4

From Quantum Entanglement to Spatiotemporal Distance

Alyssa Ney

Abstract

There is an influential research program in quantum gravity developing the connection between quantum entanglement and spatiotemporal distance. Through a series of well-confirmed results, it has been shown how these facts about the entanglement entropy of component systems may be connected to facts about spatiotemporal distance. Physicists are seeing these results as yielding promising methods for better understanding the emergence of (the dynamical) spacetime from more fundamental quantum theories, and for the development of a nonperturbative theory of quantum gravity. However, to what extent does the case for the entanglement entropy-distance link provide evidence that spacetime structure is nonfundamental and emergent from nongravitational degrees of freedom? I will show that a closer look at the results lends support only to a weaker conclusion: that the facts about quantum entanglement are constrained by facts about spatiotemporal distance, and not that they are the basis from which facts about spatiotemporal distance emerge.

4.1 Introduction

Within the field of quantum gravity, there is an influential research program developing the connection between quantum entanglement and spatiotemporal distance. Quantum information theory gives us highly refined tools for quantifying quantum entanglement. A common measure is the von Neumann entropy S, which can be seen as a quantum analog of the Shannon entropy (Preskill 1998, Nielsen and Chuang 2016). For a state with density matrix ρ,

$$S(\rho) = -\operatorname{tr}(\rho \log \rho). \tag{4.1}$$

Where the density matrix is diagonalizable, with eigenvalues λ_i, the von Neumann entropy may also be expressed as

$$S(\rho) = \sum_i \lambda_i \log \frac{1}{\lambda_i}. \tag{4.2}$$

The von Neumann entropy gives one a measure of how entangled a system is. So, for example, if we consider a bipartite system σ_{AB}, we may ask how entangled are its parts, σ_A and σ_B. In this case, we can refer to $S(\rho_A)$ and $S(\rho_B)$ as the parts' entanglement entropies, where ρ_A and ρ_B are the reduced density matrices of the system's parts. Suppose ρ_{AB} is a pure state. The von Neumann entropy for any pure state is identically zero. If the joint state of the system is a product state so that $\rho_{AB} = \rho_A \otimes \rho_B$, then the entanglement entropy for either part, $S(\rho_A)$ or $S(\rho_B)$, will also be zero. On the other hand, for an entangled system, the entanglement entropy of the parts will not be zero. Indeed, depending on the degrees of freedom of the system, there will be a maximum value of the entanglement entropy associated with the maximally entangled state. For a system of qubits, this will be the value associated with the Bell states, $\psi_{AB} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ and $\psi_{AB} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$.

Through a series of well-confirmed results, it has been shown how these facts about the entanglement entropy of component systems may be connected to facts about spatiotemporal distance.[1] Physicists are seeing these results as yielding promising methods for better understanding the emergence of (the dynamical) spacetime (of general relativity) from more fundamental quantum theories, and moreover as promising for the development of a nonperturbative theory of quantum gravity. These results appear to be part of a more general convergence forming from several different approaches to quantum gravity (string theory, spin foams, causal sets) around the idea that spacetime, dynamical spacetime of the kind found in general relativity, is an emergent phenomenon, a phenomenon that arises in some sense or other from a more fundamental, nongravitational description (Carlip 2012, Huggett and Wüthrich 2013, Oriti 2021). However, to what extent does the case for the entanglement entropy-distance link provide evidence that spacetime structure is nonfundamental and emergent from nongravitational degrees of freedom? I will show that a closer look at the results lends support only to a weaker conclusion: that the facts about quantum entanglement are

[1] We may note, in anticipation of this fact, that entanglement entropy will be connected with spatiotemporal distance, and that this quantity has certain features that make it apt to serve as a distance relation itself. In particular, it satisfies the triangle inequality: For a joint state ρ_{AB}, $S(\rho_{AB}) \geq |S(\rho_A) - S(\rho_B)|$ (Nielsen and Chuang 2016, 515–516). On the other hand, since, for example, $S(\rho_{AB})$ can equal zero even when $\rho_A \neq \rho_B$, the entanglement entropy has other features that make it seem unlike a distance relation.

constrained by facts about spatiotemporal distance, and not that they are the basis from which facts about spatiotemporal distance emerge.

4.2 Constructing the Metric from Entanglement Entropy in the AdS/CFT Context

Let's start by looking at results connecting facts about the spacetime metric with facts about quantum entanglement that have been motivated from within the string theory paradigm. These follow first on Maldacena's (1999) discovery of the AdS/CFT correspondence. Maldacena shows how one can establish a holographic correspondence between pairs of theories.[2] For, example one may consider first a gravitational theory, a string theory on anti-de Sitter (AdS) spacetime in $d+1$ dimensions; and next a theory without gravitational degrees of freedom, a conformal field theory, a type of quantum field theory with a kind of scaling symmetry (CFT), defined on a flat spacetime in d dimensions (see Fig. 4.1).

In this model, one may view the AdS description as defining a bulk spacetime for which the CFT lies on its boundary. Maldacena shows how one can translate back and forth between the description of the $(d+1)$-dimensional spacetime in the bulk and the d-dimensional spacetime on its boundary. This work has been extremely influential and so it is a good place to start to look for mainstream arguments on the link between quantum features and spacetime structure. Even though we don't ourselves inhabit an AdS spacetime, this correspondence shows us how one might, for at least one case, derive facts about a curved spacetime, and hence gravity, from a quantum theory without it.

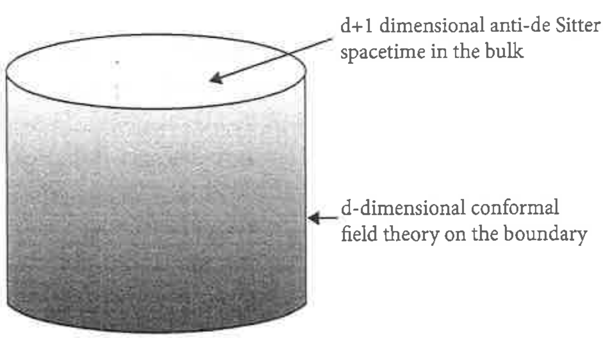

Fig. 4.1 AdS/CFT correspondence

[2] The explicit connection with holography was made in Witten (1998).

I want to look at a particular line of research that has started within the AdS/CFT context, work that aims to derive facts about the AdS metric using particula facts about the CFT on the boundary; in particular, facts about the entanglemen entropy of particular states living on the boundary.[3] In 2006, Ryu and Takayanag asked us to consider the following. First, take a CFT boundary and divide it int two regions, A and its complement, $\bar{A}$ (see Fig. 4.2).

We may then consider the reduced density matrices ρ_A, which describes th state on region A, and $\rho_{\bar{A}}$, which describes the state on the complement region $\bar{A}$ Given these density matrices, we can then compute their entanglement entropy Assuming the total boundary region is in a pure state, $S(\rho_A) = S(\rho_{\bar{A}})$. Now consider the minimal surface running through the AdS bulk that connects the tw edges dividing region A from its complement. Call this surface γ_A (see Fig. 4.3).

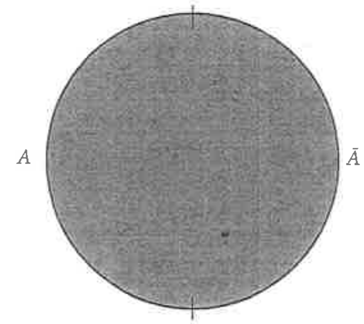

Fig. 4.2 Ryu–Takayanagi cut

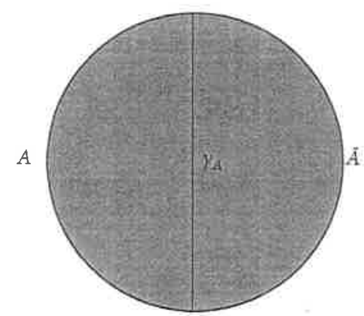

Fig. 4.3 Minimal surface

[3] Rickles (2013), Teh (2013), Dieks et al. (2015), Read (2016), De Haro (2017, 2020), Butterfiel (2021), and Huggett and Wüthrich (forthcoming) are prior philosophical works on the AdS/CFT correspondence. Jaksland (2020) discusses the Ryu–Takayanagi conjecture.

The Ryu–Takayanagi conjecture is that a particular relationship exists between the area (or in the $d = 1$ case, length) of γ_A and the entanglement entropy of the region on the boundary A:

$$S(\rho_A) = \frac{\text{area of } \gamma_A}{4G_N^{(d+1)}}, \tag{4.3}$$

where $G_N^{(d+1)}$ is Newton's constant for the relevant $(d + 1)$-dimensional space.[4] Those familiar with work on black hole thermodynamics will immediately recognize that this relationship has the same form as the Bekenstein–Hawking formula that relates the entropy of a black hole to the area of its event horizon (Bekenstein 1973, Hawking 1975):

$$S_{BH} = \frac{\text{area of the event horizon}}{4G}. \tag{4.4}$$

This is something we will come back to. Ryu and Takayanagi motivate their conjecture by appeal to this earlier work on black hole entropy, and in particular the explanation of why this formula holds developed by Sorkin and his collaborators (Sorkin 1983, Bombelli et al. 1986).

We will discuss below how Ryu and Takayanagi establish their conjecture. For now, it is just worth noting how influential this work has been in motivating a research program in quantum gravity, working within the AdS/CFT context, for seeing the connection between facts about quantum entanglement (entanglement entropy) in a quantum field theory without gravitational degrees of freedom, and facts about the metric in a curved (AdS) spacetime. To begin, Hubeny et al. (2007) generalized the result from one about a static region of the AdS spacetime to more general regions, providing a covariant account of AdS distances in quantum information theoretic terms. Later work asked whether it was possible to recover more of the AdS metric from the entanglement properties of regions on the boundary, beyond the area of the minimal surfaces γ_A. That is, how far can we succeed in 'bulk reconstruction', i.e. in gaining an understanding of "how these degrees of freedom, which are well-understood from the perspective of nongravitational quantum field theory, reorganize themselves (in an appropriate limit) into a manifestly local gravitational theory" (Bao et al. 2019, 1)? Indeed, for a complete reconstruction, we would want not only to extract the metric of the bulk, but also the dynamics.

The work on 'hole-ography' of, for example, Balasubramanian et al. (2014), Headrick et al. (2014), and Myers et al. (2014) aims to construct the metrical properties of closed surfaces embedded in the AdS space from entanglement

[4] This is expressed using units where $\hbar = c = k_B = 1$.

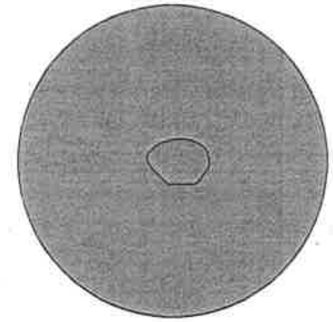

Fig. 4.4 Closed surface in the AdS bulk

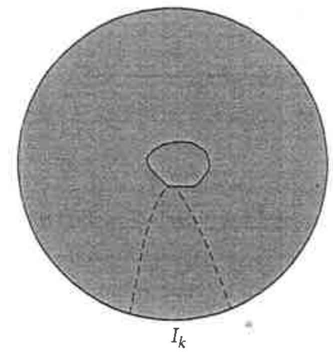

Fig. 4.5 Tangent curve extended to the CFT boundary

entropies on the boundary field theory. The strategy can be (loosely) explicated as follows. Take a closed surface in the AdS bulk (see Fig. 4.4).

Now find geodesics k that are tangent to each region on that closed surface and extend them out to the CFT boundary to get a boundary region I_k (see Fig. 4.5). The idea then is to argue that one can recover the surface's area in terms of the entanglement entropies on the boundary using:

$$E = \sum_{k=1}^{n} \left[S(I_k) - S(I_k \cap I_{k+1}) \right], \tag{4.5}$$

where E is what Myers et al. (2014) refer to as the differential entropy. The area of the surface is identified with the limit of this quantity as $n \to \infty$.[5]

[5] Differential entropy is one of many entanglement measures that may be used to probe the area of such a bulk surface. Hubeny (2014) argues that it has several limitations. In particular, as Myers et al. (2014) acknowledge, the original proposal is only well defined for three-dimensional, pure AdS geometries, and only there where the bulk surface is not too 'wiggly'. Myers et al. modify the proposal

These developments led Keeler and her collaborators to argue that entanglement entropies on the boundary are sufficient to uniquely fix the metric (up to diffeomorphism) everywhere in the neighborhood of the extremal surfaces (Bao et al. 2019). Although for the generic case, one will not be able to rule out the existence of 'entanglement shadows' in the AdS bulk, regions where the metric is left undetermined by the entropy of boundary regions, perhaps because there are regions which extremal surfaces can't cross because of singularities (cf. Headrick et al. 2014, 38), large steps have already been taken to capture a lot of information about the AdS metric in terms of quantum entanglement on the CFT boundary. Explicit reconstructions have already been successful for bulk spacetimes with large-scale symmetries (Rangamani and Takayanagi 2017).

4.3 The Entanglement Entropy-Distance Link Beyond AdS/CFT: Area Laws

As noted, our universe doesn't have an AdS geometry, and so one might ask whether these results are confined to the AdS/CFT context, or whether we may have justification for seeing a connection between entanglement entropy of the parts of a system and distance in the more general case. Let us back up and reframe a bit what we are after. So far, what we are seeing is the establishment of a series of area laws. Although in classical thermodynamics we are familiar with the idea that entropy is an extensive quantity, scaling with volume, this is not so for the cases we have been considering. In the case of black hole thermodynamics and the Ryu–Takayanagi conjecture, (Bekenstein–Hawking or entanglement) entropies scale with area. And this is what allows us to relate facts about entanglement entropy to facts about the metric.

The work we have just discussed argues for a direct correspondence between the entanglement entropy of a CFT state A on the boundary and the area of surfaces through the AdS bulk separating A from its complement. We should probe whether this connection between the entanglement entropy of a region A and the area of the surface separating it from its complement may be generalized from the AdS/CFT context to more general quantum systems. Indeed, there are indications that area laws may hold more generally outside of the AdS/CFT context, and thus there is a research program devoted to finding the conditions under which they hold. Again, this is inspired by the Bekenstein–Hawking law for black hole entropy.

to apply to more generic spacetimes, but Hubeny stresses that even the revised measures are defined only on static geometries. Her paper (2014) proposes a more robust measure: the covariant residual entropy. This is inspired by the earlier discussion of residual entropy in Balasubramian et al. (2012).

Although it is familiar from the consideration of Bell states in quantum mechanics that quantum entanglement/correlations may persist over indefinitely long spatial distances, interactions in quantum many body systems are typically local. That is, subsystems typically interact over short distances with only a finite number of neighboring subsystems. There is an intuition that interactions are short-range because these correlations are enacted through the interface surfaces[6] between regions. An area law analogous to what we've already seen provides support for this intuition.

In a survey article, Eisert et al. (2010) develop the case for area laws by illustration with systems representable as a lattice, in particular one-dimensional systems where the entire system may be represented as a chain of quantum systems L, and we focus on the entanglement entropy of a part of this chain, e.g. the block I consisting of its first n members: $\{1,\ldots n\}$. If the entanglement entropy scaled with volume, then this would mean that the entanglement entropy of I, $S(\rho_I)$ would be proportional to the cardinality of the block, i.e. n. However, it may be shown instead, for a many body quantum system, that $S(\rho_I)$ scales with the area of the boundary.[7]

For such a one-dimensional chain, the boundary is very simple. The boundary is one or two parts of the block, depending on the boundary conditions imposed. And so, we obtain an area law for the entanglement entropy just in case $S(\rho_I)$ is independent of the block size and instead is of order 1:

$$S(\rho_I) = O(1) \tag{4.6}$$

Eisert et al. emphasize that their claim is not that area laws always hold for many body quantum systems. Rather, one fact determining whether or not an area law holds is whether the system is at a quantum critical point or not. For systems near criticality, the entanglement varies with the length of the chain. And so, what we have is not an area law, but a volume law. This is not surprising given what is known about the breakdown of locality in critical systems in thermodynamics. In what follows, I summarize the argument, eliding most of the technical details.

Consider a bosonic harmonic chain. In this case, the elements of the chain are modeled as discrete versions of complex Klein–Gordon fields; that is, as spinless

[6] I use 'interface surface' here to make a distinction with the 'boundary surface' in the AdS/CFT context. What we discuss here as the interface surface corresponds to the black hole's event horizon or Ryu and Takayanagi's minimal extremal surface.

[7] It is worth noting that the notion of area here is not quite the same as in the case of the Ryu–Takayanagi area law. The area in question here is that of the boundary between the two subsystems. In the latter case, it is the area of the dual bulk surface that ends on the boundary between the two entangled subsystems.

particles with charge. We write down a Hamiltonian for this system in terms of the canonical operators x and p, and system operators X and P which describe the coupling structure. It turns out that even in such one-dimensional systems, simple though they are, the entanglement entropy is quite difficult to compute. However, there is another entanglement measure, the logarithmic negativity E_N, that is more tractable and which provides an upper bound for the entanglement entropy (at least for pure states).

Following Eisert et al., consider the simplest case where the chain L is bisected symmetrically into two parts. We will assume the Hamiltonian describes parts of the system as only able to directly interact with their nearest neighbors, so interactions are local. Here we have a gapped system, one in which there are discrete energy gaps above the ground state that depend on the coupling strength. In this case, Audenaert et al. (2002) proved that:

$$S(\rho_I) \leq E_N(\rho, I) = \frac{1}{2}\log\left(\frac{\|X\|^{1/2}}{\lambda_{\min}^{1/2}(X)}\right), \tag{4.7}$$

where $\|X\|$ is the norm of the system operator X. This was the first rigorous area law for a quantum lattice system. As one can see, the upper bound for entanglement entropy does not depend on the size of the subsystem I. It is thus independent of the block size. This result has been extended to simple higher-dimensional systems, e.g. for quasi-free systems of bosons or fermions arranged on a lattice. For these systems, unlike for the one-dimensional case, the boundary between subsystems is nontrivial. Again, one can calculate the logarithmic negativity using system operators to find that the entanglement entropy scales with the boundary area. Thus, Eisert et al. draw from this work the conclusion that under the assumption of local interactions, entanglement tracks the area of the surface separating the two subsystems. As they put it, describing the import of (4.7):

> This result suggested that the locality of the interaction in the gapped model is inherited by the locality of entanglement, a picture that was also confirmed in more generality. (2010, 5)

In the case of criticality, just as for thermal systems, we lose the locality of interactions, and so the assumptions used for deriving an area law fail. Entanglement entropy, in such cases, scales with volume, as we would otherwise expect by analogy with the thermal entropy.

More recent work by Cao et al. (2017) builds on these proofs of area laws using them (again, outside of the AdS/CFT context) to argue that we can see the spacetime metric as emerging from facts about entanglement entropy. Or, more accurately, Cao et al. (2017) use the quantum analog of the mutual information,

$I(A:B)$, which is defined in terms of the entanglement entropy, and link this with facts about spatial distance (Nielsen and Chuang 2016, 514):

$$I(A : B) \equiv S(\rho_A) + S(\rho_B) - S(\rho_{AB}). \tag{4.8}$$

The spacetime metric for Cao et al. emerges from relations of mutual information between parts of the total quantum state in Hilbert space. Eliding the details, we can just note that again, the inference isn't direct, but mediated by a claim about local interactions:

> The mutual information between regions is a measure of how correlated they are. It provides a useful way of characterizing the 'distance' between such regions *because of its relation to correlation functions between operators*. We expect that in the ground state of a field theory, correlators of field operators will decay as exponentials (for massive fields) or power laws (for massless ones). The mutual information may reflect this behavior, as it provides an upper bound on the correlation function between two operators. (2017, 4, my emphasis)

So again, for Cao et al., it is in the assumption of a local field theory that the relevant area laws obtain. We will return to this locality assumption shortly.

4.4 From Correspondence to Emergence?

The work described so far is very suggestive and, even if this is not always explicit as it is in the work of Cao et al., the establishment of such links between entanglement measures and metrical features has held out the promise for many that one can see spacetime as emerging from facts about entanglement entropy. We certainly have, in these several cases, a bridge that can take us, in certain well-defined contexts, back and forth between entanglement facts and facts about spatiotemporal distance or area. But is there reason to see the one set of facts as emerging from the other?[8]

Rickles (2013), Teh (2013), and Crowther (2014), in earlier discussions of the AdS/CFT correspondence, have warned us to be cautious before moving from

[8] As is standard in this literature, I will continue to use the language of emergence. But for these purposes, I could just as easily have framed the discussion in terms of whether we should see there being a reductive link, whether the facts about spacetime metric are reducible to facts about quantum entanglement. The central question is whether we should view the spatiotemporal facts not merely as derivable from facts about quantum entanglement, but as arising from the facts about quantum entanglement, whether the facts about quantum entanglement provide the explanation of in virtue of what the spatiotemporal facts obtain.

the existence of such a duality or bridge to a claim of emergence. And this is important: although the work we have discussed may enable us to move back and forth between two theoretical frameworks, this is compatible with the holding of several possible inter-theoretic relationships:

1. Facts about the metric emerge from fundamental facts about entanglement entropy.
2. Facts about the entanglement entropy emerge from fundamental facts about the metric.
3. Facts about the metric and entanglement entropy are equally fundamental.
4. Facts about the metric and entanglement entropy are neither fundamental, but both emerge from some more fundamental, as yet undiscovered, framework.[9]

Following Teh, we must recognize that, before we can be justified in adopting a claim of emergence (of facts in theory B from facts in theory A), we must establish that the A-facts are more fundamental—that they provide the explanation of in virtue of what the B-facts arise. Teh (2013) acknowledges that in the AdS/CFT context, often, because it is easier to define the quantum field theory than the gravity theory, there is reason to view the CFT as more fundamental (cf. Horowitz 2005). But on the other hand, others are sympathetic to the possibility that the AdS and CFT theories both emerge from some more fundamental theory (Teh 2013, 309).

Another issue that needs to be considered that is relevant to the question of emergence concerns whether it is reasonable to believe the AdS/CFT correspondence, and in particular the connection between entanglement entropies and spacetime quantities, is exact. Dieks et al. (2015, 208) note that although the AdS/CFT duality is often treated as an exact correspondence, it is not clear whether this is actually so.[10] If it is not exact, then experiments could, in principle, decide the question of which of the two theories is the more fundamental and settle a question of emergence. In the case in which it turns out the correspondence is exact, Dieks et al. are more skeptical that what we have is a situation in which either theory is more fundamental:

> in this case that the two theories completely agree on everything that is physically meaningful, the two sides of the duality should be taken as different representations of one and the same physical theory. (2015, 209)

[9] This is the alternative both Rickles (2013) and Crowther (2014) express sympathy for when discussing the AdS/CFT correspondence.

[10] Harlow (2020) argues that this is confused, and that the AdS/CFT duality is exact. What is not exact, Harlow notes, is the relationship between the facts on either side of this duality and facts in some low-energy effective gravitational field theory.

Today, there is no established consensus on whether one side of the AdS/CFT duality is more fundamental, or indeed whether neither is more fundamental than the other. And so, I argue, we shouldn't be basing a judgment of which of (1)–(4) is correct on what is the scientific consensus about AdS/CFT. Moreover, some have argued that even if it is reasonable to think the AdS/CFT correspondence supports a claim of emergence of spacetime from a more fundamental quantum theory, it doesn't itself explain *how* dynamical spacetime might emerge from a quantum field theory on flat spacetime (cf. Van Raamsdonk 2010). It is precisely here that concepts from quantum information theory are supposed to come to the rescue. The connection with quantum information theory may provide insights on which of (1)–(4) best describes the metaphysical structure of our world, insights that may generalize beyond the AdS/CFT context.

4.5 Gluing with Entanglement?

As just mentioned, Van Raamsdonk (2010) notes that just from the AdS/CFT correspondence alone, we can't see how dynamical spacetime emerges from a nongravitational quantum field theory. However, he believes that if one appeals to quantum information theoretic facts about entanglement measures, then it can be made clear how what we have is a case of emergence. His argument works by considering two cases. The first involves a set of disconnected AdS spacetimes that we can then regard as connected by virtue of the entanglement relations in their dual CFTs. The second case involves a single vacuum CFT in which we gradually remove the entanglement between its component degrees of freedom. As this happens, we can see the corresponding regions in the dual AdS spacetime as becoming pushed away from each other until, when their mutual information goes to zero, the distance between them becomes infinite, and they effectively pinch apart.

These cases are extremely suggestive of the idea that there is a genuine explanatory connection between the metrical features of an AdS spacetime and the entanglement entropies of its dual quantum field theory. And especially in Van Raamsdonk's discussion of the first case, we see a good reason to think that there isn't merely a correlation between the two theories, but that one set of facts is arising because of the other. However, as I will explain, it is not clear to me that these cases are illustrating emergence as in Option (1) of the previous section.

Let's begin with the second case first, as it's simpler, consisting of just one CFT and its dual global AdS spacetime. Here, Van Raamsdonk's discussion of the case provides a fascinating picture of how the entanglement of degrees of freedom in a CFT track distance in its holographic dual (see Fig. 4.6).

Consider a point x_A located on the edge of the bulk near the midpoint of region A on the boundary, and another point $x_{\bar{A}}$ located on the other edge of the bulk

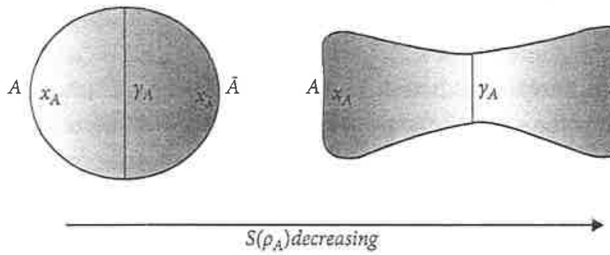

Fig. 4.6 Minimal surface variation

near the midpoint of region $\bar{A}$. What we see is that as the entanglement between region A and its complement is decreased, the area of the region through the bulk AdS spacetime separating A from $\bar{A}$, what we earlier denoted γ_A, decreases, and the distance between points x_A and $x_{\bar{A}}$ increases. Eventually as $S(\rho_A) \to 0$, x_A and $x_{\bar{A}}$ pinch off from one another altogether.

So Van Raamsdonk's second case illustrates the existence of a correlation between entanglement entropy and distance. Indeed, this correlation is a natural one given that as the entanglement entropy decreases, the mutual information $I(A:\bar{A})$ also decreases, and (as Cao et al. 2017 also note) the mutual information has several features (positivity, symmetry, the obeying of a triangle inequality) that make it particularly apt to be tracking distance relations. But what reason is there to think this correlation is due to the *emergence of the facts about distance from the facts about entanglement*? Van Raamsdonk's case for the correlation and corresponding images comes simply from an appeal to the Ryu–Takayanagi conjecture (2010, 3). And so, to evaluate whether this connection comes from the fact that the spatiotemporal relations are emerging from entanglement on the boundary (in support of my Option (1)), we must go back to see what justifies the Ryu–Takayanagi area law in the first place.

We will return to that momentarily, but first let us consider Van Raamsdonk's first case in which we see a pair of disconnected AdS spacetimes becoming spatiotemporally connected, 'glued', using quantum entanglement. First, Van Raamsdonk asks us to consider a pair of CFTs, which we may call CFT_1 and CFT_2, each defined on $S^3 \times \mathbb{R}$. In a product state, their combined quantum state may be written as a tensor product, $\rho_{AB} = \rho_A \otimes \rho_B$. In this case, there is no entanglement between the degrees of freedom of the two CFTs. And so their individual entanglement entropies are identically zero. Here, CFT_1 and CFT_2 describe two separable systems, and so, Van Raamsdonk argues, the dual AdS spacetimes AdS_1 and AdS_2 are disconnected.[11]

[11] One might already question this inference, at least insofar as it is supposed to generalize beyond the case of AdS/CFT, where we may assume each CFT gives a complete description of an AdS spacetime.

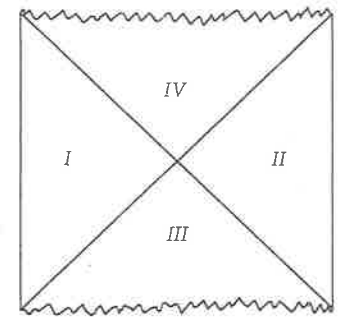

Fig. 4.7 Eternal AdS black hole spacetime

However, what if we take two such unentangled CFTs and combine them into a quantum superposition? Van Raamsdonk asks us to consider the following superposition of product states:

$$|\psi\rangle = \sum_i e^{\frac{-\beta E_i}{2}} |E_i\rangle \otimes |E_i\rangle. \tag{4.9}$$

This state, the Hawking–Hartle state, has been shown by Maldacena (2003) to correspond to a classically connected spacetime, the eternal AdS black hole spacetime, which can be represented by the above Penrose diagram (Fig. 4.7): Here, diagonal lines represent the light cone structure. Regions I and II correspond to the duals of the reduced states for CFT_1 and CFT_2, respectively. Jagged lines represent the past and future singularities.

Van Raamsdonk concludes how remarkable it is that such a complicated quantum state can give rise to a spacetime with such a simple geometric description in classical terms:

> ...we have a remarkable conclusion: the state $|\psi\rangle$ which clearly represented a quantum superposition of disconnected spacetimes may also be identified with a classically connected spacetime. In this example, *classical connectivity arises by entangling the degrees of freedom in the two components.* (2010, 2)

But is this the correct order of explanation? In the eternal AdS black hole spacetime, the causal futures of the two asymptotic regions AdS_1 and AdS_2 intersect past the event horizon. This is their joint causal future, Region IV. As Van Raamsdonk notes, it follows from the AdS/CFT correspondence that if we take a measurement

Outside of the AdS/CFT context, we have no problem conceiving separable states of quantum systems where the component states ρ_A and ρ_B inhabit a common spacetime.

on a state described by CFT_1, the dual of the result of that measurement will eventually enter region IV. This is the causal future of the dual to CFT_2. This suggests that measurement of CFT_1 affects the state of CFT_2, and supports the claim that the two are entangled. Thus, it seems it is the spatiotemporal connection between AdS_1 and AdS_2 that partially grounds the entanglement between the dual CFT_1 and CFT_2. But if this is correct, then Van Raamsdonk's discussion of the eternal AdS black hole seems most suggestive of the idea that the order of explanation goes in the opposite direction than what he wants. It is the spacetime structure that explains why it is that the measurement in the dual CFT_1 has an effect on the measurement in the dual CFT_2, and so explains the entanglement of the two component states. If this is right, then it is not that the geometric facts emerge from the quantum facts, but rather that the former constrain the latter.

Return to the four metaphysical options presented in Section 4.4.

1. Facts about the metric emerge from fundamental facts about entanglement entropy.
2. Facts about the entanglement entropy emerge from fundamental facts about the metric.
3. Facts about the metric and entanglement entropy are equally fundamental.
4. Facts about the metric and entanglement entropy are neither fundamental, but both emerge from some more fundamental, as yet undiscovered, framework.

My claim is that Van Raamsdonk's first case does not support Option (1) over its rivals, even if it does support the existence of an explanatory connection between facts about entanglement entropy and facts about the metric. The explanation of the entanglement relations in this case presupposes facts about the metric.

4.6 Back to Ryu–Takayanagi and Black Hole Thermodynamics

Let's try to seek out arguments for the emergence claim that are more promising by returning to the Ryu–Takayanagi conjecture and seeing in the first place why this conjecture turns out to work.

Recall again that Ryu and Takayanagi propose the following area law:

$$S(\rho_A) = \frac{\text{area of } \gamma_A}{4G_N^{(d+1)}}, \tag{4.10}$$

where $G_N^{(d+1)}$ is Newton's constant for the relevant $(d+1)$-dimensional space. It's easiest to understand the case for this area law in the context of a specific physical

scenario. Let's consider the simplest case of $d = 2$. Here we are considering an AdS_3 gravity theory that is dual to a $(1+1)$-dimensional CFT. For an AdS_3 spacetime with radius of curvature R, the dual CFT has central charge $c = 3R/2G_N^{(3)}$.

Now note that the metric of the AdS_3 spacetime is:

$$ds^2 = R^2\left(-\cosh\rho^2 dt^2 + d\rho^2 + \sinh\rho^2 d\theta^2\right). \tag{4.11}$$

The proper distances diverge as ρ goes to infinity. Although this divergence is not problematic on its own, for there to be a finite boundary on which the dual CFT will yield values for physical quantities that do not diverge, we impose a cutoff ρ_0 and restrict the space of interest to $\rho \le \rho_0$. In the dual CFT, this corresponds to imposing a UV cutoff where there is a minimal distance between field values of a, the lattice spacing (cf. Susskind and Witten 1998). We can then think of the CFT as living on the boundary of AdS_3 where $\rho = \rho_0$. This makes the $(1+1)$-dimensional spacetime for the CFT a cylinder (t, θ) with radius ρ_0.

Ryu and Takayanagi consider the static case where the CFT lies on a single time slice t (see Fig. 4.8). As mentioned, Hubeny et al. (2007) later extended this argument to more general regions to give a covariant result. To give the main idea of how this is established, I will focus on the static case and closely follow the 2006 discussion. Take a particular time-slice of the cylinder and divide it into a region A and its complement, such that A corresponds to $0 \le \theta \le \frac{2\pi l}{L}$, where l is the length of A and L is the length of the total boundary. The minimal surface γ_A then will be the geodesic with fixed t that connects the points $\theta = 0$ and $\theta = \frac{2\pi l}{L}$. This will be a geodesic running through the AdS_3 spacetime.

Ryu and Takayangi compute the length of the geodesic from the metric as $1 + 2\sinh^2\rho_0 \sin^2\frac{\pi l}{L}$. And then using this for the area of γ_A, they use the proposed area law (4.10) to get an expression for $S(\rho_A)$. The neat thing is that the result given by

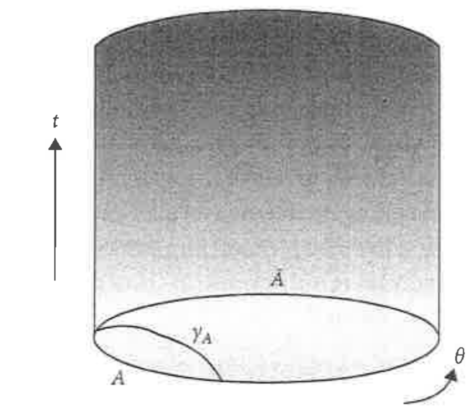

Fig. 4.8 Static case

the area law is confirmed by independent work on the entanglement entropy of many body $(1+1)$-dimensional CFTs. As Ryu and Takayanagi note, Calabrese and Cardy (2004) showed (generalizing results of Holzhey et al. 1994) that for $(1+1)$-dimensional CFTs, the entanglement entropy of a quantum many-body system (with periodic boundary conditions) is given by:

$$S(\rho_A) = \frac{c}{3}\log\left[\frac{L}{\pi a}\sin\left(\frac{\pi l}{L}\right)\right], \tag{4.12}$$

where again L is the length of the total system, l is the length of A, a is the lattice spacing, and c is the CFT's central charge. Recalling that our CFT has central charge $c = 3R/2G_N$, we then recover:

$$S(\rho_A) = \frac{c}{3}\log\left(e^{\rho_0}\sin\frac{\pi l}{L}\right), \tag{4.13}$$

assuming $e^{\rho_0} \gg 1$.

The form of argument Ryu and Takayanagi use may be summarized as follows. Using the AdS_3 metric, we can compute the area of a minimal surface connecting the edges of region A through the bulk. If we then assume the area law, this allows us to calculate a certain formula for the entanglement entropy $S(\rho_A)$ in the language of conformal field theory. This turns out to be the correct formula for $S(\rho_A)$ within a $(1+1)$-dimensional CFT, given independent results. This thus confirms the area law. Ryu and Takayanagi develop similar arguments for more complicated setups in higher dimensions (2006a, 2006b).

So we get confirmation of Ryu and Takayanagi's conjecture of an area law, but this does raise the question of why such an area law obtains. What grounds the connection between $S(\rho_A)$ and the area of γ_A? Discussions of Ryu and Takayanagi's paper often take their result to support the emergence of spacetime from an underlying quantum theory without gravitational degrees of freedom, but although their arguments certainly confirm an entanglement-distance link, they don't provide the explanation of why this link should hold, and as such cannot support Option (1) over its three rivals. Here I would suggest it is helpful if we think about the Bekenstein–Hawking law, which inspired Ryu and Takayanagi's conjecture. Bombelli et al. (1986) develop an illuminating physical argument that provides an explanation of why the Bekenstein–Hawking relationship holds. As it works by explicating Bekenstein–Hawking entropy as a form of entanglement entropy, it can help us to better understand the metaphysical implications of the Ryu–Takayanagi conjecture, which essentially takes the Bekenstein–Hawking result and generalizes it outside of the black hole context.[12]

[12] It should be noted that arguments based on the claim that black hole entropy is entanglement entropy do not provide the only path to explaining the Bekenstein–Hawking formula. For example, one also finds arguments that one can arrive at the relationship just by counting states. To see how this is done in the string theory context, see e.g. Strominger and Vafa (1996). However, the form of argument I will describe and closely related arguments presented e.g. by Srednicki (1993) are the most salient if one wants an explanation that can be carried over to the Ryu–Takayanagi area law which concerns entanglement entropy.

The core idea introduced in Sorkin (1983) and developed in Bombelli et al. (1986) is to treat the source of black hole entropy as related to its entanglement entropy, and find the entanglement entropy of the black hole by considering the entanglement entropy of the region outside it.[13] This is illustrated using a simple model, where the states of the total region consist of the states of a massless scalar field in its vacuum state. To calculate the entanglement entropy, we can represent the field as a lattice of harmonic oscillators with density $1/\ell^3$. This allows us to represent the entanglement entropy as a sum over eigenvalues λ_i for an operator Λ, using the previously stated relation:

$$S(\rho) = \sum_i \lambda_i \log\frac{1}{\lambda_i} \tag{4.14}$$

This operator Λ encodes the correlations between points in the lattice and importantly depends on a division of points in the lattice into those internal and external to the black hole. See Bombelli et al. (1986) for the details and Sorkin (1983) for an overview, but the core idea is that assuming a finite number of points in the lattice, the main contribution to the entanglement entropy will come from the values of Λ closer to the horizon (cf. Carlip 2009). And so, given how Λ is constructed, "the leading term in S will be proportional to $\frac{A}{\ell^2}$ where A is the area of the surface ('horizon') separating H^{int} from H^{ext}" (Sorkin 1983, 3).[14] The entropy of a black hole thus scales with the area of its event horizon. This is because the sum in (4.14) is dominated by terms describing the states of the field near the boundary, and thus the size of the boundary between the regions inside and outside of the black hole is imposing a constraint on the leading contribution to $S(\rho_{ext})$.

We can see why this is by considering that any physically acceptable state of the field will be a Hadamard state.[15] As noted in Unruh and Wald (2017, 2), the leading order behavior for Hadamard states Ψ of a free scalar field ϕ at two points x_1 and x_2 is:

$$\langle\Psi|\phi(x_1)\,\phi(x_2)|\Psi\rangle \sim \frac{U(x_1,x_2)}{(2\pi)^2\sigma(x_1,x_2)}. \tag{4.15}$$

This is the two-point function of the field, where $U(x_1, x_2)$ is some smooth function and $\sigma(x_1, x_2)$ is the squared geodesic distance between the two points. From (4.15),

[13] Sorkin introduced the idea of entanglement entropy in this 1983 paper.

[14] H^{int} and H^{ext} are the two spacelike hypersurfaces we are considering inside and outside of the black hole.

[15] Wald (1994, chapters 2–3) argues that the requirement that states be Hadamard ensures that the stress-energy tensor is defined.

it is easily seen that the two-point function diverges as x_1 and x_2 approach one another. And so, as the operator Λ encodes the correlations between points inside and outside of the event horizon, the largest contributions to (4.14) will come from those right at the event horizon.

Bombelli et al.' s derivation of an area law for the entanglement entropy of a black hole, at least the contribution to this entropy given by a massless scalar field, justifies the posited area law of Bekenstein and Hawking. And so we can use this justification to ask about the justification for the area law of Ryu and Takayanagi.

4.7 No Support for Emergence

It is clear from the above discussion of black hole entropy that there is no antecedent reason to view the area of the event horizon as constructed out of or emerging from the entanglement of the reduced quantum state ρ_{BH}. The most natural way to interpret the argument is that the horizon area is providing a constraint on the black hole entropy. It is not emerging from the entropy.[16] Insofar as the Ryu–Takayanagi conjecture is providing a generalization of the black hole area law, in that case too there doesn't seem to be any reason to view the area of the minimal surface γ_A as constructed out of the entanglement entropy. It is more natural again to view the area of the minimal surface as providing a constraint on the entanglement entropy. And so, the burden of proof is on one who wants to interpret the area law of Ryu and Takayanagi in a very different way metaphysically from the area law it was aimed at generalizing. This point carries over to the work following Ryu and Takayanagi (e.g. Hubeny et al. 2007, Bao et al. 2019, and so on). It is worth noting that Ryu and Takayanagi do not speculate about metaphysics in their original paper. Moreover, in a longer work (2006b) that followed it, expanding on its implications, they present their results not as motivating the emergence of spacetime but rather in an instrumental way: since it is often mathematically very difficult to compute the entanglement entropy from the quantum field theory side of things, one can make use of this proposed area law to get a good approximation of the entanglement entropy (2006b, 2–3).

One might argue that although the Ryu–Takayanagi conjecture was *inspired by* the Sorkin–Bombelli explanation of the area law of Bekenstein and Hawking, and although there are important analogies between black hole entropy and entanglement entropy on a CFT boundary, this does not entail that there is

[16] This is the most natural interpretation of 't Hooft's (1993) holographic proposal for black hole entropy as well, as he argues that the black hole entropy is maximal when it reaches $S_{max} = \frac{1}{4}A$. "[The entropy] is saturated when one black hole has the largest possible size that still fits inside our area" (5).

an absolute alignment between the two cases.[17] Although in the case of the Bekenstein–Hawking law, facts about spatiotemporal distances do explain the entanglement entropy, one might argue that in the case of the AdS/CFT correspondence, facts about entanglement entropy are fundamental and instead explain the facts about spatiotemporal distance. This is a coherent possibility. However, what I am trying to argue is that a more natural attitude is to view the Ryu–Takayanagi result as supporting the generalization of the Sorkin–Bombelli framework outside of the context it was initially intended to describe. And thus that in general, entanglement entropy is constrained by interface (event horizon or minimal surface) area. Interface area doesn't emerge from entanglement entropy in either case.

To make the case for Option (1), one needs to point to arguments in which entanglement entropies are plausibly being used to explain metrical features, without thereby assuming them. One may immediately think of the influential paper by Jacobson (1995), which derives the Einstein Field Equation from facts about entanglement entropy. But Jacobson himself relies on the Bekenstein–Hawking relationship (and the Sorkin–Bombelli explanation of it) in his argument. So this doesn't give us what we are looking for. Work by Padmanabhan (2010a,b) comes closer, as it attempts to provide an explanation of the spacetime metric using a more basic statistical mechanical framework for understanding facts about entanglement entropy. Padmanabhan explicitly frames his work as a part of a project in emergent gravity:

> In this paradigm, one considers spacetime (described by the metric, curvature etc.) as a physical system analogous to a gas or a fluid (described by density, velocity etc.). The fact that either physical system (spacetime or gas) exhibits thermal phenomena shows there *must* exist microstructure in either system . . . We do not yet know what are the correct microscopic degrees of freedom of the spacetime, but the horizon thermodynamics provides a clue . . . (2010a, 3)

Padmanabhan (2010b) shows how one can use the equipartition law $\Delta E = (1/2)(\Delta n)k_B T$ to derive the existence of these fundamental microscopic degrees of freedom and their relationship to the spacetime metric such that:

[17] There is one clear disanalogy between the cases that is worth mentioning. This is that the area that appears in the Bekenstein–Hawking relation is the area of the boundary between the two subsystems (interior and exterior of the black hole's event horizon), but the area that appears in the Ryu–Takayanagi relation is not the area of the boundary between the two subsystems (A and its complement). In the latter case, the area in question is the area of the minimal surface through the bulk that ends on the boundary between A and its complement. This disanalogy, however, does not undermine the above argument, as the boundary between A and its complement itself plays a role in constraining the minimal surface area.

$$\Delta n = \frac{\Delta A}{l_P^2}, \tag{4.16}$$

where l_P is the Planck length. The idea seems to be that the facts about entanglement entropy motivate the postulation of these fundamental degrees of freedom, which in turn ground the structure of gravity and emergent spacetime. This is an intriguing possibility, but at this stage, it is worth noting, given we don't have any idea what these microscopic degrees of freedom are, or a strong motivation for believing in them, how much more speculative these arguments are than the arguments we saw above that show how facts about the entanglement between components of quantum systems are constrained by facts about the interface surface between these components. Then again, as Carlip (private communication) points out, the latter kind of arguments have had a 'big head start'.

Let's finally return to the case for area laws outside of the AdS/CFT context, the argument we find in the results summarized in Eisert et al. (2010) and followed up on by e.g. Cao et al. (2017). This route into the entanglement-distance link also fails to provide support for the view that the facts about spatiotemporal distance emerge from facts about entanglement entropy. As we saw, the way area laws were derived for quantum many body systems relied on an initial assumption of local interactions. The idea is that when influences are only able to act locally, then the limits on correlation and hence entanglement will be restricted by the region's surface area. In this case again, there doesn't seem to be support for the emergence hypothesis. Again, what seems supported, indeed presupposed, by the whole discussion is that the area of the boundary is constraining the entanglement of the two component regions (for Eisert et al., these are I and its complement), by constraining the quantity of direct interactions. It is certainly much more exciting and radical to view the entanglement-distance link as resulting from a fact about emergence—that the spatiotemporal manifold is not fundamental, but ultimately results from deeper, quantum information theoretic facts about quantum entanglement. But the way these results are motivated does not support this radical view. In the framework in which the results are articulated, the spatiotemporal facts are fundamental. They then constrain facts about interactions, which then in turn constrain the facts about entanglement entropy.

4.8 Conclusion

What I have hoped to show is that there is strong support both from within and without the AdS/CFT context for the existence of an important link between entanglement entropy and spatiotemporal distance. However, these results do not lend obvious support for the speculation that the dynamical spacetime of general relativity is emergent from facts about quantum entanglement; at least, this is not supported by work to date. One has to interpret results like those of Ryu and Takayanagi in a way that departs significantly from the intuitions that make them comprehensible to have them support the claim that spacetime emerges from the facts of quantum entanglement.

This isn't to say that one cannot use the results of Ryu and Takayanagi and others to support the claim that the metric emerges from facts about quantum entanglement. In a recent book, Carroll suggests we do just that. He says:

> ...following our reverse-engineering philosophy, we can *define* the 'area' of a collection of degrees of freedom to be proportional to its entanglement entropy. In fact, we can assert this for every possible subset of degrees of freedom, assigning areas to every surface we can imagine drawing within our network... If the degrees of freedom are highly entangled, we *define* them to be nearby... A metric on space has emerged from the entanglement structure of the quantum state. (2019, 284)

Of course, this is a move one can make, and certainly the attendant emergence hypothesis is one we should keep on the table, especially as it seems supported by other routes into developing a quantum theory of gravity (cf. Wüthrich 2017). However, the goal of this chapter has been to make clear that this is not what area laws most directly support, when we pay attention to how they have typically been justified in physics.[18]

References

Audenaert, K., J. Eisert, M. B. Plenio, and R. F. Werner (2002). Entanglement propertie of the harmonic chain. *Physical Review A 66*(4), 042327.

Balasubramanian, V., J. de Boer, M. P. Heller, B. Czech, and B. D. Chowdhury (2014 A hole-ographic spacetime. *Physical Review D 89*, 086004.

Bao, N., C. Cao, S. Fischetti, and C. Keeler (2019). Towards bulk metric reconstructio from extremal area variations. *Classical and Quantum Gravity 36*(18), 185002.

Bekenstein, J. D. (1973). Black holes and entropy. *Physical Review D 7*(8), 2333.

Bombelli, L., R. K. Koul, J. Lee, and R. D. Sorkin (1986). Quantum source of entrop for black holes. *Physical Review D 34*(2), 373–383.

[18] Thanks to Steve Carlip, Veronika Hubeny, Nick Huggett, and an anonymous referee for comme and discussions that substantially improved this chapter.

Butterfield, J. (2021). On dualities and equivalences between physical theories. In C. Wüthrich, B. Le Bihan, and N. Huggett (Eds.), *Philosophy Beyond Spacetime*. Oxford: Oxford University Press. This volume.

Calabrese, P. and J. Cardy (2004). Entanglement entropy and quantum field theory. *Journal of Statistical Mechanics: Theory and Experiment 2004*(06), P06002.

Cao, C., S. M. Carroll, and S. Michalakis (2017). Space from Hilbert space: Recovering geometry from bulk entanglement. *Physical Review D 95*(2), 024031.

Carlip, S. (2009). Black hole thermodynamics and statistical mechanics. In E. Papantonopoulos (Ed.), *Physics of Black Holes*, pp. 89–123. Springer.

Carlip, S. (2012). Challenges for emergent gravity. *Studies in History and Philosophy of Modern Physics 46*(2), 200–208.

Carroll, S. (2019). *Something Deeply Hidden: Quantum Worlds and the Emergence of Spacetime*. New York: Dutton.

Crowther, K. (2014). *Appearing out of nowhere: The emergence of spacetime in quantum gravity*. Ph.D. thesis, University of Sydney.

De Haro, S. (2017). Dualities and emergent gravity: Gauge/gravity duality. *Studies in History and Philosophy of Modern Physics 59*, 109–125.

De Haro, S. (2020). Spacetime and physical equivalence. In N. Huggett, K. Matsubara, and C. Wüthrich (Eds.), *Beyond Spacetime: The Foundations of Quantum Gravity*, pp. 257–283. Cambridge: Cambridge University Press.

Dieks, D., J. van Dongen, and S. de Haro (2015). Emergence in holographic scenarios for gravity. *Studies in History and Philosophy of Modern Physics 52*, 203–216.

Eisert, J., M. Cramer, and M. B. Plenio (2010). Colloquium: Area laws for the entanglement entropy. *Reviews of Modern Physics 82*(1), 277.

Harlow, D. (2020). What black holes have taught us about quantum gravity. In N. Huggett, K. Matsubara, and C. Wüthrich (Eds.), *Beyond Spacetime: The Foundations of Quantum Gravity*, pp. 105–114. Cambridge: Cambridge University Press.

Hawking, S. W. (1975). Particle creation by black holes. *Communications in Mathematical Physics 43*(3), 199–220.

Headrick, M., R. C. Myers, and J. Wien (2014). Holographic holes and differential entropy. *Journal of High Energy Physics 2014*(10), 149.

Holzhey, C., F. Larsen, and F. Wilczek (1994). Geometric and renormalized entropy in conformal field theory. *Nuclear Physics B 424*(3), 443–467.

Horowitz, G. T. (2005). Spacetime in string theory. *New Journal of Physics 7*(1), 201.

Hubeny, V. E. (2014). Covariant residual entropy. *Journal of High Energy Physics 2014*(9), 156.

Hubeny, V. E., M. Rangamani, and T. Takayanagi (2007). A covariant holographic entanglement entropy proposal. *Journal of High Energy Physics 2007*(07), 062.

Huggett, N. and C. Wüthrich (2013). Emergent spacetime and empirical (in)coherence. *Studies in History and Philosophy of Modern Physics 44*(3), 276–285.

Huggett, N. and C. Wüthrich (forthcoming). *Out of Nowhere: The Emergence of Spacetime in Quantum Theories of Gravity*. Oxford: Oxford University Press.

Jacobson, T. (1995). Thermodynamics of spacetime: The Einstein equation of state. *Physical Review Letters 75*(7), 1260.

Jaksland, R. (2020). Entanglement as the world-making relation: Distance from entanglement. *Synthese*. https://doi.org/10.1007/s11229-020-02671-7.

Maldacena, J. (1999). The large-n limit of superconformal field theories and supergravity. *International Journal of Theoretical Physics 38*(4), 1113–1133.

Maldacena, J. (2003). Eternal black holes in anti-de sitter. *Journal of High Energy Physics 2003*(04), 021.

Myers, R. C., J. Rao, and S. Sugishita (2014). Holographic holes in higher dimensions. *Journal of High Energy Physics 2014*(6), 44.

Nielsen, M. A. and I. L. Chuang (2016). *Quantum Computation and Quantum Information*. Cambridge: Cambridge University Press.

Oriti, D. (2021). Levels of spacetime emergence in quantum gravity. In C. Wüthrich, B. Le Bihan, and N. Huggett (Eds.), *Philosophy Beyond Spacetime*. Oxford: Oxford University Press. This volume.

Padmanabhan, T. (2010a). Finite entanglement entropy from the zero-point area of spacetime. *Physical Review D 82*(12), 124025.

Padmanabhan, T. (2010b). Surface density of spacetime degrees of freedom from equipartition law in theories of gravity. *Physical Review D 81*(12), 124040.

Preskill, J. (1998). Lecture notes for physics 229: Quantum information and computation. *California Institute of Technology 16*. http://theory.caltech.edu/~preskill/ph229/.

Rangamani, M. and T. Takayanagi (2017). *Holographic Entanglement Entropy*. Springer.

Read, J. (2016). The interpretation of string-theoretic dualities. *Foundations of Physics 46*(2), 209–235.

Rickles, D. (2013). AdS/CFT duality and the emergence of spacetime. *Studies in History and Philosophy of Modern Physics 44*(3), 312–320.

Ryu, S. and T. Takayanagi (2006a). Aspects of holographic entanglement entropy. *Journal of High Energy Physics 2006*(08), 045.

Ryu, S. and T. Takayanagi (2006b). Holographic derivation of entanglement entropy from the anti-de sitter space/conformal field theory correspondence. *Physical Review Letters 96*(18), 181602.

Sorkin, R. D. (1983). On the entropy of the vacuum outside a horizon. In B. Bertolli, F. de Felice, and A. Pascolini (Eds.), *Tenth International Conference on General Relativity and Gravitation (Padova, 4–9 July, 1983), Contributed Papers*, Volume 2, pp. 734–736.

Srednicki, M. (1993). Entropy and area. *Physical Review Letters 71*(5), 666–669.

Strominger, A. and C. Vafa (1996). Microscopic origin of the Bekenstein–Hawking entropy. *Physics Letters B 379*(1–4), 99–104.

Susskind, L. and E. Witten (1998). The holographic bound in anti–de sitter space. arXiv hep-th/9805114.

't Hooft, G. (1993). Dimensional reduction in quantum gravity. In *Conference on Highlights of Particle and Condensed Matter Physics (SALAMFEST)*, pp. 284–296. World Scientific.

Teh, N. J. (2013). Holography and emergence. *Studies in History and Philosophy of Modern Physics 44*(3), 300–311.

Unruh, W. G. and R. M. Wald (2017). Information loss. *Reports on Progress in Physics 80*(9), 092002.

Van Raamsdonk, M. (2010). Building up spacetime with quantum entanglement. *General Relativity and Gravitation 42*(10), 2323–2329.

Wald, R. M. (1994). *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*. University of Chicago Press.

Witten, E. (1998). Anti de sitter space and holography. *Advances in Theoretical and Mathematical Physics 2*(2), 253–291.

Wüthrich, C. (2017). Raiders of the lost spacetime. In D. Lehmkuhl, G. Schiemann, and E. Scholz (Eds.), *Towards a Theory of Spacetime Theories*, pp. 297–335. Basel: Birkhäuser.

5

Taking Up Superspace

The Spacetime Setting for Supersymmetric Field Theory

Tushar Menon

Abstract

Supersymmetry (SUSY) is a proposed symmetry between bosons and fermions. The structure of the space of SUSY generators is such that the distinction between internal and spacetime symmetries is blurred. As a result, there are two viable candidates for the spacetime setting for a flat supersymmetric field theory—Minkowski spacetime and superspace, an extension of Minkowski spacetime to include (at least) four new dimensions, coordinatized by 'supernumbers', i.e. numbers with nontrivial commutation properties. This chapter argues for two theses: first, that one standard set of arguments, related to universality of symmetry behaviour, that motivate a particular choice of spacetime structure in familiar spacetime theories motivates the choice of superspace as the appropriate spacetime for SUSY field theories; and second, that the metaphysical utility of the concept of spacetime requires more than just the satisfaction of this universality condition; in supersymmetric theories, the spacetime concept is not as useful as in special relativity.

5.1 Introduction

Supersymmetry (SUSY) is a proposed symmetry between quantum fields of integer spin (bosons) and quantum fields of half-integer spin (fermions). SUSY is a bizarre and brilliant idea, and it deserves to be scrutinized by philosophers. In this chapter, we will merely dip our toes into its deep waters. The motivation for the project of looking at SUSY from a philosophical perspective is not as radical as it might appear at first. Philosophical intuitions are heavily influenced by, and tied to, particular frameworks of physical theories. Exploring the logical space of such possibilities has value for the light it sheds on the constraints imposed by our current theorising. As Friedman (2001) argues, physical theories are embedded in a package of background formal and conceptual assumptions, which can be