

Bertrand Russell's Philosophical Logic and its Logical Forms

By Nikolay Milkov*

From 1901 to 1919, Russell persistently maintained that there were two kinds of logic and distinguished between one and the other as mathematical logic and philosophical logic. In this paper, we discuss the concept of philosophical logic, as used by Russell. This was only a tentative program that Russell did not clarify in detail; therefore, our task will be to make it explicit. We shall show that there are three (-and-a-half) kinds of Russellian philosophical logic: (i) “pure logic”; (ii) philosophical logic investigating the logical forms of propositions; (iii) philosophical logic exploring the logical forms of facts: in epistemology and in the external world. In particular, Russell’s program or philosophical logic of the facts of the external world remained less than sketchily outlined.

Keywords: *Russell, mathematical logic, philosophical logic, Wittgenstein*

Symbolic/Mathematical Logic vs. Philosophical Logic

Russell was engaged in setting up perfect logic on the pattern of which not only philosophy but also mathematics and science could be built up from the time of his Neo-Hegelian project for “Logic of the Sciences” (1896–8). With this idea in mind, he worked on logic and the foundations of mathematics for years. At the same time, however, from 1901 to at least 1919, Russell persistently maintained that there were two kinds of logic and he sharply distinguished them as mathematical logic and philosophical logic (1901b, p. 74, 1914b, p. 67). He insisted here that philosophical logic was clearly different from mathematical logic:

What is now required is to give the greatest possible development to mathematical logic, to allow to the full the importance of relations, and then to found upon this secure basis a new *philosophical logic* (1901a, p. 379; italics added).

In this paper, we shall discuss the concept of “philosophical logic” as used by Russell, not as it is used today—as formal logic that goes beyond the classical propositional and predicate logic (Goble 2001, Jacquette 2002).¹ Our first step in this direction will be to outline the tasks of Russell’s mathematical logic.

*Professor, University of Paderborn, Germany.

¹In late 20th century Britain, some efforts were made to continue Russell’s project for philosophical logic (Strawson 1967, Grayling 1984, Wolfram 1989). Unfortunately, this topic has been neglected for decades now.

Mathematical Logic—Science or Instrument?

Russell's understanding of the role of mathematical logic in philosophy was twofold. On the one hand, he maintained that it could replace philosophy; on the other, he averred that it was only an instrument for achieving better results in philosophy:

(i) As if opposing his vision of two kinds of logic, in some places, Russell firmly maintained that “Mathematical logic has resolved the problems of infinity and continuity” (1911b, p. 294).² Apparently, this was a positivistic program for solving philosophical problems with the help of mathematical logic. In this sense, Gregory Landini maintains that for Russell, mathematical logic is “an *informative science* capable of studying all relational structures” (2019, p. 208; italics added). Landini further holds that exactly in this sense, “logic, that is, cp-logic,³ was the essence of philosophy” for Russell (*ibid.*).

In order to make this project better known, Russell drew up a program for bringing philosophically important sides of the new discoveries in pure mathematics and in symbolic/mathematical logic to the general public. The first clear statement of this program is to be found in the 1901 paper, “Recent Works in the Principles of Mathematics”, quoted above. However, Russell started thinking to realize this project only after Whitehead and he had finished *Principia Mathematica* in March 1911. In the upcoming months, Russell included in his literary plans a book on “Advanced Logic”.⁴ It was designed as something like a textbook of logic/mathematics for philosophers, in which the ideas of *Principia* would be presented, “together with some new ideas” in an abridged, non-technical form (p. 183). Because of the outbreak of the First World War, however, this project was only realized in 1919, when *An Introduction to Mathematical Philosophy* was published.

(ii) At the same time, Russell contended that the mathematical logic only “enable[s] us to see quickly what is the smallest store of materials with which a given logical or scientific edifice can be constructed” (1914b, p. 51; italics added). In other words, the mathematical logic only provides an improved quasi-optical instrument that can help us by treating philosophical problems. It is a kind of “a mental telescope” (1903, p. xv) and, therefore, only an “instrument for research” (1910, p. vii). It cannot automatically solve philosophical problems and thus delivers no information. To be sure, Russell underlined that mathematical logic only “gives the method of research in philosophy, just as mathematics gives the method in physics” (1914b, p. 243). Russell was even clearer on this point in the “Preface” to *Principia Mathematica*, where he wrote: “Without its [of the mathematical logic] help we shall have been unable to perform the requisite reasoning” (1910, p. vii). In this way, philosophy is an autonomous discipline and

²Significantly, these problems were central in Hegel's philosophy of mathematics. After his 1898 turn against Hegel, Russell pursued further their exploration but tried to solve them in a new, “correct” way.

³Landini's cp-logic, or “comprehensive principle logic”, signifies Russell's mathematical logic exclusively.

⁴Russell also called this project “Logic”, “Philosophy of Mathematics” (1992, p. 442), or “Popular Logic” (p. 458). The content of these plans is set out in (1984, p. 183); see also (1984, p. xxiii).

cannot be replaced by this instrument, that is, by mathematical logic exactly as physics cannot be replaced by its research instrument—mathematics.

Significantly, Russell spoke about (i), that is about the replacement of philosophy with new achievements of mathematical logic, mainly before 1903. Afterward, he rather saw mathematical logic as (ii), as a useful instrument for doing philosophy.

What did Russell Mean by “Philosophical Logic”?

In the *Principles of Mathematics*, Russell held that the realm of study of the “philosophical logician” was a few simple, foundational topics of mathematics and mathematical logic. To be more explicit, philosophical logic is concerned with “the discussion of indefinables” of mathematical logic, or of “the fundamental concepts which mathematics accepts as undefinable”: judgment, class, implication, logical constants (1903, p. xv). This is a philosophical task since the indefinables of mathematics and mathematical logic are obtained in it in a process of regressive analysis, which means that “we seek to pass from the complex to the simple, from the demonstrable to its indemonstrable principles” (1903, p. 3), from the clear body of the available mathematics to its foundations.⁵ Russell’s philosophical logic, as clearly different from mathematical logic, was also mentioned in *Principia Mathematica*, wherein Russell claimed that it discussed intensions, while mathematical logic discussed extensions (1910, p.72).

The conception of philosophical logic was also discussed in *Our Knowledge* (1914b, pp. 50–51) despite the fact that at that point in time, Russell’s philosophical work also had other objectives. What was new in Russell’s philosophical writings of the early 1910s was his explicit insistence that the “philosophical logical forms” were to be discovered in a philosophical effort—they could not be a product of mathematical logic alone. It brings us to an important aspect of Russell’s philosophical logic. It proceeded in philosophical contemplations, deliberations and, as we already know, in discussions. It also explains why Russell contended that:

When everything has been done that can be done by method [i.e. by mathematical logic], a stage is reached where only direct philosophical vision can carry matters further. Here, only genius will avail. What is wanted, as a rule, is some new effort of *logical* imagination, some glimpse of a possibility never conceived before (1914b, p. 245; italics added).

It is a very difficult task indeed—which is why Russell used to say “Logic is Hell”—which is to be accomplished in “direct contemplation of facts” discarding language (1921, p. 212) and mathematical logic. *Pace* Landini, the structures

⁵Russell explored the importance of the method of regressive analysis in mathematics in detail in his paper “The Regressive Method of Discovering the Premises of Mathematics” (1907). The distinction between regressive and progressive analysis, however, was not new in philosophy. Among others, it was discussed by Pappus of Alexandria and in Kant’s Jäsche *Logic* (1800, § 105).

brought to light by mathematical logic cannot help in this region at all, despite the fact that the discoveries made by the philosophical logicians are to be articulable in accordance with the rules of mathematical logic.

Russell's Philosophical Logic

In the lines above, we have shortly outlined Russell's program for philosophical logic, following his own words. Apparently, this was only a tentative program that Russell did not clarify in great detail. That is why it needs an explication; in fact, this will be our task in the lines below. To be more specific, we shall see that there are three (-and-a-half) kinds of Russellian philosophical logic: (i) "pure logic"; (ii) philosophical logic investigating the logical forms of propositions; (iii) philosophical logic exploring the logical forms of facts: (α) in epistemology; (β) in the external world. We are going to discuss them in this order as follows.

"Pure Logic"

The first question we shall try to answer in this section is: How did Russell ever get the idea that there was something like philosophical logic?

Our tentative answer is that this was a result of his "idealist apprenticeship". It occurred through reading the works of F. H. Bradley and through discussions with his tutor in Cambridge, James Ward, and his elderly friend and Cambridge Apostle, J. E. M. McTaggart. Also important in this respect was Russell's study of the works of Hermann Lotze (Milkov 2008).⁶ These lessons persuaded Russell to see "pure logic" as the discipline that tried to set out the transcendental question: What makes a subject of human knowledge intelligible or thinkable at all?—in his case, which makes mathematics understandable.⁷

We are going to start our analysis of this influence with the historical remark that in 1898–1901, Russell was involved in extensive talks with his fellow student, fellow Cambridge Apostle and friend G. E. Moore. These conversations reassured him how important philosophical exploration of the foundations of logic and mathematics was. Among other things, they found expression in Russell's letter to Couturat from July 1900 where he stated: "My friend G. E. Moore is, in my opinion, the most subtle in pure logic" (Preti 2019, p. 190). Moore, however, had no idea about mathematical logic. In what sense, then, did Russell mean that Moore was his teacher in "pure logic"? In fact, in the "Preface" to *The Principles* Russell is quite informative on this count. Russell maintains that he

have accepted from him [G. E. Moore ...] the pluralism which regards the world, both that of existents and that of entities, as composed of an infinite number of mutually independent entities, with relations which are ultimate, and not reducible to adjectives of their terms or of the whole which these compose (1903, p. xviii).

⁶In fact, the British idealists were considerably influenced by Lotze as well (Milkov 2023).

⁷Bradley, for example, maintained to the contrary (1893, Chapter 3) that the very idea of a plurality of objects standing in relations was unintelligible.

Apparently, Russell meant here Moore's atomistic metaphysics.⁸ In fact, this was the kernel of his doctrine of logical atomism that assumed that there were entities that were relations and entities which were not relations ("non-relations"), the latter being universals and particulars (1911a, p. 107); or, that "there are 'things' which have properties and have, also, relations to other 'things'" (1959, p. 117). For decades, Russell was convinced that " x is something different from the sum of all its properties" (ibid, p. 119). Here it is also clear that in these years, but also later, Russell closely connected mathematics and mathematical logic with the world and, therefore, with metaphysics.

This explains why Russell's atomistic metaphysics could also be articulated in logical terms: It corresponded to the atomistic logic that was based on logical atoms—the smallest elements necessary for any logical operation and for thinking in general. In fact, Russell merged metaphysical and logical problems in the very beginning. For example, he discussed the logical properties of the spatio-temporal relations, meaning with logical properties "properties such as can be expressed in logical terms, not only such as can be proved by logic"⁹ (1959, p. 75). This explains why Russell's "pure logic" explored both the foundations of mathematical logic and the logical properties of the world. Significantly, this kind of logic grounds all logico-philosophical ideas of Russell and, in this sense, has priority over them.

Russell's main argument for adopting non-reducible entities in metaphysics was closely connected with the acceptance of the numerical diversity in arithmetic. To be more explicit, Russell found place for entities in his metaphysics only because he believed that counting was theoretically sound only if we accepted the identity of indiscernibles, assuming the numerical diversity of every item or term. Russell's argument was that "if a and b have all their properties in common, you can never mention a without mentioning b or count a without at the same time counting b " (1959, p. 115); in such a case, we cannot distinguish a from b . Much earlier, in his "Preface" to *The Principles of Mathematics*, Russell wrote:

Before learning these views from him [Moore], I found myself completely unable to construct any philosophy of arithmetic, whereas their acceptance brought about an immediate liberation from a large number of difficulties which I believe to be otherwise insuperable (1903, p. xviii).

This was clearly a transcendental argument.

It was not a surprise, therefore, that it was challenged in the *Tractatus* 5.5302–3, where Wittgenstein rejected the identity of indiscernibles. To be sure, Wittgenstein's theoretical intuitions were different—he lacked Russell's "idealistic apprenticeship". In particular, Wittgenstein had no interest in the real world and in metaphysics at all, but in language and thinking. The "objects" and the "world" in the *Tractatus* only signify the ontological (not metaphysical) commitment of language that is connected with its intrinsically depictive character (Milkov 2022).

⁸See, in particular, Moore's paper "Identity" (1901).

⁹Following Russell, in the *Tractatus*, Wittgenstein spoke about "formal properties of objects and atomic facts" (4.122, 4.124, and 4.126).

*Propositional Philosophical Logic*What Is It?

Russell's "pure logic" is not to be confused with the exploration of the logical forms of propositions, or of their "grammatical form" (1903, p. 48), which he later called "philosophical logic" as well. Russell introduced the concept of logical forms of propositions in connection with his increased interest in philosophy of language evident in *The Principles of Mathematics*. To be more explicit, Russell understood the importance of the propositional logical forms only after he had fully realized the implications of his 1900 turn from mereology to mathematical logic (Milkov 2016). One consequence of this turn was the introduction of the conception of "denoting phrases" (1903, pp. 53 ff.). It holds that "denoting phrases" have contextually determined meaning, that is, meaning defined through their "grammatical form". Two years later, in "On Denoting", Russell specified that the denoting phrase "is denoting solely in virtue of its form" (1905, p. 41). This was a new conception of form different from the logical form generally understood in logic.

By way of historical remark, it should be noted that Russell's new linguistic turn went hand-in-hand with the introduction of the method of quantification in logic by Frege in 1879. Russell adopted it after August 1900 when he met Peano at the International Congress of Philosophy in Paris. However, despite the fact that the analytic philosophy of language was first advanced by Frege, the latter was neither as explicit nor as programmatic about the propositional philosophical logic as Russell was. In particular, Russell clearly insisted that it was a new kind of logic—philosophical logic—that had its specific logical forms.

Now Russell suggested a new discrimination between philosophical and mathematical logic.

[Logic] consists of two parts. The first part [philosophical logic] investigates what propositions are and what forms they may have; this part enumerates the different kinds of atomic propositions, of molecular propositions, of general propositions, and so on. The second part [mathematical logic] consists of certain supremely general propositions, which assert the truth of all propositions of certain form. This second part merges into pure mathematics (1914b, p. 67).

It also deserves notice that in spite of the fact that the conception of propositional logical forms was first introduced in 's logic in 1903–5, Russell's interest in it significantly increased after he had started to teach Wittgenstein and then to work together with him on problems of logic, on their joint "theory of symbolism" (Wittgenstein 1979a, p. 121). As a matter of fact, Russell realized its real importance only after the series of "discoveries" of new logical forms made in 1913,¹⁰ for example, that in a logically perfect language facts cannot be named, as Frege maintained. This is the case since to every fact two propositions—true and

¹⁰Wittgenstein realized that these remarks were not "discoveries" only in the process of composing the *Tractatus* (Milkov 2020c).

false—correspond, while to every object corresponds only one name (1913, p. 97). Propositions and names have two completely different logical forms.

Impressed with this and also with other “discoveries” (insights) presented in “Notes on Logic”, Russell declared that philosophy “is concerned with the analysis and enumeration of logical forms” in this sense (1914a, p. 85). Encouraged by Russell, in his *Notebooks* and in the *Tractatus*, Wittgenstein continued recording new logical forms. In fact, however, in this exploration, Wittgenstein only clarified and further developed what Russell now understood to be the “philosophical logic”.¹¹ This is supported by the fact that when Wittgenstein finished what was later called *Tractatus logico-philosophicus*, Russell initially suggested as its title *Philosophical Logic* (Monk 1990, p. 206). Neither G. E. Moore nor Wittgenstein considered this title appropriate. However, in the *Tractatus*, Wittgenstein did not maintain that he explored logical forms. The book remained a program for clarifying language (thinking) through a newly suggested form of logical symbolism, which, however, disappeared at the end (“the general propositional form is a variable” 1922, 4.53); and thus became dispensable—in the same way a ladder becomes superfluous after we have reached with its help a new level (Milkov 2017).

More on Russell’s Propositional Philosophical Logic

In general, Russell’s propositional philosophical logic maintains that human knowledge is an amalgam of ordered “knots” of known particulars, universals, and logical forms (Milkov 2003, p. 61). In human discourse, different individuals—particulars and universals—and terms are *combined* in particular forms, thereby producing the specific variants of human understanding and language. One can conceive of human knowledge as a stock of *information* that can also be *communicated* through language. Here is this understanding expressed in Russell’s own words:

In order to understand a sentence, it is necessary to have *knowledge* both of the constituents and of the particular instance of the form. It is in this way that a sentence conveys *information*, since it tells us that certain known objects are related according to a certain known *form*. Thus some kind of knowledge of *logical forms*, though with most people it is not explicit, is involved in all understanding of discourse. It is the business of *philosophical logic* to extract this knowledge from its concrete integuments, and to render it explicit and pure (1914b, p. 53; italics added).

This conception of Russell’s brought him close to elements of Frege’s philosophy of language which were also adopted by Wittgenstein (he took them up directly from Frege). Recall, however, that Frege calls the form in which the constituents of the proposition are connected together the sense (*Sinn*) of the proposition, as different from its truth-value (*Bedeutung*). Wittgenstein also

¹¹This gave rise to the belief that “had Wittgenstein never existed, there would be nothing inexplicable about Russell’s work being much as it in fact was” (Sainsbury 1979, p. 12). Unfortunately, this statement is scarcely true. It is not likely that without Wittgenstein’s nudge, Russell would have seen this dimension of his own conception in full.

maintains that “what is grasped of a thought is its sense” (1979b, p. 235). To this he adds, in Russell’s sense, that

[the language] uses signs that go proxy for the elements of a situation and represents the situation itself by means of a combination of the signs in question. Thus it reconstructs the structure of a situation by combining signs in the appropriate way. A proposition—like a model—shows us *how* the elements of a situation are connected (ibid, pp. 235 f.; italics added).

Similar to Russell, for the early Wittgenstein, the form, that is the way the words are knotted in the proposition, presents its sense (Milkov 2020b). It conveys information since it shows *how* (the way in which) the objects are connected to one another in a state of affairs. Russell, on his side, maintains that the logical form is something like “the way [in which] the constituents [of the proposition or inference] are put together” (1914b, p. 52).

The intimate connection between Russell’s philosophical logic by way of “pure logic” and his propositional philosophical logic was that both were engaged with the ultimate problem of intelligibility. But whereas the former discussed the intelligibility of mathematics, the latter explored the intelligibility of language and thinking.

It should also be observed that one of Russell’s motives for embracing the theory of propositional logical forms was his desire to decrease the role that logical objects (in the first place, the logical constants (1919, p. 201) and also numbers, classes, propositions, relations, etc.) played in *The Principles* (and partly also in *Principia*), where it was assumed that these were all subsisting entities. This was an ambitious program for “economy of thought”¹² that was also the kernel of the project for analytic philosophy—at least, as Russell understood it (Milkov 2013). To be sure, Russell’s objective was to eliminate all superfluous theoretical entities and terms. Among other things, it motivated him to formulate his own form of Ockham’s razor (1924, p. 326).

Wittgenstein developed this understanding further. His well-known criticism of the logical objects began in 1912, and was explicated in full in the *Tractatus* with the words, “At that point it becomes manifest that there are no ‘logical objects’ or ‘logical constants’ (in Frege’s and Russell’s sense)” (5.4). In fact, this was nothing but adopting and further developing Russell’s conception of “propositional logical forms” to its fullest. Its ultimate result is Wittgenstein’s claim that “the general form of a proposition is: This is how things stand” (4.5). It perfectly harmonizes with his claim, that the logical symbolism of the *Tractatus* at the end disappears. In the final reckoning, also the lead objective of Wittgenstein’s *Tractatus*, to eliminate any “body of doctrine” in philosophy and to turn it into a practice of elucidating our language and thinking (4.112), follows Russell’s pursuit of “economy of thought” but in most radical form.

¹²Russell’s program for economy of thought was clearly related (but not necessarily influenced by) the development for economy of thought in philosophy supported not only by Ernst Mach but also by Edmund Husserl. Apparently, it was *idée fixe* in Western philosophy at the *fin de siècle*.

Logical Forms of Facts

With regard to the metaphysical status of the forms of propositions, Russell's position was somewhat ambiguous. While in his manuscript, *Theory of Knowledge* (1913), he did not consider logical forms to be autonomous constituents of propositions, after Wittgenstein's criticism of Russell's MS in June 1913, Russell accepted them as such. A few months later, however, after receiving Wittgenstein's "Notes on Logic" (Sept. 1913), Russell turned back to the view that logical forms were not constituents of propositions; and he also adopted this position in *Our Knowledge* and in "The Philosophy of Logical Atomism". In the latter work, Russell made the additional remark that while the propositional logical form was not a constituent of the proposition itself, "it may possibly be a constituent of general statements about propositions that have that form, so I think it is *possible that* logical propositions might be interpreted as being about forms"¹³ (1918, p. 239).

Russell could also not conclusively decide exactly which elements the logical forms put together. Sometimes he maintained that they kept together propositions, sometimes that they kept together facts. Correspondingly, he gave different definitions of philosophical logic. In *Our Knowledge*, for example, he claimed that philosophical logic "investigates what propositions are and what forms they may have" (1914b, p. 67). However, in "The Philosophy of Logical Atomism", he wrote: "I think one might describe philosophical logic...as an inventory, or if you like a more humble word, a 'zoo' containing all the different forms that *facts* may have" (1918, p. 216; italics added). Apparently, Russell understood the "philosophical logical forms" as constitutive elements not only of propositions, but of facts as well. One of the advantages of this clearly rationalistic equipollence of facts and propositions in logic adopted by Russell was that it made it possible to make conclusions as to the forms of propositions from the forms of facts which they asserted—not only from the forms of propositions to the forms of facts, which is what the thesis of logical atomism is usually assumed to be (Urmson 1956, p. 6).

As a result, Russell developed an original philosophical logic of facts in two variants: (i) as an exploration of the logical forms in epistemology—to be sure, in *Our Knowledge* he clearly stated: "The problem of the nature of judgment or belief may be taken as an example of a problem whose solution depends upon an adequate inventory of logical forms" (1914b, p. 67); and (ii) as an investigation of the logical form of the facts of the external world. Russell thus came back to his metaphysical project that he had before 1903 of exploring logical forms of the real world,¹⁴ but it was now developed in a somewhat different form.

Logical Forms in Epistemology

The best example of the Russellian logical forms in epistemology are the propositional attitudes that are nothing but epistemological facts. The description

¹³We are going to return to this problem below.

¹⁴Mutual penetration of logic and the world was also adopted in *The Principles of Mathematics* (1903).

of their forms advances further knowledge of the possible forms of the propositions that can be made on their basis. Russell developed this position in the clearest form in his *Theory of Knowledge* (1913) where he postulated three propositional attitudes: understanding, desiring, and willing. In a *logical classification*, subclasses of understanding are believing, disbelieving, doubting, analyzing, and synthesizing propositions. These are different cognitive relations (attitudes), every one of which has a specific logical form. Thus, “*sensation* (including the apprehension of present mental facts by introspection) is a certain relation of subject and object” (1913, p. 64); “*immediate memory* is a certain relation of subject and object” (p. 65). Russell explained the difference between, for example, present mental experience and memory as entirely based on the difference in the cognitive relation. Here is another example of sharp discrimination of logical forms of this type:

The *logical form* of perception will be different from the *logical form* of believing . . . [and] volition differs from desire *logically*, in a way strictly analogous to that in which perception differs from belief (1918, p. 228; italics added).

In fact, this was the practice of conceptual analysis, conducted by the academic rationalistic philosophers for centuries, which was also practiced by Frege and Wittgenstein (Milkov 2012, 2020a, pp. 195 ff.). The later Wittgenstein repeatedly maintained that “philosophical investigations are conceptual investigations” (1984, i, § 949). It is not difficult to notice that one of the tasks of Wittgenstein’s *Philosophical Investigations* was the exploration and criticism of epistemological forms, as a study of philosophical psychology, that were somehow similar to that of Russell from 1913–18.¹⁵ It is a pity that so far, only a few interpreters (Engelmann 2012) have followed Saul Kripke’s remark on Russell’s influence on Wittgenstein’s *Philosophical Investigations*. To be more explicit, Kripke maintained that “Wittgenstein’s philosophical development was influenced” considerably by Russell’s *The Analysis of Mind* (1982, p. 25 n. 19). The influence of Russell’s philosophy of psychology on Wittgenstein has much older roots, though. It is to be traced back at least to *Theory of Knowledge* and *Our Knowledge of the External World*. This is evident from the fact that already in the *Notebooks 1914–1916*, especially in the months between June 1916 and January 1917, Wittgenstein intensively explored epistemological logical forms: “Is belief a kind of experience? Is thought a kind of experience?” (1979a, p. 89).

It deserves notice, however, that the objective of Wittgenstein’s analyses in philosophical psychology—avoiding the traps into which language lures the naïve philosophical psychologist—was markedly different from that of Russell. Be that as it may, both had as an objective the description of the logical (grammatical) forms.

Logical Form of Facts of the External World

In the mid-1910s, Russell advanced two more programs for philosophical logic. They explore the logical form of the facts of the external world: the program

¹⁵An account of these analyses is to be found, for example, in Budd (1989, pp. 10–15).

for logical constructions, and the somewhat obscure program for logical analysis of the data of the external world—in particular, the data of science.

(i) *Logical Constructions*

In his unfinished paper “On Matter” (May 1912), Russell started a program for “logical construction” of matter.¹⁶ This was a program for explicating how a scientific picture of the world could be constructed in an epistemologically aseptic way. It was closely followed by the logical positivists, in particular, by Carnap in his *Logical Construction of the World* (1928) and partly also by Russell himself in (1921, 1927, 1948).

Russell’s theory of logical constructions eliminated the physical objects and events, in a way similar to that in which he tried to eliminate the logical objects in mathematical logic.¹⁷ Instead, he held that one could describe the world only in terms of sense-data. Furthermore, with the help of the new logic developed in *Principia Mathematica*, the sense-data can be put in different constructions. The language that refers to physical objects and events can be translated in terms of sense-data *salva veritate*.

Significantly, Russell’s project for logical constructions of the external world was not a program for examination of the data of scientific observation. Thus, it had little to do with the problems science was confronted with in his time but more with its epistemological interpretations. Even in *The Analysis of Matter* (1927), where Russell replaced the phenomenalism of “On Matter” with a kind of structuralism, “the philosophical problems of modern physics with which Russell deals seem remote from the perspective of post-positivist philosophy of physics” (Demopoulos and Friedman 1985, p. 622).

(ii) *Analyzing the Facts of the World I*

Perhaps for this reason, Russell was not fully satisfied with his program for logical construction of the external world and looked for a new approach. Indeed, a few months after he had advanced the program for logical constructions, in October 1912, Russell produced an obscure manuscript of only two pages on “What is Logic?”, in which he saw logic as

the study of the form of complexes. ... Logic is not concerned with *judgments*, which are psychological. ... Logic is not concerned with [language, with] forms of words ... with propositions. ... *True* and *False* are extra-logical (1912, p. 55).

Unfortunately, a few days later, he wrote in a letter to Ottoline Morrel: “I can’t get on with ‘what is logic?’, the subject is hopelessly difficult, and for the present I am stuck. I feel very much inclined to leave it to Wittgenstein” (p. 54). As a matter of fact, “Wittgenstein had developed a keen interest in the themes [of

¹⁶Importantly enough, Russell elaborated this program working together with his student Wittgenstein. The latter, however, abandoned it after he had visited Frege in December 1912 (Milkov 2013). See, on this, this section, (iii).

¹⁷Practically at the same time, in November 1912, Wittgenstein discovered that there was only one logical constant (Milkov 2013). In the *Tractatus* (5.4) also, this sole logical constant is eliminated.

Russell's "What is Logic?" ...] and one suspects it would have been a frequent subject of discussion between them" (Klement 2015, p. 218).

Our guess is that this was a program to bring back logic together with the world (with metaphysics) in the way Russell did it between 1896 and 1903 but in a new form. Apparently, what Russell had in mind with "complexes" in this program were the phenomena, events, and facts of the external world, the facts of science included; they all were to be explored by a new variant of his philosophical logic that Russell later defined as a classification, an "inventory of the logical forms of facts" (1914b, p. 61). One is reminded here that the program for exploring the logical forms of the facts and events that science investigated was already set out in *The Principles of Mathematics*,¹⁸ where Russell maintained that philosophical logic investigated fundamental objects such as "the nature of ... space, time and motion", but also of causality, dynamics, etc. (1903, § 2, p. 3). Such objects lie on the borderline between science and philosophy.

To be more specific, in "What is Logic?", Russell's philosophical-logical exploration concentrated itself on the mode of combination of the constituents of the complexes. Russell wrote: The logical form is "the way the constituents are put together" (1912, p. 55). By the way, a little bit later, this idea was repeated in *Our Knowledge* in connection with the propositional logical form.

What was unique in this short paper, however, was his insistence that "logic [is] the class of logical complexes [and that] a complex is logical if it remains a complex whatever substitutions may be effected in it" (pp. 55 f.). Moreover, Russell believed that logical forms of this type were *entities* (Klement 2015, p. 217), despite the fact that in 1912, he did not accept that they were constituents of the propositions in which they occurred. Apparently, Russell's logical forms of 1912 were somehow related to the Platonic Forms.

(iii) *Analyzing the Facts of Science II*

The guess we are going to substantiate in this section of the paper is that Russell did not abandon this program altogether. But it underwent substantial changes. In June 1913, Wittgenstein's critique of Russellian complexes dealt it a big blow. Wittgenstein's argument followed Frege's remark made in December 1912 in a discussion in Jena. Frege criticized Russell's, and—at that point in time—also Wittgenstein's move to identify complexes with facts, pointing out that a "complex is not like a fact. For I can, for example, say of a complex that it moves from one place to another, but not of a fact" (Wittgenstein 1974, p. 199). Frege also questioned Wittgenstein about whether if an object were a part of a fact about it, the fact would be larger than the object. Frege obviously held that whereas "a complex is a spatial object, composed of spatial objects" (p. 200),¹⁹ a fact is not.

What especially struck Frege as mistaken was the idea that when people understood propositions, they grasped spatial complexes. Instead, he argues that in such cases, we understand *one* thing that is not spatial, namely the sense of the

¹⁸Russell repeats this position in "Philosophical Implications of the Mathematical Logic" (1911b, p. 294).

¹⁹We find an echo of this critical remark of Frege in Wittgenstein's "Notes on Logic": "the idea of a complex is taken from space" (Wittgenstein 1913, p. 93).

proposition that we grasp, which can be either true or false. It really differs from the spatial complex, which is segmented.

It seems that Russell finally understood and adopted Wittgenstein's argument when he edited Wittgenstein's "Notes on Logic" in October 1913. The implications of these theoretical changes are clearly discernible in his Lowell Lectures he wrote from September till November 1913, which he delivered in April 1914 in Harvard, and which were published later that year as *Our Knowledge of the External World*. In it, he stopped to speak of complexes and, instead, discussed facts.

This change in Russell's philosophy brought with itself the idea of philosophical logic that explored logical possibilities. The possibilities discovered by the philosophical logic are *philosophical-logical forms* of facts including the facts of science. They are not identical with the *symbolical logical forms*, for which language is of prime importance, but can be expressed with its help.

This was a program for advancing "logical alternatives" by looking for solution of any cognitive—including scientific—problems. Exactly in this key, Russell claims that whereas the old logic was normative, suggesting unique valid logical forms, modern logic is liberal and heuristic. It

has the effect of enlarging our abstract imagination, and providing an infinite number of possible hypotheses to be applied in the analysis of any complex fact. In this respect it is the exact opposite of the logic practiced by the classical tradition (1914b, p. 68).

The new logic assumes that there can be many possible solutions of the problems under analysis. Its task is not simply to criticize some of them but also to advance new possible theories. In other words, the task of the new logic is not to serve as a Procrustean paradigm, in the mold of which all available theories are to be rectified, but to provide a better orientation and new directions in them.

In contrast, the old logic was dogmatic inasmuch as it was believed that it dealt with the only one possible world. It was also seen as the only true logic, not as suggesting a variety of alternative logics. The new logic, in contrast, is interested mostly in finding out new "logical worlds", not in a recurrent investigation of a single, already discovered logical form.

Our claim is that when Russell spoke about "the old logic", and also about "the classical tradition" in logic, he also meant the mathematical logic of his time. To be sure, it alone, without the help of philosophical reflection, could not advance new *hypothesis* about *facts*: and that was what Russell considered as the object of the new philosophy that adopted scientific method.

Our suggestion is to read in this key Russell's instruction that contemporary

Physics, with its principle of relativity and its revolutionary investigation into the nature of matter, is feeling the need for that kind of novelty in fundamental hypotheses which scientific philosophy aims at facilitating (1914b, p. 246).

The task of the philosopher of physics is to process the data of science with his/her metaphysical expertise in order to suggest "fruitful hypotheses" in science.

That is why philosophy, by way of being philosophical logic, must be developed parallel to it. In this connection, Russell dreamed of

The creation of a school of men with scientific training and philosophical interest, unhampered by traditions of the past, and not misled by the literary methods of those who copy the ancients in all except their merits (ibid).

It can be argued here that for Russell, as a philosophical realist, the program for direct analysis of the data of science was far more appropriate than the project for investigating the propositional logical forms. It was appropriate to closely connect philosophy to science and not simply to cast philosophy in the mold of science. The real objective was to help science with truly philosophical explorations. Unfortunately, this project by Russell remained a “road less traveled”—both by Russell himself and by his interpreters.

To be more explicit, according to this short-lived and extremely truncated project of Russell, the philosophical logician advances *hypotheses* built up with the help of mathematical logic in regions of science which are still not susceptible to scientific exploration. Importantly enough, this idea was alive in Russell all the time. As late as in 1960, he defined philosophy as advancing conjectures in areas in which science still cannot go with its exact methods: “Philosophy consists of *speculations* about matters where exact knowledge is not yet possible” (p. 11; italics added).

(iv) *The Motivation of Russell's Program for Exploring the Logical Form of Facts*

Russell's new program for exploring the logical forms of facts can be better understood with reference to the dramatic change of his philosophy after he put *Principia Mathematica* in print. In short, this was a transition from looking for certainty in human knowledge that led Russell to the project of setting up a perfect, sound logic with the help of which human knowledge could be founded or justified, to search for creative uncertainty that advanced and critically examined alternative hypotheses. In the *Problems of Philosophy*, Russell openly declared that “the value of philosophy is, in fact, to be sought largely in its uncertainty” (1912, p. 242).

Arguably, this change was connected with Russell's new endeavor, which he made after March 1910, to connect philosophy with problems of *conditio humana*, with his belief that “the philosopher, by virtue of his more refined philosophical perspective, is able to detect errors that the ordinary citizen overlooks” (Schwerin 2019, p. 5). The new logic, in particular, must “assist *philosophers* in their attempts to clarify and more fundamentally grasp the issues endemic to the puzzles that bedevil ordinary citizens” (p. 15).

Russell's new approach in philosophy had two important implications. First of all, it brought him to maintain that there were *many languages* and also many worlds, whereas *Principia Mathematica* referred to only one world (van Heijenoort 1967). In his assumption, Russell was followed by Carnap again, but opposed by Frege and Wittgenstein from the *Tractatus*, who stuck to the dogma of one language (Milkov 2020a, p. 196). Secondly, it posed the epistemology at the center of his interest (Milkov 2018) at the cost of analytic metaphysics. *The*

Problems of Philosophy can be seen as a manifesto of this turn. The program for exploring the logical forms of the facts we outlined in (ii) and (iii) above in this section were developed in a rather rudimentary fashion.

Epilogue

Some keen observers of the time saw this tendency in Russell's writings in a rather negative light. George Santayana, in particular, who closely followed Russell's work, observed: "The epistemological project that Russell's *Problems* epitomizes is diseased" (1922, p. 216). In another writing of his, Santayana exclaimed: "Mr. Russell's philosophy is a dire failure" (1913, pp. 93 f.). And further on:

Of all my friends, of all persons belonging at all to my world, Bertrand Russell was the most distinguished. He had birth, genius, learning, indefatigable zeal and energy, brilliant intelligence, and absolute honesty and courage. His love of justice was as keen as his sense of humor. He was at home in mathematics, in natural science, and in history. He knew well all the more important languages and was well informed about everything going on in the world of politics and literature. He ought to have been a leader, a man of universal reputation and influence. He was indeed recognized to be a distinguished man, having made his mark in mathematics and logic, and largely inspired the new philosophical sect of "logical realists". Yet on the whole, relatively to his capacities, he was a failure. He petered out (1944, p. 440).

Furthermore, Santayana commented that Russell's assignment was to renew Francis Bacon's project for an *instauratio magna* of all sciences. To be more explicit, his task was the study of the logical form of most obscure facts of the world, including the facts of science. Unfortunately, he was involved in abortive epistemological problems of science instead.²⁰

This is a pity since, as we have already seen in the lines above, Russell also experimented with philosophical programs that were more closely connected with the real world, real science, and with facts. Unfortunately, they remained less than sketchily outlined. The task of this paper is also to make them explicit in the hope that they can start a new life.

References

- Bradley FH (1893) *Appearance and reality*. London: Swan Sonnenschein.
 Budd M (1989) *Wittgenstein's philosophical psychology*. London: Routledge.
 Demopoulos W, Friedman M (1985) Bertrand Russell's *The Analysis of Matter*: its historical context and contemporary interest. *Philosophy of Science* 52: 621–639.

²⁰The epistemological leaning of Russell's philosophical works on the facts of science is clearly seen in the compendium of his writings on the subject. They are, above all, epistemological discussions (Russell 1965).

- Engelmann M (2012) Wittgenstein's new method and Russell's *The Analysis of Mind*. *Journal of Philosophical Research* 37: 283–311.
- Goble L (Ed.) (2001) *The Blackwell guide to philosophical logic*. Oxford: Blackwell.
- Grayling A (1984) *An introduction to philosophical logic*. Oxford: Blackwell.
- Jacquette D (Ed.) (2002) *A companion to philosophical logic*. Oxford: Blackwell.
- Klement K (2015) The Constituents of the propositions of logic. In D Wishon, B Linsky (eds.), *Acquaintance, Knowledge and Logic: New Essays on Bertrand Russell's Problems of Philosophy*, 189–229. Stanford (CA): CSLI Publications.
- Kripke S (1982) *Wittgenstein on rules and private language*. Oxford: Blackwell.
- Landini G (2019) Russell's logic as the *Essence* of philosophy. In R Wahl (ed.), *Bloomsbury Companion to Bertrand Russell*, 205–236. London: Bloomsbury.
- Milkov N (2003) *A hundred years of English philosophy*. Philosophical Studies Series, vol. 94. Dordrecht: Kluwer.
- Milkov N (2008) Russell's depth to Lotze. *Studies in History and Philosophy of Science, Part A* 39(2): 186–93.
- Milkov N (2012) The construction of the logical world: Frege and Wittgenstein on fixing boundaries of human thought. In E Nemeth et al. (eds.), *Crossing Borders: Thinking (Across) Boundaries*, 151–161. Vienna.
- Milkov N (2013) The joint philosophical program of Russell and Wittgenstein and its demise. *Nordic Wittgenstein Review* 2: 81–105.
- Milkov N (2016) The 1900-turn in Bertrand Russell's logic, the emergence of his paradox, and the way out. *Siegener Beiträge Zur Geschichte und Philosophie der Mathematik* 7: 29–50.
- Milkov N (2017) The method of Wittgenstein's *Tractatus*: toward a new interpretation. *Southwest Philosophy Review* 33(2): 197–212.
- Milkov N (2018) Russell's first epistemology. In *idem*, *The Philosophy of Logical Atomism* (in Russian), 189–199. Saint Petersburg: Nauka.
- Milkov N (2020a) *Early analytic philosophy and the German philosophical tradition*. London: Bloomsbury Academic.
- Milkov N (2020b) Wittgenstein's ways. In S Wuppuluri, N da Costa (eds.), *Wittgensteinian (adj.): Looking at Sciences from the Viewpoint of Wittgenstein's Philosophy*, 7–19. Berlin: Springer.
- Milkov N (2020c) The composition of Wittgenstein's *Tractatus*: interpretative study. In K Lozev, M Bakalova (eds.), *130 Years Ludwig Wittgenstein*, 67–87. Blagoevgrad: Bon.
- Milkov N (2022) Mauro Luiz Engelmann: *Reading Wittgenstein's Tractatus*. Cambridge: Cambridge University Press, 2021. In *Nordic Wittgenstein Review* 11.
- Milkov N (2023) *Hermann Lotze's influence on the twentieth century philosophy*. Series "New Studies in the History and Historiography of Philosophy", vol. 12. Berlin–New York: de Gruyter.
- Monk R (1990) *Ludwig Wittgenstein. The duty of the genius*. London: Jonathan Cape.
- Moore GE (1901) Identity. In T Regan (ed.), *The Early Essays*, 121–146. Philadelphia: Temple University Press.
- Preti C (2019) "What Russell meant when he called Moore a logician?" In S Lapointe (ed.), *Logic from Kant to Russell*, 189–205.
- Russell B (1901a) Recent works in the principles of mathematics. In *Collected Papers*, volume 3, 366–379.
- Russell B (1901b) Mathematics and the metaphysicians. In 1918a, 59–74.
- Russell B (1903) *The principles of mathematics*. London: Allan & Unwin.
- Russell B (1905) On denoting. In 1956, 39–56.
- Russell B (1907) The regressive method of discovering the premises of mathematics.

- Russell B (1911a) On the relations of universals and particulars. In 1956, 103–124.
- Russell B (1911b) Philosophical implications of mathematical logic. 1973, 284–294.
- Russell B (1912) What is logic? In Slater (1992): 54–56.
- Russell B (1913) *Theory of knowledge*. London: Routledge, 1984.
- Russell B (1914a) On the scientific method in philosophy. In (1918): 75–93.
- Russell B (1914b) *Our knowledge of the external world*. London: George Allen & Unwin. (2nd Edition 1926).
- Russell B (1918) The philosophy of logical atomism. In (1956): 175–281.
- Russell B (1919) *Introduction to mathematical philosophy*. London: Allen & Unwin.
- Russell B (1921) *The analysis of mind*. London: Allen & Unwin.
- Russell B (1924) Logical atomism. In (1956): 321–343.
- Russell B (1927) *The analysis of matter*. London: Allen & Unwin.
- Russell B (1948) *Human knowledge, its scope and limits*. London: Allen & Unwin.
- Russell B (1959) *My philosophical development*. London: Allen & Unwin.
- Russell B (1960) *Bertrand Russell speaks his mind*. Cleveland and New York: World Publishing Company.
- Russell B (1965) *On the philosophy of science*. Edited by CA Fritz Jr. New York: Bobbs–Merrill.
- Russell B (1984) *The collected papers of Bertrand Russell*, vol. 7: *theory of knowledge: the 1913 manuscript*. Edited by ER Eams. London: Routledge.
- Russell B (1992) *The collected papers of Bertrand Russell*, vol. 6: *logical and philosophical papers, 1909–13*. Edited by JG Slater. London: Routledge.
- Russell B, Whitehead AN (1910) *Principia Mathematica*, volume 1. Cambridge: Cambridge University Press.
- Sainsbury M (1979) *Russell*. London: Routledge.
- Santayana G (1913) *Winds of doctrine: studies in contemporary opinion*. New York: Scribner's. London: Dent.
- Santayana G (1922) *Soliloquies in England and later soliloquies*. New York: Scribner's; London: Constable.
- Santayana G (1944) *Persons and places: the background of my life*. New York: Scribner's; London: Constable.
- Schwerin A (2019) Did Russell experience an epiphany in 1911? *Principia* 23(1): 1–17.
- Strawson P (Ed.) (1967) *Philosophical logic*. Oxford: Oxford University Press.
- Urmson J (1956) *Philosophical analysis*. Oxford: Oxford University Press.
- van Heijenoort J (1967) Logic as a calculus and logic as a language. *Synthese* 17: 324–30.
- Wittgenstein L (1913) Notes on logic. In 1979a: 93–107.
- Wittgenstein L (1922) *Tractatus logico-philosophicus*. Translated by F Ramsey, K Ogden. London: Kegan Paul.
- Wittgenstein L (1974) *Philosophical grammar*. Edited by R Rhees. Oxford: Blackwell.
- Wittgenstein L (1979a) *Notebooks 1914–1916*. 2nd Edition. Edited by GEM Anscombe, GH von Wright. Oxford: Blackwell.
- Wittgenstein L (1979b) *Wittgenstein and the Vienna Circle*. Edited by BF McGuinness. Oxford: Blackwell.
- Wittgenstein L (1984) *Bemerkungen über die Philosophie der Psychologie*. (Remarks on the philosophy of psychology). 2 Volumes. Frankfurt: Suhrkamp.
- Wolfram S (1989) *Philosophical logic: an introduction*. London: Routledge.

