Aristotelian Potential Infinity

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Introduction: Overarching research questions

- How might the potential vs. actual infinite distinction help us in philosophy of mathematics and science?
    - What is relation between PI and AI in mathematics??
    - How does PI/AI distinction help resolve set-theoretical paradoxes?
    - Why does Aristotle's contribution matter for PM and science?
    - What parts of the Aristotelian framework are salvageable for modern science, and which to be discarded?
- What are the resources of Aristotelian metaphysics for characterizing mathematical infinities?
- What did Aristotle mean by saying that the infinite (*ton apeiron*) was always potential, not actual? What kind of logic is appropriate?

Outline of the talk

- I. Quick reminder of some **Kantian antinomies** involving infinity.
    - With a view to rehabilitating the Aristotelian notion of potential infinity in resolving the antinomies.
- II. Metaphysical **potentialism vs. actualism, and the status of the Domain Principle (DP).** Domain Principle (DP): that every potential infinite (within V) presupposes the existence of an actual infinite.
    - Cantor and Zermelo's best argument for the DP.
    - How to interpret potentiality and actualism in PM as an Aristotelian.
    - Aristotle, Zermelo on LEM.
- III. Running commentary on **Aristotle's *Physics*.**
    - Aristotle's association of potential infinity with matter, rejection of atomism, rejection of Eleatic view.
    - Aristotle as looking at the geometry necessary to pursue cosmology of his day.
    - Aristotle rejects the DP for potentially infinite sequences of moments of time, points in space, numbers and magnitudes. Why?
    - Zeno's paradoxes as the real reason Aristotle rejects actually infinite totalities.

Part I. Why we still need the Aristotelian notion of potential infinity

Consider a Kantian antinomy of pure reason

- Thesis: The world (universe) is (spatially) finite.
- Antithesis: The world (universe) is (spatially) infinite.
- Synthesis: (?)

Notice that the thesis is the Aristotelian view.
The antithesis is a modern view. We think Aristotle is wrong about this. Kant suggests we ought to refrain from thinking of the world as a whole.
In fact a modern better view is that the world is *potentially infinite*—it is--updating the physics--indefinitely extensible in *space-time*.
In modern cosmology, we say that since the Big Bang 14 billion years ago, the universe is expanding--at an accelerating rate!

Another Kantian cosmological antinomy

- Thesis: Spatial magnitude is only finitely divisible. (It has minimum parts or atoms).
- Antithesis: Spatial magnitude is infinitely divisible.
- Synthesis: ?

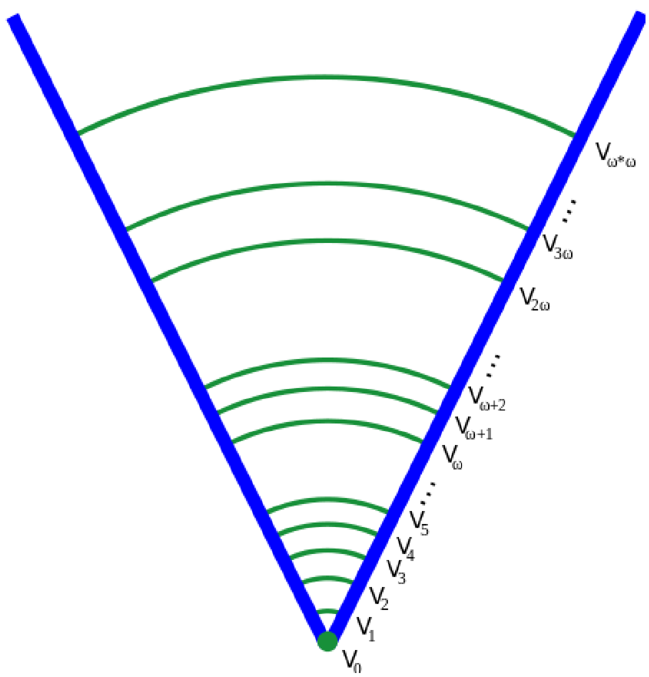
The thesis is perhaps ancient atomist Democritus' view. The antithesis is the modern view, that magnitude is infinitely divisible into uncountably many real numbers with a certain structure and is continuous.
A possible synthesis view is that the idea of a continuum is an idealization, and while we can have a continuum in mathematics, physical *space-time* may in fact be grainy and have clumpy bits. Planck length sets a limit to what we can measure in quantum mechanics.

A final modern mathematical antinomy

Thesis: The set theoretic universe is at most (transfinite K) infinite. Antithesis: Clearly, the set theoretic universe V is (beyond-K) infinite.

(i) Suppose for contradiction that V is ω-long. But $\omega+1$ comes after ω, is "longer" than ω . $\omega+1$ will be in V. So the length of V is greater than ω-long.
(ii) Suppose for contradiction V is the size of $\aleph_0$. But by Cantor's Theorem,P(0א) is greater than ($\aleph_0$) and P($\aleph_0$) is in V. The size of V must be > $\aleph_0$

Synthesis: **The set theoretic universe is potentially, not actually absolutely infinite**. **The synthesis is close to the axiomatised Zermelian view (c. 1908, 1931), a kin of the cumulative hierarchy of sets (Von-Neumann axiomatization of set theory).**



$$V_\alpha := \bigcup_{\beta<\alpha} \mathcal{P}(V_\beta)$$

Diagram of V, the set theoretic universe (above) (Details on wiki)

<u>Preventing antinomies</u>

- What blocks the class V from being a set?
- **The Axiom of Separation**: If S is a set already then there is a S'={x ϵ S: F(x)} where the propositional function F(x) is ***definite***, i.e. the relations on domain of F(x) plus axioms and logic determine whether F(x) or ¬(Fx). (LEM holds).
- V is not already a set. At each stage of the hierarchy, we cannot really say V *exists as a whole*.
- We think of successive V-stages as being generated, growing in time if you like.
- V is a PI without being an AI.
- So a certain Aristotelian-Zermelian view is vindicated on this theory.
- This does mean that V eludes us. As V is not a set already, we cannot say that V is in V or that V is out of V. V is a vague growing thing. Anything we want to say about V, we had better say about V_α instead.

Potential infinity and mathematical ontology

- What the resolution to these antinomies has in common is the Aristotelian idea of **potential infinity.**
    - (i) One issue concerns whether the potential infinity of natural numbers can be gathered as a whole in thought into a thought-object, a set. This set is actually infinite.
    - (ii) A further issue concerns whether being committed to a countably infinite set, the set of natural numbers, we should be committed to the existence of uncountably infinite sets.
    - (iii) Another issue concerns the relation of mathematics to physical reality.
    - (iv) A further issue concerns continua, are they actually divisible into points (actually infinite point-sets) or only potentially so.
- *Strict Finitists* are largely concerned with (i). *Cantorians*, logicists, intuitionists, liberal finitists and formalists are mainly concerned with (ii).

Neo-Aristotelian Philosophy of Maths (PM)

- **Where do Aristotelians fit in? What is (neo) Aristotelian PM? Is it intuitionism? But how can that be as Aristotelians advocate classical logic and intuitionists reject it, especially Law of the Excluded Middle (LEM)??**
- **Potentialists (aka neo-Aristotelians) don't have to be intuitionists. Linnebo & Shapiro (2019) show how using a particular possible world semantics can make sense of claims about potential infinities while maintaining LEM.**
- **However, Aristotle's own concept of potentiality is different from that in the many possible worlds view in modal semantics (e.g. David Lewis's modal realism, fictionalism etc). So Linnebo & Shapiro depart from an Aristotelian view while preserving LEM, and that's fine with them.**
- The ***genuine Aristotelian view of potentiality*** is connected with his notion of matter, power, change, and events in the actual world. In particular, there **are irreducible potentialities** (e.g. Sophie's potential to become a proficient pianist is not fully realised by the time she dies.). (N.B.: Linnebo and Shapiro 2019 make ***heuristic use of possible worlds***, so they are not committed to a particular metaphysics of modality. They are not Lewisians or strictly Aristotelian as far as I can see.)
- Moreover, Aristotle opposes the view that every that could potentially happen does in fact at some point in time happen. He has a concept of an **open future**.—at certain points in time, certain genuine potentialities are present and may be realised or not at all at a future time in this world, they can come to be or pass away. (Cf. *De Int*., on the sea battle).
- So Aristotle wouldn't put potentialities into different possible worlds. He keeps them in the actual world. However, his potentialities are temporalised and indexed for time. So maybe a temporal modal logic will work for him?
- But we don't want the truths of mathematics to essentially refer to space and time do we?

Part II. The Domain Principle

Cantor's Domain Principle:
Every potential infinity presupposes an actual infinity. (Every PI presupposes an AI).

- But allowing actual absolute infinities without qualification might open the way to antinomies.
- **Can we make sense of an irreducible potential infinity or must potential infinity be grounded in ontologically prior actuality?**
- **AIs are ontologically prior given Cantor's realism and theism, but this is not an independent argument for DP.**

Cantor's argument for the DP

> "**There's no doubt we cannot do without variable quantities I the sense of the potential infinite, and from this can be demonstrated the necessity of the actual infinite.** In order for there to be a variable quantity in some mathematical study **the 'domain' of its variability** must strictly speaking be known beforehand through **a definition. However, this domain cannot itself be something variable, since otherwise each fixed support (*feste Unterlage*) for the study would collapse**. Thus **this domain is a definite, actually infinite set of values** [*Wertmenge*]. Thus, each potential infinite, if rigorously applied mathematically, presupposes an actual infinite" (Cantor 1887 in Cantor, ed. Zermelo 1932, pp.410, trans. Hallett 1984, p.25)

Example

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots.$$

Statement: lim S(n) where n approaches ∞ = 2 , where S is the series above with index n.

Cantor's DP suggests we should think of there being an actually infinite set of rational numbers that is the truthmaker for the statement "lim S(n) where n approaches ∞ = 2".

This is the set { $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16},$ ....}.

Aristotelians say you cannot think of this set as finished and complete. What you have is a process for generating members of a geometric series. This doesn't mean the series exists separately from the process of generating it in time.

Zermelians say it is okay to think of that set as complete, because it belongs to a much larger existing set (such as Q, R etc.)

It's important that in practice each domain is taken to be things of a given definite kind. DP says that universal generalisations where a quantifier x ranges over a domain of objects [*of a certain kind*] presuppose that the domain of such objects is itself a set. It's important that the objects are *of a certain kind*, or else DP would collapse into an unrestricted comprehension principle.

Cantor's use of DP is *in practice* usually restricted in this way: x ranges over all natural numbers, all rational numbers, all real numbers etc. But not over all sets, all alephs, all ordinals, or does it? (*This introduces the problem of absolute infinities, which we set it aside for now.)

<u>Comment on the Cantor's argument for DP</u>

- This is not a clear argument. I interpret it with help from philosophy of language (Dummett 1973 , Priest 1995).
- The argument seems to be that if we quantify over a domain, say (x) P(x) where x ranges over elements in domain D, we cannot get a determinate truth-value for "(x)P(x)" all the time, if that domain is variable, waxing or waning.
- We must reject this world of domains of *Becoming* in favour of *Being*, Cantor would say, because otherwise the truth-values of statements about that domain will be undetermined at certain times.
- 'S is a child' may lack a truth value because (except in law) it's vague when someone turns from child into adult.
- Then given [x: x is a child] it is vague, it is indeterminate what belongs in that set. So then we can't say ((child(*a*) or not-child(*a*)) is in S for some *a*, and instances of LEM fail.
- Note: LEM is a bedrock law of logic according to Aristotle. (Meta.1011b23: For a subject s and predicate P: either P(s) ∨ ¬ P(s). *Tertium non datur.*)
- In modern logic, (LEM): (P ∨ ¬P).

<u>Zermelo on LEM in 1920s</u>

In Zermelo's view in 1920 all mathematical systems refer to an existing hypothetical **<u>domain</u>** of objects (e.g. all the natural numbers, all the real numbers etc.). The mathematical system S forms "a disjunctive system" for which $(\varphi \in S) \vee (\neg\varphi \in S)$.
Zermelo said in a Warsaw lecture, ***" Mathematics without the Law of the Excluded Middle would no longer be mathematics".*** Very Aristotelian, but naïve.

By 1930 Gödel's Incompleteness Theorems (and model theory) would upset this simple view. If we let M be a model of a sufficiently strong and consistent theory S (PA-2) then there's a Gödel sentence G in system S, asserting in essence (not Prov-G in S (n,m)). And for this Gödel sentence it cannot be determined within S—proved by the axioms of S-- if it is in a model M of the consequences of axiomatic system S or not. The model M is incomplete. M is ***agnostic*** about the Godel sentence S. This Gödel sentence G is effectively undecidable (in S).

The main philosophical issue

Stated roughly in modern logic (which kind?). Suppose DP: (x)(PIx →AIx).

Given PIx.

So, AIx.

PIx: x is potentially infinite. AIx: x is actually infinite. x is a set in domain V.

Neo-Aristotelian-Zermelian view: Rejects DP: (x) (PIx → AIx) where x is V but accepts for x $= V_a$.

Neo-Aristotelian finitist view: Rejects DP: (x) (PIx → AIx) where x is some smaller subset of V, say V_ω.

Stated in Aristotelian logic. Rejects (All PIs are AIs)

- Let x range over:
  - -moments of time (nows)
  - -points on a line (cuts)
  - -numbers
- Aristotle holds that the DP fails for some such x.
- Cantor & Zermelo hold that for any domain D in the set-theoretic universe V, DP holds for D. Neo-Aristotelians hold that DP can fail for D. There is no need to view D as completed.

Part III. Back to Aristotle

- <u>What is it "***to be*** **infinite**"</u>**?**
- **Review of Aristotle's ontology at *Physics* III.1**
- **Physics III.1: As change (kinesis) is taken to be continuous, the infinite (*ton apeiron*) first appears in the continuous (*toi sunechei*).**
- **For Aristotle, things are said "to be" either actually (entelecheia) or potentially (dunamei), or sometimes both, as an individual substance, a quantity, a quality, etc. (depending on one's list of categories of being). This is Aristotle's basic ontology.**
- **Only actual things can change.** Change itself is defined as "the actuality of what is potentially". **Aristotle's position against the Eleatics is that change is after all real, not an illusion, and doesn't lead to paradoxes.**

Aristotle's *Physics* Book III.4

Physics III.4

- Key argument to consider for the existence of the infinite is **"because they do not give out in thought, number and mathematical magnitudes and what is outside the heavens all are thought to be infinite**"(c.203b25)
- The background context is that Aristotle canvasses his predecessors' theories about the infinite by (i) **Pythagoras** and **Plato**, and (ii) the natural philosophers (physicists) **Anaxagoras, Anaximander**, and the atomist **Democritus**, and he rejects them all, and proposes a different theory. He rejects the view (i) that the infinite is a substance or principle of things, and (ii) he rejects the atomists' view that there are infinitely many minimum divisible units (atoms) into which everything extended is divisible.

Aristotle's strange finite cosmos, *Physics* III.5

- Limitation: Aristotle refuses to think of the infinite as existing **separately (in abstraction)** from the objects of sense-perception (**204a8**). It will follow that it cannot be a substance or principle as (Aristotle says) the Pythagoreans and Platonists think.

- In particular, there cannot be an infinite body (*soma apeiron*), perceptible by sense. All such bodies have **a natural place.** But there's no natural place for it to be.

- ***"Will it then occupy the whole of the place? How could it?" (205a8ff.)***

- ***"And how of what is infinite can there be an above and a below, or an extreme and a centre?"(205b30).***

Schema huius præmissæ diuisionis Sphærarum.

(Picture of geocentric Ptolemaic universe, not actually Aristotle's own.)

Potential vs Actual, Phys.III.6

"Being is said on the one hand potentially (dunamei) and in actuality (entelecheiai)" (Phys. 206a14-16)

Typical Aristotelian Examples:

- **A builder actualises his potential for building when engaged in the activity of building.**
- **A learner actualises her potential for knowing geometry when she is doing geometry.**
- **Bronze matter has the potential to take on the form of a horse statue.**

BUT The infinite exists *dunamei* in thought, not as something *separate.*

(Yes, but why? Aristotle probably thinks conflict results from taking an infinite as a separate AI. Existing 'in thought' is okay apparently because a particular Aristotelian mind (nous) cannot run through an infinite. Yes, but what about the unmoved mover? Surely, it can.)

Phys. III.6

'We say that things are potentially (*dunamei*) and in actuality (*entelecheiai*), and the infinite is by addition (*prosthesei*) and by division (*afairesei*).

We have said that the magnitude that is in actual operation (***kat' energeian***) is not the infinite (*ouk estin apeiron*). It is divisible (*diaresei*) (it's not hard to refute atomic lines (*atomos grammas*)). **And so it results that the infinite is potential.**

(*Leipetai oun dunamei einai to apeiron*).

- Things thought to be infinite are: number, time, and spatial magnitude. (N.B.: No space-time for Aristotle).
- Natural things are bodies, have spatially extended magnitude, and change over time.Things are infinite either by progression (addition) or division (206a16)

Potential existence and "Potentially"

- "**We must not take 'potentially' here in the same way in which if it is possible for this to be a statue, it actually will be a statue, and suppose that there is an infinite which is in actual operation. Since "to be" has many senses, just as the day is, and the contest is, by the constant occurring of one thing after another, so too with the infinite….In general, the infinite is in virtue of one thing's constantly being taken after another—each thing taken is finite, but it is always followed one by another**; but in magnitudes what was taken persists, [whereas] in the case of time and the race of men the things taken cease to be, yet so that [the series] does not give out."(Phys. III.6, 206a27-206b2, Hussey transl.).

- Aristotle rejects that PI→AI for the infinite diachronically. (Some contemporary scholars think he holds there could be AIs synchronically)

- The infinite exists potentially doesn't always imply there's a time when the infinite actually exists.

- *There are unrealised potential infinities that never come about fully in actuality.*

The infinite is not the whole.. (Conclusion of Physics III.6, back to cosmology)

Aristotle is doing cosmology and physics, and a bit of geometry as a tool for studying the cosmos.

- "So one must judge Parmenides to have spoken better than Melissus: the latter says the infinite is whole, the former that the whole is finite, "evenly balanced from the middle.'' **The infinite is a different kettle of fish from *the universe* or *whole*—**yet it is from this that people derive the dignity attributed to the infinite, that it surrounds everything and contains everything in itself because it has some similarity to the whole. **In fact the infinite is the material of the completeness of magnitude, and is that which is potentially but not actually whole, being divisible by process of reduction and by inversely corresponding addition…"**(Hussey trans., Phys. III. 6, 207a15ff)

An Aristotelian Table of Opposites, implicit in Physics III.

INFINITE (*ton apeiron*)	FINITE
Matter	Form
Unknowable	Knowable
Potential (dunamis)	Actual (energeia, entelecheia)
Incomplete, indefinite	Whole, completed, definite

Cutting a line segment

- Cutting a line segment (x 1/2d) is in modern terminology an infinity generator.
- ' but it (magnitude) is infinite in division' (Phys III.6 206a15ff).
- The intention seems to be that the infinity generated is PI, not AI.
- Cf. Phys. VIII.8, 263b3-8 Reply to Zeno's dichotomy paradox.
- *Aristotle's view*: There's no limit to the process of making divisions or cuts, but no need to think of all the cuts as actually existing at once (in actuality).

Cantorian Platonist view: What guarantees there's no limit to dividing the segment in principle us that the line is not just potentially infinitely divisible but actually is composed of infinitely many points (point-individuals).

Textual evidence for Aristotle's view, Phys. III.7

- 207b10 • ἐπὶ δὲ τὸ πλεῖον ἀεὶ ἔστι νοῆσαι. ἄπειροι γὰρ αἱ διχοτομίαι τοῦ μεγέθους. ὥστε δυνάμει μὲν ἔστιν, ἐνεργείᾳ δ' οὔ• ἀλλ' ἀεὶ ὑπερβάλλει τὸ λαμβανόμενον παντὸς ὡρισμένου πλήθους. ἀλλ' οὐ χωριστὸς ὁ ἀριθμὸς οὗτος [τῆς διχοτομίας], οὐδὲ μένει ἡ ἀπειρία ἀλλὰ γίγνεται, ὥσπερ καὶ ὁ χρόνος καὶ ὁ ἀριθμὸς τοῦ χρόνου.

- But in the direction of largeness it is always possible to think of a larger number: for the number of times a magnitude can be bisected is infinite. Hence this infinite is potential, never actual: the number of parts that can be taken always surpasses any assigned number. But this number is not separable from the process of bisection, and its infinity is not a permanent actuality but consists in a process of coming to be, like time and the number of time.

Cutting the line segment 2

- The line segment [a,b] does presuppose an actual infinity of points on the classical Cantorian view—it's an infinite point set with a certain structure on the real numbers, known as the continuum.
- The classical conception of a real number represents a real number as having an actual infinity (AI) of decimal places.
- On a(neo)-Aristotelian approach we can just have as many decimal places in the specification of a number or divisions *as we like*. It's a PI not an AI. ***We have to do something—like cut a segment, or reverse direction-- to actualise a potential point***. So there's a shift of focus.
- Aristotle thought his view didn't deprive mathematicians [of his day] of anything they needed. (*Phys*.207b27).

Textual evidence: Phys. 207b27

- 207b27 οὐκ ἀφαιρεῖται δ' ὁ λόγος οὐδὲ τοὺς μαθηματικοὺς τὴν θεωρίαν, ἀναιρῶν οὕτως εἶναι ἄπειρον ὥστε ἐνεργείᾳ εἶναι ἐπὶ τὴν αὔξησιν ἀδιεξίτητον• οὐδὲ γὰρ νῦν δέονται τοῦ ἀπείρου (οὐ γὰρ χρῶνται), ἀλλὰ μόνον εἶναι ὅσην ἂν βούλωνται πεπερασμένην• τῷ δὲ μεγίστῳ μεγέθει τὸν αὐτὸν ἔστι τετμῆσθαι λόγον ὁπηλικονοῦν μέγεθος ἕτερον. ὥστε πρὸς μὲν τὸ δεῖξαι ἐκείνοις οὐδὲν διοίσει τὸ [δ'] εἶναι ἐν τοῖς οὖσιν μεγέθεσιν.

Aristotle: "Our account does not rob the mathematicians of their science, ….In point of fact they do not need the infinite and do not use it. **They postulate only that the finite straight line may be produced as far as they wish**. It is possible to have divided in the same ratio as the largest quantity another magnitude of any size you like. **Hence, for the purposes of proof, it will make no difference to them to have such an infinite instead, while its existence will be in the sphere of real magnitudes."** (Phys. 207b27)

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Aristotelian vs Cantorian realism

- *Two different perspectives/frameworks* (cf. 'metaphors' of the astronomer and artificer, according to Dummett 1973).
- The Aristotelian focuses on (an idealised) human geometer in time, constructing lines with compass and straightedge. The Cantorian focuses on the idea of a fairly abstract mathematical continuum.
- *What is the relation of the two frameworks? Must we decide?*
- Different "language games" and different preoccupations.
- The Aristotelian is trying to do natural science, physics and cosmology from a temporal perspective. She is concerned with applied geometry.

- The Cantorian Platonist framework is concerned with mathematical ideas, aiming to abstract away from temporality and spatio-temporal intuition altogether. She works with set theory.

<u>Aristotle's other argument against PI→AI, Phys. III.8</u>

- " It is absurd to rely on thought: the excess and deficiency are not in the actual thing but in thought. Thus one might think of each of us as being many times as large as himself, increasing each of us ad infinitum; but it is not for this reason, because someone thinks it is so, that anyone exceeds this particular size that we have, but because it is the case; and that (someone's thinking it) just happens to be true [(when it is true)]."(Phys III.8, Hussey transl., 208a15ff.)

- For some magnitude M, I can conceive of M+1. But that doesn't mean M obtains in reality.

- +1 is an infinity generator in thought. Applying +1 to 0, I can generate the sequence {0,1,2,…..n….}. For any finite size k in N, it seems that the size of this sequence taken as a whole is bigger than k. There is no finite number k that measures the length of this sequence.

- But that's okay with Aristotle. It's Cantor who wants to invoke ω as the number corresponding to this sequence, conceived of as the limit of iteration of the +1 operation.

- But, why, suppose that ω exists? Modern Aristotelians might agree we can think of ω, but would deny that it exists independently of thought, as an actual thing. Cantorians and Platonists simply hold that it is good enough that ω exists as part of what modern Cantorian Rudy Rucker called the "Mindscape". So what appears to be at stake in deciding whether PI→AI is one's stance on ontology, i.e., whether there are mind-independent universals.

<u>Ontology in Phys III.8, Phys. VIII.8</u>

- **"Time, change, and thought are infinite things which are of the kind in which what is taken does not persist throughout."** (208a20)
- Cf. Reply to Zeno, Phys. VIII.8: "You cannot traverse an actual infinity of divisions, but you can of potentially existing ones" (263b3-6).
- Somewhat problematically, Aristotle allows a PI of time, not an AI. But there's an AI of possible *nows.*
- Problem: -1 day before *now* is an infinity generator. (So is -1/2d before here.)
- Aristotle cannot really rule out an AI of possible nows (Priest 1995). Sorabji, defending Aristotle, tries to say it's a PI of infinitely many possible nows (Sorabji 1983).
- Priest's argument depends on a kind of domain principle (DP), which he argues for on the grounds that the *determinacy of sense* requires it. Assigning truth-values to statements about domains requires completed domains. Otherwise laws of logic like Aristotle's **Law of Excluded Middle** (LEM) fail where the totality is vague and not well-defined.
- Aristotelian way out is to plead the fuzzy vague nature of some sequences, like ''all the endlessly many days before now".

Answering the original guiding questions

- ***What did Aristotle mean by saying the infinite is potential, not actual?***

- Aristotle means that any infinite sequence doesn't have all of its members existing at once. The temporal metaphor is inescapable here.

- It's pretty clear that Aristotle rejects a DP that suggests PI→AI in many cases.

- It's not at all clear that Aristotelian potentiality is the same as logical possibility (modality). In fact, Aristotelian potentiality is connected with matter and a lack of complete form. So it's a natural science notion, not purely logical.

- Aristotle cautions that potential existence of the infinite is not to be understood along the lines of the potentiality of a lump of matter to become a certain statue. Unlike the statue, the infinite can never be complete and finished and existent as a whole. Its parts are never fully actualised—not in this world, our only world.

- Aristotle's actual preoccupations are with human geometers immersed in time performing mathematical operations, and also with moving physicists onto a better view than the existing ancient cosmological theories. It's a totally different framework from contemporary maths, logic, and PM.

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