionals ($\rightarrow$):

$$[\![\Delta\phi]\!]^{w,i} = 1 \text{ iff } \mathbb{P}_i(\Omega_i \cap [\phi]^i) > .5,$$
$$[\![\phi \rightarrow \psi]\!]^{w,i} = 1 \text{ iff } i^\phi \text{ accepts } \psi.$$

We model consequence as preservation of acceptance: Σ entails ϕ if every information state accepting all $\psi \in \Sigma$ also accepts ϕ.[5] This consequence relation validates *modus ponens*: every information state accepting ϕ and $\phi \rightarrow \psi$ also accepts ψ. However, it does *not* validate Chancy *modus ponens*: there are information states which accept $\Delta\phi$ and $\phi \rightarrow \psi$ but do not accept $\Delta\psi$.[6] This demonstrates that Chancy *modus ponens* and *modus ponens* can

[4]This means that for all $X \in \mathscr{F}_i$, we have $\mathbb{P}_i^\phi(X) = \mathbb{P}_i(X \mid [\phi]^i) = \frac{\mathbb{P}_i(X \cap [\phi]^i)}{\mathbb{P}_i([\phi]^i)}$. Note that the update operation is only defined if $\mathbb{P}_i([\phi]^i) > 0$.

[5]Note that this way of understanding consequence is common in the dynamic semantics literature (Veltman 1996; Willer 2015).

[6]For a countermodel, consider the information state which models the horse race example. Let $\Omega_i = \{a, b, c, \}$, where a is a world where horse A wins, b is a world where horse B wins and c is a world where horse c wins. Let $\mathscr{F}_i = \mathcal{P}(\Omega_i)$, and let $\mathbb{P}_i$ be the (unique) probability measure such that $\mathbb{P}_i(\{a\}) = .55$, $\mathbb{P}_i(\{b\}) = .3$ and $\mathbb{P}_i(\{c\}) = .15$. It is an easy exercise to verify, using the semantics in (Yalcin 2012: 1018), that this information state i accepts $\Delta\texttt{red}$, where $\mathcal{I}(\texttt{red}) = \{a, c\}$, and also accepts $\texttt{red} \rightarrow (\neg\texttt{a} \rightarrow \texttt{c})$, where $\mathcal{I}(\neg\texttt{a}) = \{b, c\}$ and $\mathcal{I}(c) = \{c\}$ but does not accept $\Delta(\neg\texttt{a} \rightarrow \texttt{c})$ or $(\neg\texttt{a} \rightarrow \Delta\texttt{c})$.

come apart.

4 McGee on Modus Ponens

I have argued that we can validate *modus ponens* without validating Chancy *modus ponens*. We can go even further and argue that the counterexamples against *modus ponens* defended by McGee (1985) are actually better understood as counterexamples to Chancy *modus ponens*. Here is how McGee describes his point:

> [...] there are occasions on which one has good grounds for believing the premises of an application of *modus ponens* but yet one is not justified in accepting the conclusion. [..] Sometimes the conclusion of an application of *modus ponens* is something we do not believe and should not believe, even though the premises are propositions we believe very properly. (McGee 1985: 462-463, my italics)

McGee claims that sometimes, one has 'good grounds' for believing ϕ and *if* ϕ*, then* ψ, yet one is 'not justified in accepting' ψ. I take it that at least sometimes, one has good grounds for believing ϕ if ϕ is (very) probable. Now given this assumption, we can explain the phenomenon that McGee is describing as a failure of Chancy *modus ponens* rather than a failure of *modus ponens*: McGee is describing a situation in which one has good grounds for believing *if* ϕ*, then* ψ, one also has good grounds for believing ϕ, because ϕ is very probable, yet one has no good grounds for believing ψ, because ψ is not probable.

On the view presented here, McGee-style counterexamples to *modus ponens* highlight a somewhat counterintuitive feature of conditional probability. The unconditional probability that team red wins is high. Further, supposing that team red wins, the conditional probability that horse C wins, given that horse A doesn't win, is 1. However, it does not follow that the conditional probability that C wins, given that A doesn't win, is high. It is perfectly possible that, after conditionalizing on a (very) probable event X, the conditional probability of Z given Y is 1, while the current conditional probability of Z given Y is pretty low.[7]

This story also explains why McGee-style counterexamples to *modus ponens* become less compelling once we stipulate that we know *for certain* that the premises are true. If a perfectly reliable oracle tells me that team red

[7] A similar point is made by Stern and Hartmann (2018).

wins, I can surely infer that if it's not horse A who wins, it must be horse C. But in our actual example, there is no perfectly reliable oracle, only fallible statistical evidence. This is another piece of evidence that McGee-style examples actually refute Chancy *modus ponens* and not *modus ponens*.

Note that, for all I have said, McGee's case might still be a genuine counterexample to *modus ponens*. My point here is that we can explain all of McGee's data points about what is reasonable to believe by the invalidity of Chancy *modus ponens*. Thus, there is little remaining motivation for giving up the validity of *modus ponens*.

5 Restricted Validity

We are left with a problem: If Chancy *modus ponens* is not valid, then why does it *seem* valid in many cases? Now, assuming that the semantics by Yalcin (2012) is on the right track, we can give an elegant answer to this question. While this semantics does not validate Chancy *modus ponens* in full generality, it predicts that many instances of Chancy *modus ponens* preserve acceptance.

Let us say that ψ is *information insensitive* if the semantic value of ψ does not depend on the information state parameter, so we have $[\psi]^i = [\psi]^j$ for all information states i and j (Kolodny and MacFarlane 2010: 141). Now, we can show the following:

Theorem. *If ψ is information insensitive, then any information state which accepts $\Delta\phi$ and $\phi \to \psi$ also accepts $\Delta\psi$.*

Proof. Pick any information state i and suppose that i accepts $\Delta\phi$ and $\phi \to \psi$, where ψ is information insensitive. By our semantics, $\mathbb{P}_i(\Omega_i \cap [\phi]^i) > .5$ and i^ϕ accepts ψ, so $\Omega_i \cap [\phi]^i \subseteq [\psi]^{i^\phi}$. Now since ψ is information insensitive, $[\psi]^{i^\phi} = [\psi]^i$. (This is the crucial step in the proof, which fails if ψ is not information insensitive.) We have shown that $\Omega_i \cap [\phi]^i \subseteq [\psi]^i$, so $\Omega_i \cap [\phi]^i \subseteq \Omega_i \cap [\psi]^i$. Probability is monotonically increasing with respect to entailment, so $\mathbb{P}_i(\Omega_i \cap [\phi]^i) \geq \mathbb{P}_i(\Omega_i \cap [\phi]^i) > .5$, so $\mathbb{P}_i(\Omega_i \cap [\phi]^i) > .5$, whence it follows that i accepts $\Delta\psi$. □

I have shown that on the semantics sketched above, there is a restricted sense in which Chancy *modus ponens* is valid. This means that there is some restricted class of sentences (viz. those which are information insensitive) such that for any ψ which belongs to this class, the argument from $\Delta\phi$ and $\phi \to \psi$ to $\Delta\psi$ is valid in the sense that any information state which accepts $\Delta\phi$ and $\phi \to \psi$ also accepts $\Delta\psi$.

Note that this prediction fits with the data we have seen so far. In the counterexamples we discussed above, ψ contains indicative conditionals and deontic modals, both of which are arguably information sensitive.[8] Thus, we can reconcile the invalidity of Chancy *modus ponens* with the fact that many applications of Chancy *modus ponens* in ordinary reasoning are perfectly acceptable. We can give an explanation of why Chancy *modus ponens* seems valid: Because it actually *is* valid in a wide range of cases, and we make the natural but erroneous generalization that it is valid in all cases.[9]

References

Adams, E. 1965. The Logic of Conditionals. *Inquiry: An Interdisciplinary Journal of Philosophy* 8: 166–97.

Forrest, P. 1981. Probabilistic Modal Inferences. *Australasian Journal of Philosophy* 59: 38–53.

Kolodny, N., and J. MacFarlane. 2010. Ifs and Oughts. *Journal of Philosophy* 107: 115–43.

McGee, V. 1985. A Counterexample to Modus Ponens. *Journal of Philosophy* 82: 462–71.

Moss, S. 2015. On the Semantics and Pragmatics of Epistemic Vocabulary. *Semantics and Pragmatics* 8: 1–81.

— . 2018. *Probabilistic Knowledge*. Oxford: Oxford University Press.

Stalnaker, R. C. 1970. Probability and Conditionals. *Philosophy of Science* 37: 64–80.

Stern, R., and S. Hartmann. 2018. Two Sides of Modus Ponens. *Journal of Philosophy* 115: 605–21.

Van Fraassen, B. 1976. Probabilities of Conditionals. In *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science*, ed. by W. H. C. Hooker, 261–308. Dordrecht: Springer.

[8]For the view that indicative conditionals are information sensitive, see Yalcin (2012) and Moss (2018). For the view that deontic modals are information sensitive, see Kolodny and MacFarlane (2010) and Willer (2014).

[9]Thanks to audiences at the 11th California Universities Semantics and Pragmatics Conference (CUSP 11) and Berkeley's Richard Wollheim Society, where earlier versions of this material were presented. Special thanks to John MacFarlane, Mathias Böhm, Ethan Jerzak, Alexander W. Kocurek, Sarah Moss, Rachel Etta Rudolph, Seth Yalcin and an anonymous referee for helpful comments and discussion.

Veltman, F. 1996. Defaults in Update Semantics. *Journal of Philosophical Logic* 25: 221–61.

Willer, M. 2015. An Update on Epistemic Modals. *Journal of Philosophical Logic* 44: 835–49.

— . 2014. Dynamic Thoughts on Ifs and Oughts. *Philosophers' Imprint* 14: 1–30.

Yalcin, S. 2012. A Counterexample to Modus Tollens. *Journal of Philosophical Logic* 41: 1001–24.

— . 2007. Epistemic Modals. *Mind* 116: 983–1026.

— . 2010. Probability Operators. *Philosophy Compass* 5: 916–37.