Counterpossibles, Consequence and Context

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Abstract:

What is the connection between valid inference and true conditionals? Many conditional logics require that when A is a logical consequence of B, "if B then A" is true. Taking counterlogical conditionals seriously leads to systems that permit counterexamples to that general rule. However, this leaves those of us who endorse non-trivial accounts of counterpossible conditionals to explain what the connection between conditionals and consequence is. The explanation of the connection also answers a common line of objection to non-trivial counterpossibles, which is based on a transition from valid arguments to the corresponding conditionals. It also contributes to the wider project of illuminating the connections between contexts of utterance, on the one hand, and the truth-conditions of conditionals uttered in those contexts, on the other.

Some conditionals with necessarily false antecedents strike most of us as false, even when we are aware the antecedent cannot be true. While there is an ongoing debate about whether "counterpossibles" are all trivially true (see e.g. Berto et al 2018, Williamson 2020 ch 15, Kocurek 2021 and the references therein), I am sympathetic to the view that some conditionals with necessarily false antecedents are true, and some are false, and specifically that some conditionals of a counterfactual form with necessarily false antecedents are true, and some are false.[1] Once that is established, attention should naturally turn to the question of *which* of these so-called counterpossibles are true, and which are false. This can seem an even more puzzling question than the already difficult question of which counterfactuals with possibly true antecedents are correct: our judgements about counterpossibles are often unclear and easily swayed, even when we think we have all the relevant information we need about the contingent goings-on of the actual world. Counterfactuals in general quickly give rise to puzzles that are hard to solve, and counterpossible conditionals share this feature.

Once non-trivial counterpossibles are countenanced, it is natural, but not compulsory, to think that some counter-*logical* counterpossibles are non-trivial as well. "If intuitionistic logic were

[1] This is also the supposition that conditionals, and specifically counterfactuals, have truth values at all: though an expressivist who thinks that they do not have truth values, even relative to conversational contexts, can probably reword much of the discussion in this paper in terms of counterfactuals with necessarily false antecedents being *acceptable* or *unacceptable* in the appropriate way. I also maintain that indicative conditionals with necessarily false antecedents do not have trivial truth conditions, though it is easier to discuss counterfactuals with this feature when it is clear to writer and reader that the antecedent *is* indeed impossible.

correct, non-contradiction would not be a theorem" is false: while excluded middle can fail in standard intuitionistic systems, non-contradiction does not. (If you are an intuitionistic logician, pick a different example!) If we accept non-trivial counterlogical conditionals, then we have to rethink the connection between valid arguments, on the one hand, and conditionals, on the other. It is natural to think that when an argument from P to C is valid, the corresponding conditional P →C is true. Indeed, in many logical systems, the corresponding conditional will itself be a theorem, and demonstrating that a logic supports a so-called "deduction theorem", ensuring that valid arguments are matched in various ways by conditionals that are theorems, is among the first meta-logical results one would demonstrate. (For an interesting history of deduction theorems and their role in reasoning about symbolic logic systems in the early- to mid-twentieth century, see Franks 2021.)

Those of us who countenance non-trivial counterlogical conditionals should provide an alternative account of the relationship between logical consequence, on the one hand, and true conditionals, on the other. There is an appeal in moving from observing that an argument from a premise to a conclusion is valid, on the one hand, to asserting the corresponding conditional, on the other. (Though not vice versa: if it rains I will take an umbrella, but I am not tempted to think that "I take an umbrella" is a logical consequence of it raining, if only because I sometimes am caught out in the rain without one.) The account I prefer appeals to the context of utterance: there is a natural way of setting the context of an utterance in which one can, in that context, utter conditional claims truly provided they correspond to valid consequences. (Though other conditionals will often be fine in such contexts too.)[2]

This does not give the game away for the non-vacuity of counterlogical and countermathematical conditionals, provided there are also contexts of utterance where conditionals corresponding to deductively valid inferences are not all correct. One good way to show that there are such contexts is to give specific examples, with enough scaffolding so that people are able to see how to go on to evaluate other counterlogicals or countermathematicals. After discussing the kind of

[2] Vander Laan 2004 also claims that we need to appreciate the context-sensitivity of counterpossibles to understand their behavior, and on p 263 claims that there is a setting of context that will make true every counterfactual with a consequent that is either entailed, or strictly implied, by its antecedent. Much of the first half of this paper can be seen as spelling out and defending a similar suggestion. Berto et. al. 2018 also stress the context-sensitivity of counterpossibles, as I will discuss below.

context that does allow us to transition from deductive arguments to conditionals, I will discuss two kinds of contexts where this is not unrestrictedly allowed. As well as giving a better sense of how contexts for counterpossibles sometimes work, this will also give us the resources to defuse a popular argument for the triviality of counterpossibles, using transitions from deductive connections to counterfactuals. While you can establish a context in which many, many counterlogicals and countermathematicals are true, that does little to show that counterpossible conditionals of this sort are true no matter what linguistic context we employ.

1. Counterfactuals and Context

In my view, one reason why it can be tricky to come to a firm view in many places about the truth-value of a given counterfactual, whether with a possible or impossible antecedent, is that counterfactual conditionals are context-dependent.[3] At the very least, the *acceptability* of counterfactual conditionals seems to depend greatly on the conversational context, and plausible stories of the truth-conditions of counterfactual conditionals typically build in a contextual parameter to their evaluation. This phenomenon has been discussed more in the case of counterfactual conditionals than indicative conditionals, but there are a range of cases that suggest that, given the same facts, the utterance of a conditional sentence can be correct or incorrect depending on the conversational context, whether that conditional is counterfactual or indicative.[4] One fairly clear case is the case of "backtracking" counterfactuals. It is not safe for me to leap from a third-story window, since if I were to leap from a third-story window, the fall would seriously injure or kill me. But if I were to leap from a third-story window, I would only do so if there were adequate safety equipment in place. In a context where I first aver to my caution and the potential for there to be safety equipment, "if I were to leap from a third-story window, I would escape serious injury, due to the presence of appropriate safety equipment" looks correct, and "If I were to leap from a third-story window, the fall would seriously injure or

[3] This proposal appears to have been first made in the contemporary literature by Chisholm 1946 p 303, at least if Chisholm's "context of inquiry" is enough like contexts as we understand them. Thanks to Chris Daly for the reference.

[4] There are a range of other conditional constructions in English that would need to be covered by a full treatment of context dependence. While some argue that there is a "material implication" conditional in English which would not be context-sensitive, I take it that this is often a contestable *theory* about standard indicative conditionals rather than corresponding by stipulation to a part of untechnical English. (Which is not to say that we could not introduce a material construction by stipulation in an extension of ordinary English.)

kill me" looks incorrect. But this second sentence is just the one I asserted correctly before changing the context. (See Lewis 1979 pp 456–8 on backtrackers.)[5]

There are well-known cases of context-dependence that do not rely on backtracking. Plausibly, the consequent of conditionals can inform us about what context to employ. In the famous Caesar-in-Korea examples (Quine 1960 pp 222), when we consider how various ruthless generals of history would have used the resources available to the US in the Korean war, "If Julius Caesar were in Korea, he would have used the atomic bomb" looks like a very plausible thing to say. On the other hand, were we to be comparing the armaments available to armies at different times and places, perhaps because we are thinking of wargaming-out a scenario, "If Julius Caesar were in Korea, he would have used catapults" seems like a plausible and perhaps correct thing to say. But uttering those two conditionals together, without a change of context, does not look right: a Caesar with weapons like the atomic bomb and other modern armaments would not have wasted time with catapults, and a Caesar limited by Ancient Roman military technology would not have had any atomic bombs, no matter how ruthless he was. There are other kinds of examples of contextual sensitivity of counterfactuals as well[6], but I do not expect to convert the few remaining anti-contextualists about counterfactuals here. In light of the kinds of cases given, I will treat the truth of counterfactual sentences as context-sensitive for the rest of this discussion, though those who might still disagree may wish to treat this paper as an exploration of what to say about counterpossible conditionals on this hypothesis.

Since counterfactual conditionals in general are context-sensitive, we might well expect the same of counterfactual conditionals with impossible antecedents: so-called *counterpossible* conditionals. This is particularly so if we adopt truth-conditions for counterpossible conditionals which do not treat all counterpossibles as having the same truth value. Stalnaker 1968 and Lewis 1973, for example, treat all counterfactuals with metaphysically impossible antecedents as being

[5] My discussion of context will presuppose so-called "static" approaches to the effect on truth-conditions by context, as opposed to so-called "dynamic" approaches. But those who favour dynamic approaches should be able to translate many of the points I wish to make in my discussion of context, without loss, into typical dynamic frameworks.

[6] Another good case for the context-sensitivity of counterfactuals is provided by the phenomenon of so-called Sobel sequences and reverse Sobel sequences, where the same counterfactual sentences can seem to change in truth-value depending on the order in which they are uttered: see Gilles 2007, Moss 2012. Sobel sequences and reverse Sobel sequences also appear to arise for indicative conditionals (see Lycan 2001 pp 58-63, Willer 2017), suggesting we want a similar story for the interaction of indicative conditionals and context.

true, but a simple modification of their account allows us to extend their strategy to counterpossible conditionals as well. I will focus on this closeness-of-worlds approach in what follows, though there are other approaches to context-sensitivity for counterfactuals and indicative conditionals. There are of course other promising approaches to accounting for the effect of context on the truth-values of conditionals. Gillies 2007 and other "contextually variable strict conditional" accounts use context to rule out ranges of possibilities, rather than to order them: roughly, a conditional is true just in case, in every world not ruled out by the context where the antecedent is true, the consequent is true as well. I prefer to think of context as giving us something more graded, so that a world where the antecedent is true can be not entirely ruled out by the context, but not have to be one where the consequent is also true for the conditional to be true, provided there are other worlds that do even better by contextual standards where the antecedent is true. But adjudicating between these two approaches, or between them and others that employ context in evaluating conditionals such as the approach of Kratzer 2012, is no simple matter. The remarks I will make about context in this paper can be easily transposed into these other general approaches when those frameworks are suitably extended with impossible worlds, and do not seem to me to depend on the details of closest-worlds approaches as opposed to the others mentioned.

Lewis's version of the closest-worlds view, roughly speaking, is to employ a "closeness" ordering on worlds, usually understood as an ordering of worlds by similarity, and using []-> to symbolise the counterfactual conditional, whether "A []-> B" is true at a world W if and only if all the closest A-worlds (worlds where A is true) are B-worlds (worlds where B is true). This "closeness" is spelled out in terms of similarity in various respects: intuitively, we look at scenarios which are like the actual world, except that A is true, to see whether B is true at them as well. Lewis employs only possible worlds, so this condition holds vacuously in his system when there are no possible worlds where A is true. Extending his account to include impossible worlds as well, we can allow that some counterfactuals A []-> B where A is impossible are false, while others are true. It is just a matter of which impossibilities are more dissimilar from the actual world than others. See Nolan 1997 for my preferred development of a closest-worlds semantics in this way.

Crucially for current purposes, Lewis allows that which respects of similarity between worlds are relevant for the truth of a counterfactual is a context-dependent matter. This point is sometimes missed because Lewis famously offered an account of the similarity ordering for counterfactuals when they receive their "standard resolution" (Lewis 1979). But this "standard resolution" is one contextual setting among others, and while Lewis thought it was in some sense standard, and was the one relevant for expressing his counterfactual theory of causation, he did not think there was a unique resolution for all non-back-tracking cases, as can be seen in his discussion of Caesar-in-Korea cases in Lewis 1979 p 457.

Once we notice that counterpossible conditionals are context sensitive, we are in a position to explain some apparent conflicts in our initial judgements about which counterpossible conditionals are correct. Set the context up one way, and a conditional will sound incorrect, while if we set the context up a different way, that very conditional sentence will be fine after all. "If wishes were horses, beggars would ride" is proverbial: and it seems correct to say when we reflect on how easy wishing is. (Or at least it did seem correct when the proverb started, and horses or other riding animals were the sorts of things beggars might have wanted, and would have rode if available.) On the other hand, when we consider the difficult economic circumstances of most beggars, and the way they must give up many things of value just to eat, "if wishes were horses, beggars would not even be able to afford wishes anymore" can seem like a correct thing to say. (See Nolan 1997 p 570 ftnt 24). Saying both together sounds odd, though, and someone uttering the second conditional can easily sound as if she is disagreeing with someone who utters the first.

Plausibly, however, each claim is correct in the circumstances in which it would most naturally be uttered: the proverb reflects the fact that wishing is easy and those who do not have much else have many wishes; and in the second kind of context, the conditional reflects the fact that horses are expensive to acquire and maintain, and beggars, as a rule, have nowhere near enough resources to maintain horses as well as cover their basic needs. Speakers uttering each of the conditionals may well be faced with the same set of facts: the difference in what they say is not because sometimes, beggars would ride if wishes were horses, and sometimes not. Instead, the difference seems to be in the linguistic context of the two utterances. Using a most-similar-antecedent-worlds understanding of counterpossible conditionals, this translates into a difference in which similarities are relevant between our world and various impossible worlds. (I am assuming that it is impossible for wishes to be horses.) Worlds where wishes are easy to have, and those without much else tend to have many wishes, *and* wishes are horses are ones where beggars have horses. Or rather, at least the impossibilities that differ as little in relevant respects as possible besides this odd impossibility are like this. On the other hand, worlds where wishes are horses, but horses are expensive to maintain and beggars must trade what they have of value to survive, are ones where beggars lack horses, and so lack wishes too. Or again, this is true of the impossibilities where wishes are horses that are not more dissimilar from the actual world than they need to be.

Of course there are some impossibilities where wishes are horses, wishing is easy etc. and nevertheless it fails to be the case that beggars ride. I think that for any combination of propositions such that they could not all be true together, there is an impossible world where *just those* propositions are true. Even if you wanted to insist that impossible worlds were more well behaved than that, e.g. that they must be maximal, or they must be closed under some weak logic, there is unlikely to be any connection between wishes being horses and beggars riding that is so intimate it must obtain no matter which impossible world we inspect. However, only the impossibilities that are closest to actuality matter for the truth-value of these counterpossibles in a context at the actual world, so the existence of impossible worlds where the antecedent is true and the conclusion false is not a threat to the truth of the counterfactual. (Compare: suppose I truly say, in a given context and circumstance, "If I had not noticed the fridge was unplugged, our food would have spoiled". The mere fact that there are possible worlds where the fridge is unplugged, this goes unnoticed, but no food ever spoils does not show the counterfactual is wrong. For the counterfactual to be wrong, there have to be possible worlds sufficiently relevantly similar to the actual world where I do not notice but the food remains unspoiled.)

I will not try to give a comprehensive account of the effect of context on counterpossible conditionals in this paper. (I suspect that such an account would have to be at least as complex as a general theory of shared conversational purposes, for example. I do not even know of a successful and comprehensive theory of the effect of context on non-counterpossible conditionals.) Instead, I want to demonstrate how certain classes of counterpossibles behave in three different kinds of contexts, with an account of those contexts specific enough to make predictions about the truth-values of the relevant counterpossibles in those contexts, together with specifications of relevant non-conditional facts. I will first argue that there is a distinctive kind of context, which I will label the "logic classroom context", where almost any valid argument can be transposed into a true conditional, with the conjunction of the premises as the antecedent and the conclusion as the consequent. Recognising this distinctive context resolves a number of puzzles about counterpossibles. These include what to make of various "collapse" arguments against counterpossibles, and to explain what a theorist committed to non-trivial counterpossibles can put in the place of "deduction theorems" linking valid arguments and logically true conditionals.

Many of my remarks here will carry over to the question of which *indicative* conditionals with impossible antecedents are true and which are false. We are also often happy to transition between a proof that we take to be valid from A to B to the indicative claim "if A then B". The inference from "roses are red and violets are blue" to "roses are red" is clearly valid: and this seems ample reason to say that if roses are red and violets are blue, then roses are red. If this kind of transition were fully general, indicative conditionals would suffer from a number of the so-called "paradoxes of strict implication". (See e.g. Lewis and Langford 1932 pp 248–250). In classical logic at least, from a logical falsehood anything follows, and anything whatever entails a logical theorem. But in many contexts of utterance, not every indicative conditional with a logical falsehood as an antecedent is true, nor every conditional with a logical theorem as a consequent. So there is an appearance of non-trivial counterpossible indicative conditionals, just as there is of counterfactual ones.

My own view is that indicative conditionals have a similar closest-worlds semantics to counterfactual conditionals (Nolan 2003), and that they obey the same core logic, and lessons about how counterpossible counterfactual conditionals behave in the logic classroom will largely transfer over to the case of counterpossible indicatives. Even those who do not think that the two varieties of conditionals should receive the same semantic treatment are likely to think that there are many similarities in their behaviour, so a plausible story about the relationship between the counterfactual and recognised consequence may well carry over to the indicative, no matter what exactly the relation between the two turns out to be.

One general question that arises for any non-trivial account of the truth-conditions of conditionals with impossible antecedents is why we should be motivated to explain differences in which counterpossibles we accept in terms of semantic features like truth-conditions, rather than pragmatic features or the ability of hearers to otherwise recover useful material from utterances. Our most secure data often comes from which conditional utterances, against a factual background, a hearer finds acceptable, and which unacceptable, together with data about which conditionals speakers tend to produce and which they do not. But just given this pattern of acceptability and unacceptability, it is an open question whether to offer a primarily semantic or a primarily pragmatic explanation of the patterns we find. (Or some other explanation, such as Williamson 2020's account that involves attributing widespread error to competent speakers.)

I do not have space to defend the view that we should seek a more semantic explanation of the acceptability of conditionals in general, but I do want to make a conditional claim: *if* the semantic approach to explaining acceptability is a good one in the case of non-counterpossible conditionals, we should seek to treat counterpossible conditionals in a similar way.[7] The process by which we formulate and evaluate conditionals seem very similar in both cases, and we seem to have similar powers of understanding new conditional utterances, and the point of speakers making them, in both cases. (Some counterpossible statements leave us at a loss to know what they are for or what to do with them, but the same is true of some counterfactuals with possibly true antecedents.) So treating both counterpossibles and non-counterpossibles in a unified way is desirable, if it can be achieved.

As a contribution to the project of developing an account of the acceptability or unacceptability of counterpossible conditionals asserted or otherwise considered in different contexts that *does* make an important appeal to differences in semantic value of different counterpossibles, let me now turn to discuss some different ways that context can influence what sorts of facts are "held fixed", or make for relevant similarity, when evaluating closest-world counterpossible

[7] Pace Emery and Hill 2017, among others.

conditionals across a few different contexts. As mentioned above, these remarks about some specific kinds of contexts are not intended to cover all the ways context may interact with counterpossibles. Still, they may do enough to defuse some standard objections to non-trivial counterpossibles and to explain the connection between valid arguments and the truth of corresponding conditionals.

2. Conditionals and the Logic Classroom

Consider a teacher leading a class through a proof in a logic class. She might begin with three premises:

> "Roses are red or violets are blue. Either roses are not red or my dog is green. Either violets are not blue or my dog is green. Now, if roses are red, and either roses are not red or my dog is green, then my dog is green, by disjunctive syllogism. And if violets are blue, and either violets are not blue or my dog is green, then my dog is green, again by disjunctive syllogism. So either way my dog is green, so by disjunction elimination we have that my dog is green. So, what we've shown is, if roses are red and violets are blue AND either roses are not red or my dog is green, AND either violets are not blue or my dog is green, then my dog is green."

(Imagine suitable pointing at a whiteboard while all this is going on, maybe with formulations of disjunctive syllogism and disjunction elimination off to the side.)

The teacher goes back and forth between conditional claims and claims about arguments and conclusions, and we can naturally interpret her as trying to demonstrate the use of disjunctive syllogism and disjunction elimination, argument rules that make no reference to conditionals. She can do this for a class that has not been asked to consider deduction theorems, or the function of the natural-language conditionals and how those conditionals might relate to logic; depending on how the course is structured, a teacher could conceivably present natural deduction rules for *and*, *or*, and *not*, all before introducing the material conditional. It is very natural to think that when we have a valid argument from premises to conclusions, that naturally licenses a corresponding conditional. Furthermore, at least in situations like this, it is natural to signal facts about what valid arguments there are with conditionals, as in the teacher's last quoted claim.

Variations on the above script are common too. One common variation is to mention rather than use premises and conclusions, or even to mention uninterpreted pieces of symbolism rather than sentences interpreted enough to have full truth conditions. As an example of the first sort, given the right sentences labelled A through C and F through H, one might say something like "We have a proof of F from A and B; and we have a proof of G from B and C. And from F and G we can prove H. So if we have A, B and C, we will have H" will not be out of place. Asked to go through that again, we might summarise "If A and B then F, If B and C then G, and B is the same in both places, so all we need for both F and G is A, B, and C. And if F and G, then H, so we will have enough". While strictly speaking these claims employ various devices to mention or quasi-mention sentences rather than assert them, what seems to underpin communication here is a willingness of both speaker and hearer to move back-and-forth between claims about arguments, on the one hand, and conditionals, on the other.

Techniques like these, passing freely from valid arguments to corresponding conditionals, seem very natural even in cases where a premise, or a corresponding antecedent, is impossible. Proving "Socrates is mortal" from "Socrates is a man" and "All men are mortal" is no more difficult than proving that "Socrates is mortal" from "Socrates is a number" and "All numbers are mortal": indeed, someone teaching predicate calculus or syllogistic might present the class with the latter argument just to make a point of validities in these systems being a matter of form, rather than any particular meaning a constant or predicate might have. And in this scenario, it would be natural to utter the related conditionals: "If Socrates is a number and all numbers are mortal, then Socrates is mortal".

In a closest-worlds framework, it is relatively easy to see what dimensions of similarity are important for the evaluation of conditionals in the logic classroom context. Worlds that match the actual world with respect to closure under logical relations are closer than those that do not.[8]

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[8] Note that when I am talking about match with the actual world with respect to logical closure, I am not here talking about what is true about logical closure *according* to the world in question. One question is what propositions are in fact true according to a world: which is part of the question of how to describe a world correctly "from the outside", as it were. Another question is what, according to that world itself, is true about which propositions are true. A world can represent that truths are closed under classical logic, for example. But if that world (call it W_c) is sufficiently impossible, it can represent that even though it is not closed under classical logic (that is, there can be counterexamples to the claim that *when B is a classical consequence of A, and A is true according to* W_c*, then B is true according to* W_c). The distinction between what is true according to a world and what is true about a world is among the most confusing in the discussion of impossible worlds, but it is crucial to avoid this confusion.

There seems to be a naturally available setting of context available in situations like the one the logic teacher is in. Call this sort of context, where it is of central importance that the relevant possibilities are closed under whatever the correct logic happens to be (and perhaps some of whatever the correct mathematical principles may be), the "logic classroom context". I should be clear that I do not think this context *only* is found in conversations in logic classrooms, nor that it is *always* the context relevant for utterances when discussing logic, pedagogically or not. As well as the kinds of discussions mentioned above, we also consider more speculative "what ifs" about logic: what if no argument had been deductively valid, or what if the relevant logic R had been the One True Logic? Obviously (or at least obviously to me) to the contextual standards for engaging in these latter sorts of discussions are rather different, even though a logic classroom might be a place to have some of those conversations as well.[9]

In this "logic classroom" context, with the sort of focus on logical relationships suggested above, it may even be that it is primarily whether a world is logically well-behaved that matters, and other standard dimensions of similarity may not hold. Suppose we are presented with:

A. All cats are mammals.
B. Some mammals are dogs.
C. No cat is a dog.
D. All mammals are cats or dogs.

A student is asked what do we get if we have A-D. If he replies "all dogs are mammals", or, more fully, "if we have A-D then all dogs are mammals", he has answered incorrectly: "all dogs are mammals" is not a deductive consequence of A-D. Still, by more ordinary standards, all dogs *would* be mammals if A-D were true. After all, dogs are essentially mammals, and there is nothing e.g. in the information about cats, or the absence of other kinds of mammals, that would lead us to suppose any influence on that fact. We are not forced to say that the student's answer is

[9] As I read Vander Laan 2004 p 163, he agrees there is such a context for evaluating counterpossibles; and may go further in holding there is also a context where every strict implication corresponds to a true counterpossible. Note that he distinguishes this context from one he says "we might expect to find, say, in a mathematics classroom". This illustrates the point that in fact all sorts of conversational contexts may be deployed in classrooms.

false here, but this being an unusual context where *only* logical behaviour of worlds makes for closeness could allow for that verdict.

It is easy to slip into what I call the logic classroom context when discussing what deductively follows from what. I think this is one important factor in the sense among logicians that an adequate logical conditional will conform to a *deduction theorem*: a theorem in the meta-theory of the logic that when a sentence A is a logical consequence of a set of sentences Σ, then the corresponding conditional, whose antecedent is the conjunction of the sentences Σ and whose consequent is the sentence A, will be a theorem of the logic. (Notice that a deduction theorem does not just establish that the conditional corresponding to a valid entailment is *true*, but also establishes that it is a *theorem*: the conditional itself holds as a matter of logic.)

I think the availability of logic classroom contexts is also *part* of the story about why logicians with a classical logic background have traditionally thought counter-logical counterpossibles are all true. In classical logic, any proposition is a deductive consequence of any contradiction. They are rightly in the habit of going from claims about logical consequence to making the corresponding conditional claims. A contradiction is the paradigm of a logical falsehood (and is logically equivalent to any other logical falsehood). So if any conditional of the form (A & ~A)→B is true (and indeed a theorem, for any conditional in a language that satisfies the relevant deduction theorem), and any logical falsehood is equivalent to a contradiction, that would strongly suggest that any conditional with a logically impossible antecedent should be true.

There is much to this line of thought, if I am right, but it goes too far. It overgeneralises from contexts where it is legitimate to transition from an entailment claim to the corresponding conditional claim, to conclusions about conditionals that are intended to hold in all contexts. I should be clear that there is often no harm in defining a connective that obeys a deduction theorem regardless of the conversational context, and regardless of whether the defined connective has other features "conditionals" are often intended to have (such as having a modus ponens elimination rule). The material conditional in classical logic and the strict conditional in the reflexive Kripke systems of modal logic are two such connectives. The risk of overgeneralisation about conditionals comes when we take lessons from formal logic to predict and explain the behaviour of natural-language "if" sentences and other conditional constructions.

Recognising the existence of logic classroom contexts explains a very natural transition we are inclined to make from valid arguments to conditionals, while still making room for non-trivial uses of counterpossibles (in other contexts).[10] It can also help to defuse one traditional style of argument against non-trivial counterpossibles, as I will now explain.

3. The Russell/Hardy/Williamson Argument for the Triviality of Counterpossibles

There is an old argument, apocryphally attributed to Bertrand Russell or sometimes G.H. Hardy, to the effect that if 2+2=5 (or 2×2=5), Bertrand Russell is the pope. (Or McTaggert is the pope, in Hardy's version.) One way to run it is as follows:

If 2+2 were to equal 5, then (2+2)-3 would be 5-3.
So, if 2+2 were to equal 5, then 1 would equal 2.
The number of members of {Bertrand Russell, the pope} equals 2.
So, if 2+2=5, then the number of members of {Bertrand Russell, the pope} would equal 1 (since 1 would equal 2).
If the number of members of {Bertrand Russell, the pope} were 1, then Bertrand Russell would be the same person as the pope.
So, if 2+2=5, Bertrand Russell would be the pope.

The earliest I have found in print so far is a variant using multiplication in Jeffreys 1931 p 18, who attributes it to G.H. Hardy, though I have most often heard it attributed to Russell in conversation. It can be adapted to suggest conditionals with impossible antecedents are all true. One such adaptation has recently been offered by Timothy Williamson (Williamson 2007 pp 171–173). Williamson is describing a case where, intuitively, a counterpossible is false. (That is,

[10] It would be convenient if logic classroom contexts ensured every conditional corresponding to a valid transition was true. If logic classroom contexts are this generous, I might face challenges elsewhere: for example, Nolan 2016 p 2636 relies on not being able to transition from a valid argument to the corresponding conditional in apparently *any* context. Space precludes discussing this conflict here, and for convenience I will talk in this paper as if logic classroom contexts ensures the truth of every conditional corresponding to a valid single-premise, single-conclusion argument.

he is describing an apparent counterexample to his view that all counterpossible counterfactual conditionals are true.) Let me call this class of arguments "Russell arguments", whatever their provenance.

Williamson asks us to consider a case where someone was asked "What is 5+7?" and answered "11". Talking about the case later, our speaker gets confused and believes he answered "13". As a result of that confusion, he might utter
"(30) If 5 +7 were 13, I would have got that sum right".

We may naturally think this is false, and instead this is true:
"(31) If 5 +7 were 13, I would have got that sum wrong".

(After all, as we might think, he said "11", so 5 + 7 would have to have been 11 for him to have got the sum right.) Williamson continues:

> "However, such examples are quite unpersuasive. First, they tend to fall apart when thought through. For example, if 5 +7 were 13 then 5 + 6 would be 12, and so (by another eleven steps) 0 would be 1, so if the right number of answers I gave were 0, the number of right answers I gave would be 1. We prefer (31) to (30) because the argument for (31) is more obvious, but the argument for (30) is equally strong." (Williamson 2007 p 171).

Notice that even if we conceded this example to Williamson, we would still be some way from having to concede that all counterpossibles are true. (Or even to concede that the judgements that some counterpossibles to be false are undermined.) Perhaps there is something special about counter-mathematical conditionals, or even counter-arithmetical conditionals, so that they are trivial even if not all counterpossibles are. Counter-mathematicals, especially ones with antecedents that are not only false but easily seen to be false, have some distinctive features that not all necessarily false antecedents share, after all, such as apparently being false *a priori* and involving propositions often thought to be logically impossible, as opposed to "merely" metaphysically impossible. There is room for a view that treats counter-mathematicals like these as all true, even though some other counterfactuals with metaphysically necessarily false antecedents are false. (E.g. "if water were infinitely divisible, it would be made up of equal parts hydrogen and carbon", or "if I were a dolphin I would not be a mammal"). It would be a hasty generalisation from Russell/Hardy examples to the trivial truth of all counterpossible conditionals.

I will pursue the more ambitious route of claiming that even countermathematical conditionals have non-trivial truth-conditions, and some counterfactuals with mathematically impossible antecedents are false. (Some of course are true as well.) Even in Williamson's case, I think we should stay true to our first thought, which is that in the situation and conversational context in which (30) is uttered, it expresses a falsehood, and instead (31) would express a truth. What Williamson is inviting us to do, in engaging with his further reasoning, is to shift to a context where "if 5 +7 were 13 then 5 + 6 would be 12" comes out true, and perhaps the other 11 counterfactuals mentioned. Then, in either the same context or a slightly different one, we also find the counterfactual about numbers of answers compelling, as uttered in that context. (I am not sure quite how to reconstruct the intended counterfactual, but perhaps "If 0=1, then if the right number of answers I gave were 0, the number of right answers I gave would be 1" would do. We would of course need to be careful to be sure this is the sort of nested conditional that behaves as Williamson needs, and not e.g. one of the nested sort discussed by McGee 1985.)

Williamson has to rely here on dubious principles of reasoning with chains of conditionals, including nested conditionals. So the argument Williamson offers here for (30) is significantly less compelling than the arguments for (31), even apart from any concerns about context shifting. But if Williamson has shifted the context in a crucial way in moving to his argument for (30), his argument may not have much bearing on whether (30) was false in the envisaged context and scenario we were originally evaluating. I should be clear that Williamson intends no sleight of hand here, and I doubt he thinks he has shifted the context relevant for evaluation of the sentence, at least not in a way that matters for his argument. Still, if this shift of context is what is behind our shifting confidence in the falsehood of (30), Williamson's argument has equivocated regardless of his own views about how context interacts with conditionals.

Notice that a crucial move in this argument is the transition from demonstrating an apparently valid inference to the conditional: in effect, we deduce from 2+2=5, plus some true side premises, that Russell is the pope, and conclude that if 2+2=5, then Russell is the pope. This suggests that something like a logic-classroom context is in play. Context may have to do even more work than in the cases I considered in the previous section, since not all the premises are explicit in the antecedent: but given that the reasoning is often found compelling, we have reason to think that in these contexts at least, not just the validity of the associated argument but the truth of the other premises is "held fixed" in the closest worlds relevant for evaluating the conditional.

Once we keep in mind that counterfactuals, and conditionals more generally, are context-sensitive, the lesson from conditionals like "if 2+2=5, then Russell is the pope" and "if 2+2 were 5, then Russell would have been the pope" is that *we can set up a context* where "if 2+2=5, then *C*" is true for any substitution instance *C*. (And indeed, plausibly this has been done by the end of giving the Russell-Hardy argument.) It does *not* follow from this observation that conditionals like this are true in every context of utterance, or even in most contexts. In particular, it does not show, or even make plausible, that "if 2+2=5, then Russell is the pope", or "were 2+2 to equal 5, Russell would be the pope" are true in other contexts besides the special logic-classroom-like contexts created by presenting the argument.

Why did this sort of argument seem compelling to philosophers and mathematicians as a general argument for the vacuity of counterpossibles? To the uninitiated, it does not feel like a contextual sleight-of-hand in the way, e.g. trying to argue that an admittedly big mouse is big compared to an elephant would be. (After all, you already conceded Nibbles was big, right? And you conceded Dumbo is small? So Nibbles is the big one of the two?) I think there are a few reasons for this. That conditionals are contextually variable is not as obvious as that "big" is contextually variable. Less obvious still is how this contextual variation works. (Williamson 2020 argues that the indicative conditional is not contextually variable, for example, and Bennett's influential introduction to the philosophy of conditionals (Bennett 2003) rejects contextual variability for indicative conditionals and even develops a contextually invariant account of counterfactuals.) So it could be easy to generalise from the behaviour of conditionals in logic-classroom contexts, where facts about logical entailment are especially salient, to other contexts.

Another thing that may have helped Russell arguments be so persuasive is that most of us have not thought much about the behaviour of conditionals with impossible antecedents, especially ones like "2+2=5". With little idea how people handle them, we are prey to the first plausible-seeming intuition pump that comes along. A final reason Russell arguments seem to have such general morals is that so many attempts to offer theories of the truth-conditions of conditionals yield trivial counterpossibles: early material-conditional accounts, strict conditional accounts, and even Lewis-Stalnaker closest world proposals. Those looking for a piece of intuitive support for a doctrine they accept on general theoretical grounds may look less closely than when a case seems to go against a result of their theory. (By the same token, of course, I have been motivated to understand Russell arguments in a way that does not doom the project of understanding many counterpossibles as non-trivial.)

Berto et al 2018 p 705 diagnose a different problem in Williamson's argument involving context. They suggest that (31) is true in a much wider range of conversational contexts that (30), given the background facts stipulated in the example. So they suggest that Williamson's argument to (30) from the background facts of the case likely fails, since there is no guarantee that Williamson has managed to set up a context where all the intervening counterfactual steps express truths. Or at the very least "for Williamson’s argument to work, he needs to argue that we are in such a context, which he does not attempt" (p 705). In a way, then, I am more sympathetic to Williamson's argument than Berto et. al. For I have been, in effect, arguing that we can easily be in the kind of context Williamson needs, with the contextual cues that Williamson gives. *In that special context*, (30) as well as (31) are true. I do agree with Berto et. al.'s broader point, of course, that context dependence of counterpossibles means that merely showing that a counterpossible sentence expresses a truth in one context of utterance does little to show that it is never false in other contexts, and I agree with them that this is an important part of explaining the behaviour of countermathematical conditionals. See Berto et. al. 2018 pp 703–4, for example, for an illuminating account of the behaviour of such conditionals in reductio proofs.

4. Cases of Non-Trivial Mathematical Counterpossibles

It should be clear by now that a contextual parameter would allow us to endorse the reasoning to "if 2+2=5, Bertrand Russell is the pope", when it occurs, without having to endorse the truth of all conditionals of the form "if 2+2=5...", in all the contexts in which they might be uttered. But at least two important questions remain. How well can we independently motivate the idea that counter-mathematical conditionals should be treated as non-trivial in some contexts? And what principles are governing the contexts of conditionals, such that we get the result that counter-mathematicals are largely (or always) true in logic-classroom contexts, but are predictably false in others?

One species of argument that countermathematicals are sometimes non-trivial, in the contexts in which they are uttered, is to provide intuitive examples. There are many of these, and I find many of them compelling. In Nolan 1997 p 544, I offered as an example of a true countermathematical "If Hobbes had squared the circle, sick children in the mountains of South America at the time would not have cared". (Squaring the circle was something Hobbes in fact repeatedly attempted, but alas it is mathematically impossible.) Various contrast cases will also sound false: "if Hobbes had squared the circle, he would have been made King of England", for example. We do not need to restrict our attention to consequents concerning the physical world. "If there were only finitely many primes, not every even number would be the sum of two primes" is very plausibly true: after all, there are infinitely many even numbers but only finitely many pairs of a finite set of numbers. On the other hand, "if there were only finitely many primes, every even number would be the sum of two primes" is plausibly false, by the same reasoning.

Examples can be piled up, but they are unlikely to convince those who have already taken sides in the debate. They also do little to give us a way of saying something informative in general about *which* countermathematical are true and which are false. It would be more satisfactory if we could add to these arguments at least a sketch of a positive proposal about when we should expect countermathematicals and counterlogicals to be straightforwardly true, when straightforwardly false, and when to expect difficulties in determining a truth value. Here I will attempt some of this sketch, for two kinds of natural contexts. (For a description of another kind of case where countermathematicals are systematically used, see Jenny 2018.)

Instead of offering a general formula for how context fixes similarity between worlds outside of logic classroom contexts, instead I want to briefly sketch two other contexts in which we might want to evaluate a class of countermathematical conditionals, and say something informative enough about communicative purposes, salience, etc. so that a reader knows "how to go on" in those contexts: what sorts of counterfactuals (and indicatives) are correct, and what sorts are incorrect, at least for a limited class of countermathematical claims in each. Each of these kinds of situations also demonstrate why getting things right with certain countermathematicals can be of systematic practical benefit for people evaluating the conditionals, beyond mere philosophers' intuition trading. Systematic use of a class of conditionals by a community pursuing a cognitive project can also help us feel we have a grip on the phenomena in a way intuition trading may not.

4.1 The Inconsistent Infinitesimal Calculus

The standard way to approach calculus problems is with the epsilon-delta (ε/δ) understanding of limits: when calculating the instantaneous rate of change of a quantity, for example, and given a curve that represents the change of that quantity over time, we can calculate the gradient of the line tangent to that curve. In turn, we can think of this as the result of a series of better and better approximations via lines through pairs of points on the curve, where the distances between the pair of points and the point we are interested in get smaller and smaller. (Of course, one can just learn what the derivative of various kinds of equations are, and crunch the numbers – but it helps to have some conception of what the derivative represents.)

There are other ways of conceiving of taking derivatives and integrals. (I will focus on differential calculus in what follows, but the same range of options is available for integral calculus.) One other way is to use sophisticated non-standard analysis involving consistent theories of infinitesimals, such as those devised by Robinson 1966 or Conway 1976. The idea behind infinitesimals in the case of taking derivatives is to determine the tangent of a curve at a point by looking at a line drawn through that point and another point "infinitesimally" along the line from the first point. Infinitesimals are intended to be quantities greater than zero but less than any real number higher than zero. A point infinitesimally along the curve from a point is supposed to be "so close" to the original point that the line through the two points is effectively just like a tangent line that passes only through the original point.

It turns out, however, that one surprisingly easy way to employ infinitesimals is to employ the "inconsistent infinitesimal calculus". In this manner of proceeding, we do not posit an infinitesimal quantity strictly between 0 and all the positive real numbers: we just posit an infinitesimal quantity that, in effect, counts as being non-zero at some points in the calculation and as equal to zero in other parts. Presentations of calculus in textbooks up to the early twentieth century often employed infinitesimals in this way, and some prominent mathematicians were on record praising this as a way of learning calculus (see Katz and Tall 2012). Some contemporary empirical research has shown that high-school level students introduced to calculus via infinitesimals do better solving certain problems and answering certain conceptual questions about differentiation than those trained in standard ε/δ techniques (Tall 1990).

As its name suggests, a natural way to formulate the inconsistent infinitesimal calculus is inconsistent. It postulates one or more quantities ε, which are greater than zero at some points in the calculation and equal to zero at others. Employing other standard principles of mathematics, this can quickly lead to disaster: from $\varepsilon = 0$ and $\varepsilon > 0$ we can derive that $0 \neq 0$. It may be somewhat surprising, given this background, that the techniques of inconsistent infinitesimal calculus can deliver the same answers to typical calculus questions as its consistent ε/δ cousin. I will make the relatively uncontroversial assumption that there are no quantities x such that $x = 0$ and $x > 0$ (pace Mortensen 1990).

Students taught infinitesimal understandings of calculus are not taught using counterfactuals. (In the experiments reported by Tall 1990, they were taught from textbooks written to be compatible with consistent theories of infinitesimals such as Robinson's. And in the bad old days inconsistent infinitesimal calculus textbooks taught that doctrine as the truth, not in a hypothetical way.) However, we can easily imagine students being given an explicitly hypothetical introduction to infinitesimals. A teacher, motivated perhaps by classical scruples or a desire to avoid later confusion, might introduce the inconsistent infinitesimal ε in a more "as if" fashion than is standard. She may begin with a speech along these lines:

> Okay class, we are going to move on to a new kind of problem. We are going to learn how to calculate rates of change of a quantity. (For instance, if we get a formula specifying an object's velocity over time, we are going to be able to calculate the object's acceleration at any given moment.) To do this, we are going to take a convenient shortcut...

Then, at test time, the test sheet could be introduced as asking conditional questions (or alternatively questions under the supposition that there are inconsistent infinitesimals of the right sort). e.g.

> (Q) If there were ε quantities of the sort described by the inconsistent infinitesimal calculus, and a car travels from rest with a velocity expressed by the equation v=7t, where t is measured in seconds and v is measured in m/s, what would the car's acceleration be when t=5 in m/s^2?

Apart from slight difficulties in parsing a longer question, I would bet students taught the methods of inconsistent infinitesimal calculus could perform as well on an exam with questions like this as students in more standard calculus exams who had been taught other methods of determining derivatives. (Or perhaps students taught the inconsistent infinitesimal calculus would do better, given the results reported in Tall 1990.)

As well as a correct answer, (7), there are also incorrect answers. A student in a mathematics exam who answers the car would have an acceleration of $0m/s^2$ at t=5, or $35m/s^2$ at t=5, would be making a mistake and would be marked down. We can even check which answers to questions like these are correct without ourselves needing to evaluate counterpossible conditionals directly, since the answers will agree with those reached by consistent methods.

It is possible to take a "logic classroom context" approach to counterfactual questions like (Q), and to insist that any numerical answer would be the right one. (After all, assume $0 \neq 0$, ...) That would not be the right way to answer differentiation questions using the inconsistent infinitesimal calculus, and students doing that would rightly get low grades. My explanation for the difference is a shift in conversational context: the context in which (Q) is posed and evaluated in a calculus exam is one where students should work out the answer using inconsistent infinitesimal calculus techniques, and where there are right and wrong answers about how to use ε. Even those who do not like that theory about what is happening, however, should be able to see how to go on, and what the right and wrong answers are supposed to be. It was a significant intellectual achievement to *avoid* thinking in terms of a quantity that was somehow larger than zero and somehow zero when developing the ideas behind calculus, so it should be no surprise that there is a coherent way to counterfactually entertain the supposition that calculus works that way, in a disciplined enough way to help with solving calculus problems.

Not only facts about the non-mathematical world, but most of mathematics is the same in the closest (impossible) worlds where there is a value like ε. I doubt the context produced by introducing inconsistent infinitesimal calculus and teaching calculus techniques using it is specific enough to yield answers to all counterfactual questions involving ε. (Were there such a quantity as ε, would my height be given by a real number or by a real number plus ε?) But that seems fine to me: even with non-counterpossible counterfactuals, not every context determines precise answers to every counterfactual question. As long as there is a class of counterpossibles that are clearly true, and another that are clearly false, the context I have been discussing will do to illustrate a natural alternative to the logic classroom context.[11]

4.2 Sociology of a Mathematics Department

A quite different kind of context may weight institutional and social facts heavily for relevant similarity, while being considerably more flexible about which mathematical principles to vary.

[11] For an attempt to more systematically work out what would be true, were the inconsistent infinitesimal theory correct, see Brown and Priest 2004. They are concerned in the first instance to characterise appropriate and legitimate reasoning against the background of adopting an inconsistent infinitesimal theory, and to offer an account of how that reasoning was carried out, when it was done well, though their account does have implications for what would be true were this theory correct. The questions addressed by Brown and Priest are ones that are left open by my account, as well as important historical questions about how people approached the calculus and how they grappled with theoretical questions about infinitesimals.

Consider a case adapted from Bernstein 2016 pp 2580–2586, and related to the case described in Jenkins and Nolan 2012 pp 739–743. Avery is a mathematician on the tenure track in a mathematics department in the US. His teaching and service are fine, and he is liked by his colleagues. His university is not under any unusual pressures, but research plays a central enough role in its values that his department and the college tenure-and-promotion committee that his tenure case will be determined largely by what the department and his tenure letter writers think of his research.

Avery has a few minor research accomplishments to his name, but the main thing Avery has been pouring his research efforts into in the final few years in the lead up to tenure has been an attempt to prove a particular bold and important mathematical conjecture: let us call it "Avery's conjecture". Other experts in the field have proved that Avery's conjecture bears on central long-standing open problems; his letter writers are excited about the conjecture; and are all of the opinion that a proof of a conjecture of this importance would be at least as valuable as the central achievements of other mathematicians who have received tenure at top departments in recent years. Unfortunately for Avery, he does not prove the conjecture. Unbeknownst to him, the conjecture is a mathematical falsehood, and so nothing he could have feasibly done would have been a proof of Avery's conjecture. (As opposed to a "proof" that people thought was a genuine proof. Alas Avery also does not come up with a non-proof that fools the community.) Avery's tenure case is unsuccessful, and he has to look for a job elsewhere.

Consider the conditional "If Avery had proved Avery's conjecture, he would have received tenure." In the scenario above, and spoken e.g. by a senior colleague aware of the ins and outs of Avery's case, it is plausibly true. And it is true even though Avery's conjecture is a mathematical falsehood: even though it was not even possible that Avery successfully proved that conjecture. Likewise, if an unduly cynical colleague uttered "If Avery had proved Avery's conjecture, he would not have received tenure", that conditional would plausibly be false in the scenario envisaged, given the natural context of utterance. If a third, deluded, colleague uttered "If Avery had proved Avery's conjecture, he would have rapidly become university president", that colleague would betray a warped understanding of how university administration worked, and would have been wrong.

To see why the Head of Department's claim is non-trivial, contrast an alternative possible scenario where things are otherwise not going as well for Avery. He is employed by a college looking to reduce lines, his teaching is considered unsatisfactory, and his colleagues find him abrasive. In addition, his other mathematical work has also been unsuccessful, and he has nothing else to show on the research front. He makes the same conjecture, but a lack of publicity means that there is much less excitement in the mathematical community about it: in particular, no other mathematicians point out that the conjecture would allow proofs of important consequences. He fails to prove Avery's conjecture and is denied tenure. In some cases like this, the counterfactual "If Avery had proved Avery's conjecture, he would have received tenure", as uttered by an optimistic colleague afterwards, would be *false*. Even if that proof had been successful, it would not outweigh the other considerations running against him, and would not sufficiently sway his college's processes.

Notice that the contexts in which these mathematicians are discussing Avery's career trajectory are different from "logic classroom" contexts. A mathematician examining the history of the event several centuries later, in full awareness that Avery's conjecture is provably a mathematical falsehood, uses straightforward techniques to show that 1=0 follows from Avery's conjecture, and then Russell/Hardy reasoning to come to utter "If Avery's conjecture were true, Avery would have been the pope". In that context, with that background, she utters a truth, in my view. (And likewise, mutatis mutandis, if she varies the reasoning to show "If Avery's conjecture were true, Avery would have been the university president", "If Avery's conjecture were true, Avery would have received tenure", etc. etc.) Even though these claims are true in her mouth, that does not make the department chair's claims trivial: our later mathematician in a logic classroom context is neither agreeing nor disagreeing with the department chair, but rather making those claims in another context.

There is a plausible story to tell about what is held fixed in the contexts where Avery's colleagues utter "If Avery had proved Avery's conjecture, he would have received tenure." in the two scenarios envisaged. We hold fixed facts about the general attitudes of the department, the tenure and promotion committee, and the tenure letter writers: though of course we envisage some *specific* attitudes being different in the relevant counterfactual scenarios where Avery is successful, since presumably in the Avery-proves worlds they see the proof, perhaps work through it, discover Avery has published it, etc. We also hold fixed various facts about how Avery's university works, who his colleagues are, who constitutes the tenure and promotion committee, the broad goals of people deciding on recommendations, etc. (It would be gratuitous to consider an impossible world where he proves the theorem but the university goes bankrupt, if there was no risk of bankruptcy in the original scenario.) What we do not hold fixed are all the mathematical facts: the relevant counterfactual worlds are worlds where Avery's theorem is a mathematical truth, since they are worlds where he proves it.

Telling the full story about how what is relevant to counterfactuals about how people would have acted is not easy: fortunately, the vast majority of us are fairly good at evaluating these sorts of counterfactuals in practice. So without an explicit theory of the contextual settings of these counterfactuals, I want to gesture at a well established practice. You know the thing we do when we try to work out what a committee would decide if they had different information? We do *that* sort of thing when reasoning about whether Avery would have got tenure, had he proven Avery's conjecture.

I think a story can also be told about why we would engage in a practice of using this sort of counterfactual. It is very important to us to predict and explain what people around us do, and would do. After-the-fact counterfactuals about how things would have gone differently, socially speaking, if things had been otherwise, are probably less important than future-oriented predictions. Still, these sorts of postmortems seem to be valuable in general for learning how the social institutions around us work, and to help us think about how people around us will behave in future.

The benefits from thinking about what people and institutions would have done, had things gone differently, do not suddenly disappear when we move to counterpossibles. Understanding how tenure works in a mathematics department is not miles away from understanding how tenure works in a microbiology or English department, even though tenure in a mathematics department can be sensitive to what candidates have proved, and that in turn is sensitive to which mathematical claims are in fact true.[12] (You cannot prove a conjecture unless it is true.) One of Avery's junior colleagues, trying to work out what she should do, can benefit from a conversation about what it would have taken for Avery to get tenure, just as much as junior faculty in areas which do not study matters that hold of mathematical necessity. I am happy to concede that there is some range of non-counterfactual facts underlying the situation Avery found himself in, and underlying the situation other junior faculty in mathematics departments trying to learn from Avery's travails. Still, the most cognitively available method to learn from what happened to Avery, and to plan accordingly, may be to consider counterfactuals about what would have happened if, for example, Avery had managed to prove his conjecture.

There is more to be learned about how exactly we negotiate the tricky business of deciding what is held fixed in these social counterfactuals: even given a particular context it might be very hard to work out which factors to include and which to ignore, and deciding that we want a context where institutional facts about Avery's case are largely respected still leaves a lot of leeway for further constraints on context. If the use of context in conversation elsewhere is a good guide, it is likely that the contexts of actual speech situations leave it somewhat indeterminate which truth conditions are associated with sentences: often we seem to muddle through in communication using contexts that settle enough for communicative purposes, and doing repair or asking for clarification if things left indeterminate need to be filled in. It seems to me, however, that these are all instances of *general* features of context which a theory of counterfactuals has to grapple with. Counterfactuals are useful for us despite their having this open-textured feature.

In this section I've looked at two fairly specific kinds of contexts for evaluating some counterpossible conditionals that are all counterlogicals or countermathematicals. In these contexts counterlogicals and countermathematicals are non-trivial, in contrast to the logic classroom context.

Countermathematicals uttered in the context of employing the inconsistent infinitesimal calculus in the sort of classrooms described behave quite differently from "logic classroom contexts". A

[12] How should we cash out this "sensitivity", given that there can be no difference in what purely mathematical propositions are true? I think we should also cash it out in terms of counterfactuals. (What would have happened, or might have happened, *if p* were true.)

significant amount of mathematical information is held fixed, and most of what is the case in the non-mathematical world is held fixed too. What position an object would have, were it subject to such-and-such accelerations from such-and-such a starting point, and the inconsistent infinitesimal calculus were also correct, turns out to be the same, (or infinitesimally close to the same), as if that object were simply subject to those accelerations from that starting point. (The fact that we get basically the same answer is part of the explanation of why solving calculus problems using the inconsistent infinitesimal calculus is useful.) A difficult student who argues that all of their answers are correct, since if there were a quantity ε=0 & ε>0, then 0~=0, and every mathematical equality would follow, is not just missing the point of the exercise: she is in effect treating the context of the counterfactual question as different from the context in which it was asked, and is mistaken about what the correct answers are to the teacher's questions *in the context in which they were posed*.

Countermathematicals uttered in "sociological" contexts like those set by discussions of the unfortunate Avery are non-trivial in a different way. Here we do not need to do calculations in a contrary-to-fact mathematical system. Instead, we focus on contrary-to-fact institutional and psychological matters, and we attempt to work out what would be the causal result of them. Apart from Avery's conjecture and some of its corollaries, insofar as we do any calculations at all when determining if Avery would have got tenure, we are likely to use mathematics we take to be in fact correct. (E.g. if we want to work out how much money the university could save from terminating Avery's line, or which side has a majority if there are 8 people in favour and 4 against on a crucial committee.)

Rather than list a set of rules for similarity of worlds determined by the two kinds of contexts just discussed, I have been relying on our pre-theoretic grasp of how to use counterfactuals to guide us in articulating some counterfactuals that are true and some that are false, given a linguistic context and a situation being described. While this procedure unavoidably leaves a lot of interesting questions unsettled, and is fallible, it can do enough to not only show that there are contexts where some counter-mathematicals are non-trivial, but also to explain why it would be useful for us to have these contexts available. It makes some sense of our practice of uttering some counterpossibles rather than others in these situations, and so makes an advance on the mere observation that some counterpossibles strike us as true and some strike us as false.

5. Conclusion

Noticing that there are "logic classroom contexts" where logical entailments correspond to true conditionals is useful for several reasons for those of us who want to maintain that some counterpossible conditionals are false. One set of advantages are dialectical. We can explain the natural tendency to move from claims about what follows from what, on the one hand, to what would be the case if..., on the other. And we can explain this without adopting a semantics of conditionals that guarantees that that every true entailment corresponds to true conditional. (Just as well, since in other contexts, there are examples of conditionals where the antecedent entails the consequent, but the conditional nevertheless strikes us as false.)

The other main dialectical advantage is that we have a straightforward, and to my mind plausible, diagnosis of the error in a family of typical arguments for the trivial truth of counterpossible conditionals. These arguments employ a logic classroom context to vindicate a range of counterpossible conditionals that would otherwise sound odd, and then suggest that the truth of these counterpossibles is in tension with the idea that there could be *any* false counterpossibles. Once the role of context in our use of counterpossibles is appreciated, going from the conclusion that many counterpossibles *in logic classroom contexts* are true to the conclusion that counterpossibles of this sort are true *in any context* is unsupported. (At least to date: perhaps future refinements of these arguments will offer something to fill in the gap here.)

The recognition of contextual variation offers us something else as well. When a context-sensitive account of the truth-conditions of conditionals is offered, something should (eventually) be said about which contexts conditionals are uttered in, and what the influence of these contexts is on the truth-value of the conditionals produced in them. My own view, illustrated here, is that the influence of context on conditionals is not a simple matter. But we can make progress in understanding the contexts of use of conditionals by understanding some case studies, and making some limited generalisations about special kinds of cases. Those more limited generalisations will be springboards for offering even more general accounts of how linguistic context is determined, and influences truth-conditions. (Eventually the story will presumably link up with accounts of the function of context for other expressions as well, both expressions with affinities with conditionals, such as natural language modal claims, and context-dependent expressions very different from conditionals.)

Those deeply invested in semantic theories of conditionals which determine that all counterpossible conditionals must get the same truth-value are unlikely to be won over simply by pointing out that their opponents have a plausible story to tell about logic classroom contexts, nor even by pointing out that their opponents have reason to remain unmoved by the Russell style arguments discussed above. However, those not yet entrenched may find these observations useful in making up their mind. And for those of us who needed little or no convincing that counterpossible conditionals are non-trivial, the suggestion defended in this paper gives us a better handle on the connection between entailment and conditionals, and takes one step towards a general theory of what contexts are found in the background of our use of conditionals and the effect of those contexts on the truth-values of conditionals on particular occasions of use.[13]

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