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Meaning, Presuppositions, Truth-relevance, Gödel's Sentence and the Liar Paradox

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Section 1 reviews Strawson's logic of presuppositions. Strawson's justification is critiqued and a new justification proposed. Section 2 extends the logic of presuppositions to cases when the subject class is necessarily empty, such as (x) ((Px & ~Px) → Qx) . The strong similarity of the resulting logic with Richard Diaz's truth-relevant logic is pointed out. Section 3 further extends the logic of presuppositions to sentences with many variables, and a certain valuation is proposed. It is noted that, given this valuation, Gödel's sentence becomes neither true nor false. The similarity of this outcome with Goldstein and Gaifman's solution of the Liar paradox, which is discussed in section 4, is emphasized. Section 5 returns to the definition of meaningfulness; the meaninglessness of certain sentences with empty subjects and of the Liar sentence is discussed. The objective of this paper is to show how all the above-mentioned concepts are interrelated.

1. Justification of Strawson's Theory of Presuppositions

P.F. Strawson is known for introducing the logic of presuppositions. According to this theory, the sentence

"The present king of France is wise." (1.1)

is neither true nor false if there is no king of France. Intuitively (1.1) is meaningful. Strawson considered the question of how a meaningful sentence can be neither true nor false as the main problem (Strawson, 1950, pp. 321-324; 1952, pp. 174-175), and he proposed a solution. He made a distinction between a sentence and the *use* of a sentence. For example (1.1) is a sentence, but it can be used differently on different occasions. If someone uttered (1.1) in the era of Luis XIV, he would be making a true assertion; if someone uttered it in the era of Luis XV he would be making a false assertion; and if somebody uttered it today it would be neither true nor false. Strawson defined meaning as follows: "to give the meaning of a sentence is to give *general* *directions* for its use in making true or false assertions." (Strawson, 1950, p. 185). He compared it to giving a meaning to "I" or "this." In short, Strawson emphasized that (1.1) was indexical.

This was too much for Bertrand Russell, who wrote:

> As regards "the present King of France", he fastens upon the egocentric word "present" and does not seem able to grasp that, if for the word "present" I had substituted the words "in 1905", the whole of his argument would have collapsed." (Russell, 1957, p. 385)

I will suggest a new justification of the logic of presuppositions using the following three definitions.

> **Definition 1:** A sentence is *meaningful* iff it or its internal negation expresses a possible state of affairs.
>
> **Definition 2:** A sentence is *true* iff the possible state of affairs it expresses corresponds to an actual state of affairs.
>
> **Definition 3:** A sentence is *false* iff the possible state of affairs expressed by its internal negation corresponds to an actual state of affairs.

Definition 1 is Ayer's interpretation of Wittgenstein. "A genuine proposition pictures a possible state of affairs." (Ayer, 1984, p. 112) Whether Ayer intended it or not, this is *not* the same as the verification principle. I consider a state of affairs possible iff we can picture it to ourselves. Contradictions are meaningful in the sense that they can be interpreted as denying the state of affairs expressed by its internal negation. For example "The apple in the basket is *red and not red*" *denies* that the apple is either *red or not red*.

Definition 2 is very similar to "In order to tell whether a picture is true or false we must compare it with reality." (Wittgenstein, 1961, p. 10.)

The *internal negation* in definitions 1 and 3 means the denial of the predicate rather than the denial of the state of affairs. A negation in this sense asserts that the king of France does not posses the property of being wise. Instead of saying "the king of France is not wise" we could say "the king of France is unwise" or perhaps even "the king of France is foolish," that is, his decisions are not well thought out and his acts often have unintended or detrimental consequences. The denial of the state of affairs is a wider concept and includes the possibility that there is no king of France at all.

Clearly

"The King of France in 1905 was wise." (1.2)

expresses a possible state of affairs. We can picture to ourselves what the sentence states. A novel could have been written in the era of Luis XIV about the French monarchy in 1905. However, if we enumerate all the things that are wise and all the things that are not wise, the King of France in 1905 will not appear on either list. Therefore the sentence is neither true nor false.

Later Strawson modified his stance and offered this definition: "It is enough that it should be possible to describe or *imagine* [emphasis added] circumstances in which its use would result in a true or false statement." (Strawson, 1952, p. 185.) When translated into our parlance, this becomes "it should be possible to picture to ourselves circumstances in which its use would result in a true or false statement." This is very similar to our theory.

There are two interesting observations about the logic of presuppositions (LP). Firstly, it is compatible with the traditional Aristotelian syllogism (Strawson, 1952, pp. 173-179.) Secondly, LP is an alternative to the Theory of Definite Descriptions (TDD). It is illuminating to contrast the two.

Both LP and TDD hold that (1.1) can be true only if there is a king of France. LP also holds that (1.1) can be false only if there is a king of France. (The subject class must be nonempty for the sentence to have a truth value.) The purpose of TDD is to elucidate the sentences where the grammatical subject is in the singular with the definite article. (It is not clear why we need to analyze the definite article considering that most languages including Latin do not have it.) LP treats such sentences and universally quantified sentences uniformly while TDD does not. According to LP both

"All the kings of Switzerland have been wise." (1.3)

and

"The present king of Switzerland is wise." (1.4)

are neither true nor false. In contrast TDD proposes that (1.4) is false, although (1.3) is usually considered [vacuously] true. I do not find this plausible. Perhaps Aristotle and Strawson were right while Russell was wrong.

2. Presuppositions & Truth Relevance

2.1. Presuppositions

In Strawson's view a sentence is neither true nor false if its subject class is empty (Strawson, 1952, pp. 163-179).

Strawson has observed that the natural language sentence

"All John's children are asleep." (2.1.1)

can be analyzed either as

$$\sim(\mathrm{E}x)(fx \ \& \sim gx) \qquad (2.1.2)$$

or as

$$\sim(\mathrm{E}x)(fx \ \& \sim gx) \ \& \ (\mathrm{E}x)(fx) \qquad (2.1.3)$$

or as

$$\sim(\mathrm{E}x)(fx \ \& \sim gx) \ \& \ (\mathrm{E}x)(fx) \ \& \ (\mathrm{E}x)(\sim gx), \qquad (2.1.4)$$

where

$$(\mathrm{E}x)(fx) \qquad (2.1.5)$$

stands for "John has children," and

$$(\mathrm{E}x)(gx) \qquad (2.1.6)$$

stands for "Something is asleep." If John does not have any children then, according to classical logic, (2.1.2) is true, (2.1.3) and (2.1.4) are false.

Accepting (2.1.3) or (2.1.4) would open the question of what the translation into English of "$\sim(\mathrm{E}x)(fx \ \& \sim gx)$" is. "All John's children are asleep" is now a conjunction of three formulas of which $\sim(\mathrm{E}x)(fx \ \& \sim gx)$ is only one. The sentence (2.1.1) and $\sim(\mathrm{E}x)(fx \ \& \sim gx)$ can no longer be equivalent.

Strawson rejected all three interpretations and proposed a different approach: The sentence (2.1.1) is *neither true nor false* if John has no children. In other words the two following conditions hold:

(a) if "All John's children are asleep" is *true* then "John has children" is true

(b) if "All John's children are asleep" is *false* then "John has children" is true

We say that (2.1.1) *presupposes* "John has children."

Note that if we accept this solution then "$(\mathrm{E}x)(fx)$" is no longer a part of the translation of "All John's children are asleep." The reason is that it is not possible to express both

(2.1.1) and its negation as a conjunction of some wff with $(\mathrm{E}x)fx$. Therefore, we can let "$\sim(\mathrm{E}x)(fx \;\&\; \sim gx)$" stand as the formalization of "All John's children are asleep." Then

(a) if "$\sim(\mathrm{E}x)(fx \;\&\; \sim gx)$" is *true* then "$(\mathrm{E}x)(fx)$" is true

(b) if "$\sim(\mathrm{E}x)(fx \;\&\; \sim gx)$" is *false* then "$(\mathrm{E}x)(fx)$" is true

"$\sim(\mathrm{E}x)(fx \;\&\; \sim gx)$" *presupposes* "$(\mathrm{E}x)(fx)$." The logic is no longer classical.

This generalizes to any Fx and Gx. In particular, we have

(a) if "$\sim(\mathrm{E}x)((Px \;\&\; \sim Px) \;\&\; \sim Qx)$" is *true* then "$(\mathrm{E}x)(Px \;\&\; \sim Px)$" is true

(b) if "$\sim(\mathrm{E}x)((Px \;\&\; \sim Px) \;\&\; \sim Qx)$" is *false* then "$(\mathrm{E}x)(Px \;\&\; \sim Px)$" is true

But "$(\mathrm{E}x)(Px \;\&\; \sim Px)$" is not true hence

$$\sim(\mathrm{E}x)((Px \;\&\; \sim Px) \;\&\; \sim Qx) \tag{2.1.7}$$

is *neither true nor false*. The same applies to its equivalent

$$(x)((Px \;\&\; \sim Px) \rightarrow Qx). \tag{2.1.8}$$

So, (2.1.7) and (2.1.8) are neither true nor false because

$$\sim(\mathrm{E}x)(Px \;\&\; \sim Px) \tag{2.1.9}$$

is always true.

The following example from arithmetic,

$$(\mathrm{x})((2 > \mathrm{x} > 4) \text{ -> } \sim(\mathrm{x} < \mathrm{x} + 1)), \tag{2.1.10}$$

is neither true nor false as well.

We will depart from Strawson by requiring that the predicate class not be universal. We note that (2.1.2) is equivalent to

$$(x)(fx \rightarrow gx). \tag{2.1.11}$$

By Modus Tollens, we obtain

$$(x)(\sim gx \rightarrow \sim fx). \tag{2.1.12}$$

Now $\sim gx$ is the subject class and it ought to be nonempty. Therefore, for a sentence of the form “~(Ex)(fx & ~gx)” to be either true or false, both “(Ex)(fx)” and “(Ex)(~gx)” must hold.

2.2 Truth Relevance

In 1981 Richard Diaz published a monograph in which he presented *truth-relevant logic* (Diaz, 1981, pp. 65-72.) It has striking similarities with the logic of presuppositions.

When evaluating formulae of classical propositional calculus, we often find ourselves using shortcuts such as

1. if p is false, then $p \rightarrow q$ is true, regardless of the value of q
2. if q is true, then $p \rightarrow q$ is true, regardless of the value of p

“There are even some formulae whose truth value may be determined in every valuation, even if we do not know the truth value assigned to one of its variables *in any valuation*. A case in point is $p \rightarrow (q \rightarrow p)$. Suppose p is true. Then $q \rightarrow p$ is true by shortcut 2, and hence $p \rightarrow (q \rightarrow p)$ is true, again by 2. If p is false, then by 1, $p \rightarrow (q \rightarrow p)$ is true” (Diaz, 1981: p. 65). The term q is not relevant to the determination of $p \rightarrow (q \rightarrow p)$. We will call this kind of relevance *truth-relevance*. A formula is truth relevant if all the variables occurring in it are truth relevant.

The shortcut tables for disjunction and conjunction are below. “x” stands for “uknown.”

v	F	x	T
F	F	x	T
x	x	x	T
T	T	T	T

Table 1.1

&	F	x	T
F	F	F	F
x	F	x	x
T	F	x	T

Table 1.2

And here is an example of how to use these tables to evaluate ~((*P* & ~*P*) & ~*Q*).

P	Q	~	((P	&	~P)	&	~Q)
T	x	T		F	F	F	x
F	x	T		F	T	F	x

Table 1.3

We do not need to use the truth value of *Q* to determine that the formula is a tautology.

“Of special interest is the set of tautologies that are also t-relevant.” (Diaz, 1981, p. 67.)

The tautologies that are *not* t relevant are the propositional counterparts of the “vacuously true” sentences of classical logic:

$$(P \,\&\, {\sim}P) \rightarrow Q,$$

$${\sim}((P \,\&\, {\sim}P) \,\&\, {\sim}Q),$$

$$({\sim}P \vee P) \vee Q.$$

An example of a t-relevant tautology is modus tollens:

$$(P \rightarrow \sim Q) \rightarrow (Q \rightarrow \sim P).$$

For more detail please see Newberry (2009).

2.3 Conclusion

We note the isomorphism between the tautologies that are *not* t-relevant and the formulas that are neither true nor false because their *subject class is empty or their predicate class is universal.*

Non t-relevant tautology	**Formula with an empty subject class**
$(P \,\&\, \sim P) \rightarrow Q$	$(x)((Px \,\&\, \sim Px) \rightarrow Qx)$
$\sim((P \,\&\, \sim P) \,\&\, \sim Q)$	$\sim(\mathrm{E}x)((Px \,\&\, \sim Px) \,\&\, \sim Qx)$

Non t-relevant tautology	**Formula with universal predicate class**
$R \rightarrow \sim(S \,\&\, \sim S)$	$(x)(Rx \rightarrow \sim(Sx \,\&\, \sim Sx))$
$\sim(R \,\&\, (S \,\&\, \sim S))$	$\sim(\mathrm{E}x)(Rx \,\&\, (Sx \,\&\, \sim Sx))$

The formulae on the right are but generalizations of the formulas on the left.

In all likelihood truth-relevant logic can be used as the basis for formalizing the logic of presuppositions.

3. When Are Relations Neither True Nor False?

When are relations neither true nor false? For example when is

$$(x)(y)\sim(Fxy \,\&\, Gxy) \tag{3.1.1}$$

neither true nor false? In classical logic,

$$(x)(y)Axy \tag{3.1.2}$$

is interpreted as follows.

For all a_i in the range of x, it is the case that

$$(y)Aa_iy \tag{3.1.3}$$

and, for all b_i in the range of y, it is the case that

$$(x)Axb_i. \tag{3.1.4}$$

When is (3.1.2) true in the logic of presuppositions? What if (3.1.3) is N for some a_i or if (3.1.4) is N for some b_i? ("N" stands for "~(T v F)".) We will simply "count" only those $(y)Aa_iy$ and $(x)Axb_i$ that are either true or false. The meaning of (3.1.2) is then defined by the following procedure. Collect all the $(y)Aa_iy$ and $(x)Axb_i$ that are either true or false. If all of them are true then (3.1.2) is true, if one is false then (3.1.2) is false. If there is no such $(y)Aa_iy$ or $(x)Axb_i$ then (3.1.2) is N. For formal semantics, please see Newberry (2013).

In the example in Figure 1, there are some

$$(y)(Fa_iy \,\&\, Ga_iy) \tag{3.1.5}$$

that are T v F. When a = t, then (Ey)Fay and (Ey)Gay. But there is no b_i in the range of y for which

$$(x)(Fxb_i \,\&\, Gxb_i) \tag{3.1.6}$$

is T v F.

In summary, (3.1.1) will be T v F iff there is an a_i such that

$$(\mathrm{E}y)Fa_iy \text{ and } (\mathrm{E}y)Ga_iy \tag{3.1.7}$$

and there is a b_i such that

$$(\mathrm{E}x)Fxb_i \text{ and } (\mathrm{E}x)Gxb_i. \tag{3.1.8}$$

Thus, it will be T v F if

$$(\mathrm{E}x)[(\mathrm{E}y)Fxy \ \& \ (\mathrm{E}y)Gxy] \ \& \ (\mathrm{E}y)[(\mathrm{E}x)Fxy \ \& \ (\mathrm{E}x)Gxy]. \tag{3.1.9}$$

This means that (3.1.1) will be T v F if F and G overlap along both axes. Figure 1 shows the case when (3.1.1) is ~(T v F), Figure 2 shows the case when (3.1.1) is true, finally Figure 3 shows the case when (3.1.1) is false. The asterisk means both F and G.

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Figure 1

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Figure 2

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Figure 3

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Let us now study a special case,

$$\sim(\mathrm{E}x)(\mathrm{E}y)(Fxy \;\&\; Gy), \tag{3.2.1}$$

such that only one $y = m$ satisfies Gy. For example let "Gy" be "$y = m$":

$$\sim(\mathrm{E}x)(\mathrm{E}y)(Fxy \;\&\; (y = m)). \tag{3.2.2}$$

There is no "x" at "G", but we can imagine that (3.2.2) is expressed as

$$\sim(\mathrm{E}x)(\mathrm{E}y)[Fxy \;\&\; ((y = m) \;\&\; (x = x))]. \tag{3.2.3}$$

The situation is depicted in Figures 4 and 5. Here there are only two cases. Either the two regions overlap (Figure 4) or they do not (Figure 5). We observe that (3.2.1) can be either false or neither true nor false; it can never be true. In case of (3.1.1), when the two regions did not overlap, there were two further subcases: either the formula was true (Figure 2) or it was neither true nor false (Figure 1.)

It is apparent that (3.2.1) will be ~(T v F) if

$$\sim(\mathrm{E}x)Fxm \tag{3.2.4}$$

is true. In this case the two regions will not overlap (Figure 5).

Let our domain be the set of natural numbers. Then

$$\boldsymbol{\sim(\mathrm{E}x)(\mathrm{E}y)[(x + y < 6) \;\&\; (y = 8)]} \tag{3.2.5}$$

is ~(T v F) (Figure 6). This is so because the two regions do not overlap along the x axis. Let us pick $y = 8$:

$$\sim(\mathrm{E}x)[(x + 8 < 6) \;\&\; (8 = 8)]. \tag{3.2.6}$$

We observe that

$$\sim(\mathrm{E}x)(x + 8 < 6); \tag{3.2.7}$$

that is, (3.2.6) is ~(T v F) analogously to (1.3); our logic is not classical. Let us pick, say,

$y = 4$:

$$\sim(\mathrm{E}x)[(x + 4 < 6) \;\&\; (4 = 8)]. \tag{3.2.8}$$

We observe that

$$\sim(\mathrm{E}x)(4 = 8); \tag{3.2.9}$$

that is, (3.2.8) is ~(T v F). It is apparent that for any choice of *y,* the corresponding sentence will be ~(T v F), hence (3.2.5) is ~(T v F). Nevertheless,

$$\boldsymbol{\sim(\mathrm{E}x)(x + 8 < 6)} \tag{3.2.10}$$

is true. **In the logic of presuppositions, (3.2.5) and (3.2.10) are *not* equivalent.**

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Figure 4

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Figure 5

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 5| F . . . . . . . . . . . . . . . . . . .
 4| F F . . . . . . . . . . . . . . . . . .
 3| F F F . . . . . . . . . . . . . . . . .
 2| F F F F . . . . . . . . . . . . . . . .
 1| F F F F F . . . . . . . . . . . . . . .
 0| F F F F F F . . . . . . . . . . . . . .
 ------------------------------------------------- x
    0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
```

Figure 6

* * * * * * *

Gödel's sentence has the same form as (3.2.1):

$$\sim(\mathrm{E}x)(\mathrm{E}y)(Prf(x,y)\ \&\ This(y)). \tag{3.3.1}$$

Prf(x,y) means that *x* is the proof of *y*, where *x* and *y* are Gödel numbers of wffs or sequences of wffs. *This(m)* has been constructed such that it holds just when *m* is the Gödel number of (3.3.1)

Assume that Gödel's sentence (3.3.1) is *not* derivable; that is, that

$$\sim(\mathrm{E}x)Prf(x,\ m) \tag{3.3.2}$$

is true. Then (3.3.1) is ~(T v F). Thus, if Gödel's sentence is not derivable, it is neither true nor false.

Let there be a hypothetical derivation system S that derives only true sentences in Strawson's sense. That is, it derives neither (2.1.10), (3.2.5) nor their negations. System S has gaps. It does not derive any of the "vacuously true" formulas of classical logic as indeed the logic of presuppositions does not regard these as true. The equivalent of Gödel's sentence in the hypothetical system S would be

$$\sim(\mathrm{E}x)(\mathrm{E}y)(Prf''(x,y)\ \&\ This'(y)). \tag{3.3.3}$$

A presupposition of (3.3.3) is

$$(\mathrm{E}x)Prf''(x,m'). \tag{3.3.4}$$

Let's now use our imagination and suppose that

$$\sim(\mathrm{E}x)Prf''(x,m') \tag{3.3.5}$$

is provable in S. (3.3.5) does two things: It asserts that (3.3.3) is unprovable, and it denies a presupposition of (3.3.3). But then (3.3.3) is neither true nor false. It is not surprising that it is not provable! Note the close similarity of this outcome with Goldstein and Gaifman's solution of the Liar paradox below.

4. Gaifman’s Solution of the Liar Paradox

The two-line puzzle is the launching point for the solution proposed by Haim Gaifman (2000). The sentences (4.1) and (4.2) below are two different sentence-tokens of the same sentence-type.

Line 1	The sentence on line 1 is not true.	(4.1)
Line 2	The sentence on line 1 is not true.	(4.2)

How do we evaluate a sentence of the form “The sentence on line *x* is not true”? Paraphrasing Gaifman (2000, p.3):

Go to line x and evaluate the sentence written there. If that sentence is true, then “The sentence on line x is not true” is false, else the latter is true.

When we evaluate the sentence on line 1 we are instructed to evaluate the sentence on line 1: We enter an infinite loop, and no truth value will ever be assigned to (4.1). Hence, (4.1) is neither true nor false. When we evaluate (4.2) we already know that the sentence on line 1 is not true, hence (4.2) is true. Thus (4.1) and (4.2) are assigned different truth values although their grammatical subjects have the same referent and their predicates the same extent.

I find Gaifman’s evaluation procedure quite convincing. Less convincing is his *unable-to-say paradox*, “which consists of our being unable to say that the line 1 sentence is not true, without repeating this very same sentence. It is the latter—the subject of this work—that necessitates an attribution of truth values to pointers” [Author: i.e. to tokens] (Gaifman, 2000, p.16). But consider the example below:

Line 1	The sentence on line 1 is not true.	(4.1)
Line 2	The sentence on line 1 is not true.	(4.2)
Line 3	The sentence next to the label "Line 1" is not true.	(4.3)

On line 3, we have replaced the grammatical subject of sentence (4.1) with a *synonym.* It is a different token than the sentence on line 1, but it is also a *different type*. Its grammatical subject has the same referent as the subject of (4.1), and its predicate has the same extent as the predicate of (4.1). Synonyms by definition have the same referent, but they alter the sentence-type. Let us now assume that the sentence-types are the truth bearers; that is, all the sentence-tokens of the same sentence-type have the same truth value. Then (4.1) and (4.2) have the same truth value, namely neither true nor false. But the sentence on line 3 is still true. It succeeds in expressing our conclusion, that sentence (4.1) is not true, without a contradiction. If we accept Gaifman's thesis that (4.1) and (4.2) are not equivalent when the sentence-tokens are the truth bearers, then we also ought to accept that (4.1) and (4.3) are not equivalent even if the sentence-types are the truth bearers.

When using a natural language, we indeed tend to interpret (4.2) as true. Nevertheless we do obtain a consistent system even if we take the sentence-types to be the truth bearers. For example, the construction of formal languages is greatly simplified when we adopt the convention that the sentence-types are the truth bearers.

* * * * * * *

Note that we can replace the grammatical subject of

This sentence is not true, (4.4)

with a synonym. For example,

"This sentence is not true" is not true. (4.5)

"This sentence" in (4.4) and ' "This sentence is not true" ' in (4.5) have the same referent, namely the sentence (4.4). The latter is true but the former is neither true nor false analogously to (4.1) and (4.3). More formally we can put

Y = “This sentence is not true”.

Then ~T(Y) is true and Y is neither true nor false. We have reached the same conclusion through different means in Newberry (2008, pp.8-10). A similar result is obtained by Lawrence Goldstein (1992, pp.1-2). Compare also (4.4), (4.5) with (3.3.1), (3.3.2).

5. Gödel and the Liar

We concluded in section 4 that given

Y = “This sentence is not true”, (5.1)

~T(Y) was true but Y was neither true nor false. In section 3, we postulated a hypothetical derivation system S of the logic of presuppositions that derived only the sentences that were true in Strawson’s sense (i.e., if their presuppositions were true). We have seen that the equivalent of Gödel’s sentence in the hypothetical system S would be (3.3.3). If (3.3.5) is provable then (3.3.3) is unprovable. We have an almost complete analogy with the Liar paradox. ("G" stands for Gödel’s sentence, “[G]” stands for Gödel number of “G”.)

Natural Lang.	Arithmetic	Value
Y	G	~T & ~F
F(Y)	(Ex)Prf(x, [~G])	F
T(Y)	(Ex)Prf(x, [G])	F
~T(Y)	~(Ex)Prf(x ,[G])	T

Table 5.1

But there is yet another parallel: Both (5.1) and (3.3.3) are meaningless. In section 1, we utilized the following definition:

Definition 1: A sentence is *meaningful* iff it or its internal negation expresses a possible state of affairs.

We further clarified that a state of affairs was possible if we could picture it to ourselves. Our main objective then was to account for the sentences that were meaningful yet neither true nor false. An example of such a sentence is

All John's children are asleep (5.2)

when John does not have any children. However, meaningless sentences also exist. For example

All round squares are large (5.3)

The sentence above does not express any possible state of affairs. It is possible for John's children to exist, but it is impossible for round squares to exist. The meaningless sentences are a proper subset of the sentences that are neither true nor false. The following sentences are meaningless as well.

$$(x)((Px \ \& \ {\sim}Px) \ {-}{>} \ Qx) \tag{5.4}$$

$$(x)((2>x>4) \ {-}{>} \ (x > x+1)) \tag{5.5}$$

$$\sim(Ex)(Ey)[(x + y < 6) \ \& \ (y = 8)] \tag{5.6}$$

Gödel's sentence (3.3.3) is meaningless if it is not derivable.

The Liar sentence is equally meaningless. Suppose I have a basket with a red apple. I form the sentence

A: The apple is red

This sentence is meaningful. To determine if it is true or false I have to compare the possible state of affairs it expresses with the actual state of affairs. Similarly in case of

B: The apple is green

If I did not understand the possible state of affairs it expressed (i.e., its meaning) I would not know what to compare it with or how to verify it. I could perhaps verify A but not falsify B. This would lead to the absurd conclusion that only true sentences were meaningful. The verification principle as a criterion of meaningfulness is too restrictive, But this does not mean that there should not be a criterion.

The existence and properties of sentences are also facts, and we can form sentences about sentences. For example

C: A is true

To determine if C is true, we apply

> **Definition 2:** A sentence is *true* iff the possible state of affairs it expresses corresponds to an actual state of affairs.

C expresses the possible state of affairs that the possible state of affairs expressed by A corresponds to the actual state of affairs. In short C is true if A is true. What if A is not true? We apply

> **Definition 3:** A sentence is *false* iff the possible state of affairs expressed by its internal negation corresponds to an actual state of affairs.

The internal negation of "A is true" is "A is not true." A can be false, neither true nor false or even meaningless. In all these cases, C is false, see Table 5.2.

A	**C**
T	T
N	F
F	F

Table 5.2

In general to evaluate a sentence of the form 'The sentence X is true' we would follow this procedure:

Go to the label X and evaluate the sentence written there. If that sentence is true, then "The sentence X is true" is true, otherwise it is false.

Using our definitions of truth, we have arrived at an exact replica of Gaifman's evaluation procedure (Gaifman, 2000, p.3). Let us apply it to

D: The sentence D is not true

In order to determine if D is true we need to determine if D is true. We are in an infinite loop and no truth value will be assigned to D. What possible state of affairs does D express? It "expresses" that the possible state of affairs that D expresses does not correspond to an actual state of affairs. An attempt to find the meaning of D leaves us in an infinite loop as well. We are not able to picture to ourselves the state of affairs D expresses. It does not express any. D is meaningless just like (5.3).

Sentence D is clearly not verifiable. Even if we use a relaxed criterion of meaningfulness such as Definition 1, D is still meaningless. Yet it appears to say that the sentence D is an element of the set of untrue sentences. Furthermore D indeed *is* an element of the set of untrue sentences. But in fact D does *not* say that it itself is an element of the set of untrue sentences because ... well ... because it is meaningless. The algorithmic procedure establishes that D is meaningless, therefore the compositional semantics is not applicable. Haim Gaifman may have put it best when he said: "*if the sentence fails to express a proposition it does not, contrary to appearance, say of itself that it is not true*" (Gaifman, 2000, p.16) Contrary to appearances, (3.3.3) does *not* say that it itself is not derivable. It says the same thing as (5.5), that is nothing. It has no truth value; it ought not to be derivable. In S there is no separation of truth and derivability.

Gaifman (2000, p.15) prefers to say that the Liar sentence is meaningful but that it does not express a proposition. The meaning of the Liar sentence is the *evaluation procedure* that reveals that the sentence has no truth value. This procedure is the *sense* of the sentence states Gaifman. What about its reference? Gaifman does not say. I can only conclude that he would agree that the Liar sentence (or token) has *no interpretation in the domain of sentences.* But is this not what we usually mean by "meaning"? Goldstein and Blum (2008, p.5) argue: "*So we can avoid all the unpalatable conclusions if we acknowledge that there is a sentence, the Liar sentence, that is grammatically correct and meaningful (we can, after all, understand it and translate it into French) but deny that it can be used to make a statement.*" The point is not well taken. Firstly, why can we not argue that if we understand it then it must be making a statement? Secondly, it does not follow that if all the components of a sentence have referents then the whole sentence does. That is there are cases such that even if there *is an object* corresponding to each term of a sentence, there is *no state of affairs* that corresponds to the whole sentence. This is the case whether the terms are in French or in English. And I would dispute that we actually understand the sentence. For one, it causes a tremendous amount confusion. It *seems* to be saying - and this is the source of the confusion - that the liar sentence does not belong to the set of true sentences, but it *does not*! We have a tendency to erroneously project the meaning of the sentence (4.3) into (4.1). But that does not mean that we "understand" it. The construct of the *proposition* or *statement* is not necessary to establish that Y does not "say" anything.

Conclusion

I have outlined a simple theory of meaning and truth, which is compatible with Strawson's logic of presuppositions. (Contrary to the current trends it is a thesis of this paper that while the verification principle is too restrictive a *criterion of meaningfulness is required*.) When this theory is applied to self-referential sentences it yields the same evaluation as the one proposed by Haim Gaifman. The sentence

X: The sentence X is not true

is evaluated as follows:

Go to the label X and evaluate the sentence written there. If that sentence is true, then "The sentence X is not true" is false, else the latter is true.

Since the procedure never ends no truth value is assigned to X, and as a result the Liar sentence is neither true nor false.

Strawson's logic of presuppositions yields the result that a sentence is neither true nor false if its subject class is empty, i.e. if ~(Ex)Fx then (x)(Fx -> Gx) is neither true nor false. When brought to its logical conclusions it implies that (x)((Px & ~Px) -> Qx) is neither true nor false. We observe that such a logic is but a quantified version of Diaz's truth-relevant logic. It can be generalized to sentences with many variables, and we find that then Gödel's sentence is neither true nor false. This outcome parallels the result we obtained above for the Liar sentence.

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