Stoic Trichotomies

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Long and Sedley point out that where we might expect a *tertium non datur*, Chrysippus and the Stoics often seem to postulate a third option as well. When discussing the theory that the Stoics may have classified some objects as being *neither* corporeal nor non-corporeal, they say:

> Such trichotomies are characteristically Stoic: cf 'true, false and neither', 31A5; 'equal, unequal and neither'. 50C5; 'good, bad, and neither' 58A; 'the same, different, and neither' 60G3.[1]

There are a number of other apparent cases of Stoics offering a trichotomy where we might expect a dichotomy, apart from the possible corporeal/incorporeal/neither classification and the other four listed by Long and Sedley. Plutarch attributes to Chrysippus the view that the ultimate parts of objects are neither finite in number nor infinite (Plutarch *Comm. not.* 1079C-D (= LS 50C)), and Plutarch attributes several other trichotomies to the Stoics: that the sum of everything is neither in rest nor in motion (Plutarch *Comm. not.* 1074A), and it is neither a part nor a whole (Plutarch *Comm. not.* 1074C), for example. Plutarch lists many more where the Stoics appear to be denying both of a pair of apparently exhaustive options, but some of these may be conclusions of arguments Plutarch offers from Stoic premises rather than conclusions the Stoics themselves

[1] A.A. Long and D.N Sedley, *The Hellenistic Philosophers* (2 volumes) ['LS'], (Cambridge University Press, 1987), Vol 1, p 165. Unless otherwise noted, translations are from LS.

drew.[2] Finally, some contemporary theorists have seen Chrysippus as offering an ontology according to which there are existing things, non-existing things, and 'not-somethings' in a third category.[3]

This pattern might suggest that the Stoics thought there was a third option besides *p* and *not-p*: things that were neither true nor false, equal nor not-equal, neither same nor not-same, etc. But this runs counter to most of what we know about Stoic logic, particularly Chrysippus's logic. Chrysippus endorsed both bivalence for assertibles/*axiomata* (Cicero *On Fate* 21 (= LS 20A)) and excluded middle in his logic (Sextus *M*. 8 282): that is, both that every assertible is either

[2] These include the claims that the sum of things is neither heavy nor light (*Comm. not*. 1074A), neither animate nor inanimate (*Comm. not*. 1074B), neither complete nor incomplete (1074C), and that some gods are neither mortal nor immortal (1075D), and later, in 1080B-C, that some circles are neither equal nor unequal to each other, and some angles and lengths and heights and bodies are neither equal nor unequal to each other.

[3] See, for example, J. Brunschwig, 'The Stoic theory of the supreme genus and Platonic ontology' ['Supreme Genus'], in Brunschwig, J. *Papers in Hellenistic Philosophy,* (Cambridge, 1994), pp 92-157, though a Chrysippean commitment to a third category of "not somethings" is rejected by V. Caston, 'Something and Nothing: The Stoics on Concepts and Universals' ['Something and Nothing'], *Oxford Studies in Ancient Philosophy,* 17 (1999), 145-213.

true or false, and that for any claim *p*, a claim of the form '*p* or not-*p*' will always be true.[4] Chrysippus is apparently as explicit as any ancient author that there is no third option here. So this pattern of postulating three options when there initially seem to be only two is especially puzzling. If Chrysippus did really support bivalence and excluded middle for all assertibles, then presumably he was not appealing to violations of excluded middle in appealing to these third options. But then what was he doing, and why did he think appealing to these sorts of third options would either help his philosophical system, or help his arguments against rival schools? While contemporary interpreters have puzzled about many of the particular cases where the Stoics employ these trichotomies, the challenge of explaining the *pattern* of trichotomies has received less attention.

This paper will diagnose what is going on in these appeals to trichotomies, through looking at a number of the particular cases where Chrysippus invokes a third option when the first two options had apparently exhausted the field. Each of these cases has been extensively discussed, and many are quite controversial exegetically. Nevertheless, perhaps ambitiously, I want to claim that it is tolerably clear what is going on in some of these cases, at least for the purposes in hand.

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[4] There may be some differences between the doctrines Chrysippus plausibly endorsed and the doctrines that go under the labels "bivalence" and "excluded middle" today: see J. Barnes, *Truth etc.: Six Lectures on Ancient Logic,* (Oxford University Press, 2007), pp 1-6. But most of these differences are subtleties that can be put aside for the current discussion: in particular, complexities due to views about *when* assertibles were true or false.

Once we have an understanding of what Chrysippus had in mind in particular cases, we can illuminate what was behind his general tendency to distinguish a third option besides the two options that at first might have seemed to cover all the cases, or which even seem to be exhaustive through the second just being the contradictory of the first (equal or unequal, same or different, finite or infinite, etc.). With a general hypothesis about what Chrysippus was doing supported by examination of the clearest cases, we can use that general understanding to help settle difficult interpretive issues in other cases where the state of our evidence makes it hard to directly reconstruct what Chrysippus had in mind. Section 2 illustrates this with an especially puzzling trichotomy: the fact that when Chrysippus discusses the paradoxes of the cone and the pyramid, he wishes to say that two surfaces are neither equal, nor unequal (that is, presumably, neither equal nor unequal to each other in area).

Explanations of puzzling Stoic trichotomies to date have tended to focus interpretive efforts on particular cases, bringing in perhaps only one or two others for illumination. However, if there is a general pattern here, interpretations of particular cases that cannot be generalised are missing something important. There are materials for a general strategy implicit in some approaches already in the literature: for example, we could, after all, think that Chrysippus thinks there are failures of bivalence or excluded middle in these cases. This could be because Chrysippus gives up one or both outright: Gould claims Chrysippus "is, in effect, negating the law of excluded middle", for example.[5] More plausibly, perhaps Chrysippus denied that the problematic class of

[5] J. B. Gould, *The Philosophy of Chrysippus*, (SUNY, 1970), p 118

sentences were either true or false, and/or denied they obeyed excluded middle, because he held that there were no assertibles associated with those sentences: this would be to generalise a solution to some interpretative puzzles about Chrysippus on vague language, generic sentences, and the Liar paradox offered by Bobzien and Caston, among others.[6] The idea is that we can abandon the claim that every assertoric *sentence* is either true or false, while keeping the doctrine that every assertible is either true or false, and so keep bivalence at least for assertibles. Treating these suggestions as *general* suggestions for handling Stoic trichotomies face serious limitations: see section 3, below.

The hypothesis of this paper is that there is a general idea behind positing an unexpected third option, but that when Chrysippus does this it not because he is endorsing any violation of excluded middle or bivalence. Instead, when he adopts a third option using this sort of paradoxical language, he does so by arguing that the first two options, which *appear* to be contradictories, are merely contraries, and the third option is a third, internally consistent, contrary of the other two. Chrysippus's pronouncements, then, have the air of paradox – he at first sounds like he wants an impossible *via media* – but then it transpires that the initial appearance that the first two options were exhaustive is misleading.

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[6] see S. Bobzien, 'Chrysippus and the Epistemic Theory of Vagueness' ['Vagueness'], *Proceedings of the Aristotelian Society,* 102.1 (2002), 217-238, and Caston, 'Something and Nothing' p 193.

If this is right, we would like to know why Chrysippus puts things in this initially off-putting way: surely it would be more effective to avoid even the initial air of impossibility? I will discuss this issue briefly after considering the evidence about what Chrysippus said, and how we might interpret it, but I am afraid the state of our evidence makes any guess about Chrysippus's purpose here speculative.

A standard note of caution: part of the argument in this paper is that the presentation of Stoic positions in this kind of trichotomy form is due to Chrysippus. In attributing anything to Chrysippus, we run into the standard problem that many of our ancient sources only attribute views to "the Stoics" in general, without making clear who exactly the ancient author had in mind. Fortunately in this area many of the testimonia mention Chrysippus explicitly, though there is always some risk that positions have been misattributed to him.

In the next section, I will discuss four of the most straightforward trichotomies that Chrysippus provides. After establishing a pattern in Chrysippus's use of these trichotomies, I will go on in section 2 to discuss a particular trichotomy (or pair of trichotomies) that are particularly puzzling, arguing that my diagnosis of the clearer cases sheds partial light on what sort of solution Chrysippus had in mind. In section 3, I will discuss the question of why Chrysippus seems to have adopted this way of putting his views, while in section 4 I will discuss and reject some alternative options for understanding the trichotomous pattern in Chrysippus's approach to philosophical problems.

1. Example Trichotomies

I will begin by discussing two rather straightforward trichotomies, where any appearance that the third option is paradoxical is dispelled once the third option is made clear. Sextus Empiricus reports the Stoics as holding that "parts are neither the same as wholes nor are they different from wholes" (Sextus *M*. 11 24, in LS60 G. See also Sextus *M.* 9 336). At first thinking that *A* is neither the same as *B* nor different from *B* may seem contradictory, if we are thinking of difference as just being not the same. But there are two things that "not the same" might mean, corresponding to the ambiguity in the English word "distinct". "Distinct" can be used to mean "not identical to", or it can be used to mean "not overlapping" —that is, *disjoint*. In this latter sense of distinct, of course, my hand, for example, is neither identical to me nor 'distinct' from me. Sextus indeed reports that the Stoics use hands as an example: "the hand is not the same as a whole man, since the hand is not a whole man, but nor is it other than the whole since the whole man is conceived as man together with his hand."[7]

[7] This manner of talking, according to which parts are neither the same nor different from wholes, appears also in Plato: see *Parmenides* 146B 2-5, and the discussion in Barnes, J. 'Bits and Pieces' in Barnes, J., *Method and Metaphysics*: *Essays in Ancient Philosophy* ['Bits'], (Oxford University Press, 2011), 429-483, p 430. Note that Stobaeus 1.179 (= LS 28D) reports a Stoic view that the "peculiarly qualified" entity is neither the same nor different as its constituent substance: if this goes back to Chrysippus, this may be another case where objects are neither the same nor different due to overlap. Barnes 'Bits' pp 459-461 agrees that "different" can be understood as "disjoint" in some apparently paradoxical Stoic claims that wholes are neither the

We appear to have a straightforward and unmysterious explanation of what is going on here. Of course, there are possible interpretations which take the Stoics to be embracing a contradiction, or embracing failures of excluded middle (the same or not the same), but there does not seem to be any reason to endorse anything so extravagant in this case, at least. Even Sextus Empiricus does not claim to see anything odd here—he mentions this position on wholes and parts in the context of setting out other, more contentious, Stoic views about the relation of "benefit" to men.

The second trichotomy is so straightforward that perhaps it does not belong in the list of problematic or paradoxical-looking trichotomies at all. This is the trichotomy according to which "some existing things are good, others are bad, and others are neither of these" (Diogenes Laertius, 7.101 (=LS 58A)). There need be no mystery here: the Stoic view is that some things

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same as, nor different from, their parts, and so agrees with the diagnosis I offer in the text for some of these cases. Barnes does go on to suggest that perhaps some other Stoics held a conceptualist view of parts which explain their use of "neither the same nor different" locutions (pp 461-3). I doubt this extra resource is needed to make sense of the testimonia we have, but a full discussion would take us too far from the focus of the present paper. It is possible that the view that a man is neither identical to nor distinct from his hand enters the Stoic tradition after Chrysippus, though I think it likely Chrysippus derived it from Plato. At any rate, I will talk as if it is Chrysippus's in what follows. If it is not Chrysippus's, then I have one fewer clear case to support the general conjecture I want to make about Chrysippus's trichotomies.

are merely *indifferent*. We also take for granted that "good" and "bad" are contraries rather than contradictories—some states could in principle fail to be either, such as Diogenes Laertius's example of whether we have an odd or even number of hairs on our head (7.104 = LS 58B). What is of course very surprising is what the Stoics took to be neither good nor bad: health, wealth, disease, poverty, beauty, ugliness, and many other things normally considered to belong to one or other category. Whatever our qualms about the correctness of that view, there does not seem to be anything *logically* incoherent or otherwise paradoxical in thinking that wealth is neither good nor bad, for example: in saying there are things neither good nor bad the Stoics are just signaling that good and bad are only contraries.

The third trichotomy I wish to discuss is more controversial, and my discussion draws on the discussion in previous work.[8] Here, too, I think it is clear that in saying something that initially sounds paradoxical, Chrysippus intends to point out that two options that might seem contradictory are only contraries, and that he maintains a third option. This trichotomy concerns the number of ultimate parts of things. We have the good fortune here to have a direct quotation of Chrysippus. Plutarch says (*Comm. not.* 1079B-C, =LS 50C 3):

> λέγει γὰρ ὁ Χρύσιππος ἐρωτωμένους ἡμᾶς εἴ τινα ἔχομεν μέρη καὶ πόσα καὶ ἐκ τίνων συγκείμενα μερῶν καὶ πόσων, διαστολῇ χρήσεσθαι, τὸ μὲν ὁλοσχερὲς τιθέντας ὡς ἐκ κεφαλῆς καὶ θώρακος καὶ σκελῶν συγκείμεθα·

[8] D. Nolan, 'Stoic Gunk' ['Stoic Gunk'], *Phronesis,* 51.2 (2006), 162-183.

τοῦτο γὰρ ἦν πᾶν τὸ ζητούμενον καὶ ἀπορούμενον· ἐὰν δ' ἐπὶ τὰ ἔσχατα μέρη τὸ ἐρωτᾶν προάγωσιν, οὐδέν φησί τῶν τοιούτων ἐστὶν ὑπολεπτέον, ἀλλὰ ῥητέον οὔτ' ἐκ τίνων συνεστάναι καὶ ὁμοίως οὔτ' ἐξ ὁπόσων, οὔτ' <ἐξ> ἀπείρων οὔτ' ἐκ πεπερασμένων. καί μοι δοκῶ ταῖς ἐκείνου κεχρῆσθαι λέξεσιν αὐταῖς, ὅπως συνίδῃς ὃν τρόπον διεφύλαττε τὰς κοινὰς ἐννοίας, κελεύων ἡμᾶς νοεῖν τῶν σωμάτων ἕκαστον οὔτ' ἔκ τινων οὔτ' ἐξ ὁποσωνοῦν μερῶν, οὔτ' ἐξ ἀπείρων οὔτ' ἐκ πεπερασμένων, συγκείμενον.

Chrysippus says that when asked if we have parts, and how many, and of what and how many parts they consist, we will operate a distinction. With regard to the inexact question we will reply that we consist of head, trunk and limbs – for that was all that the problem that was put to us amounted to. But if they extend their questioning to the ultimate parts, we must not, he says, in reply concede any such things, but we must say neither of what parts we consist, nor, likewise, of how many, either infinite or finite. I have, I think, quoted his actual words, so you may see how he conserved the common conceptions, urging us to think of each body as consisting neither of certain parts nor of some number of them, either infinite or finite.

Plutarch goes on to complain that there is no third option "intermediate" between infinite and finite. But the "actual words" quoted do not imply that there are ultimate parts that are not of infinite number and also not of finite number. They suggest, strongly in my view, that Chrysippus rejected the assumption that there are *any* ultimate parts. (Remember the Ancient

Greeks did not consider "zero" a number, so for Chrysippus there would be no number, finite or infinite, for ultimate parts when there are no ultimate parts.) In 'Stoic Gunk' I argue that the best way to understand Chrysippus's views on parts and wholes, as well as on topics such as the metaphysics of mixture and the ontology of time, is to suppose he thought that bodies (and space and time) had parts, which themselves had parts, and so on without end: that they are atomless *gunk*, to use today's technical term.

When asked about ultimate parts, there is a consistent, bivalent option other than the option that there are finitely many of them and that there are infinitely many of them: it is that there are *none* of them. (Again, we are not counting zero as a number, and so not a finite number.) There is no need for a third kind of number between finite and infinite, nor a third truth-value for claims about the parts of objects.[9]

The three trichotomies so far discussed have the clearest diagnoses, in my view. In each case, options that are apparently exhaustive are plausibly only contraries, and after choosing "neither" Chrysippus goes on to explain why there is a third option. Hands are neither the same as, nor different from, their possessors: as Chrysippus seems to be interpreting "different from" as entailing "does not overlap", hands are in this intermediate category because they overlap their

[9] Hahm, D. E., 'Chrysippus' Solution to the Democritean Dilemma of the Cone' ['Solution'], *Isis*, 63.2 (1972), 205-220 also interprets Chrysippus as denying that there are any ultimate parts here, rather than e.g. engaging in some denial of bivalence or excluded middle (p 210).

owners (my hands are parts of me). Health is neither good nor bad: it is *indifferent*. It is not that the least parts of an object are finite in number, and it is not that the least parts of an object are infinite in number: there *are no* least parts of any object. Denying that two contraries are exhaustive does not require denying the law of excluded middle, or embracing truth-value gaps, or anything this exotic. It is as if we were asked whether Cate Blanchett was American or English, and replied that she is neither, on the grounds that she is Australian.

Note also that Chrysippus does not stop simply by claiming the alternatives are contrary rather than contradictory: he apparently goes on to give a positive philosophical account of the third option (being indifferent, sharing parts, or there being no ultimate parts in division). So while there is a common form of Chrysippus's theories, there is additional work in each particular case that Chrysippus does in specifying and defending an alternative to the initial, on the face of it exhaustive, alternatives. The implementation of this strategy varies depending on considerations specific to the topic at hand, but we can see an important common thread for all that.

The fourth and final trichotomy I want to discuss in this section also seems quite straightforward, though there is some variation in how it is currently interpreted. Diogenes Laertius reports some Stoic philosophers as holding that "dialectic, since it is the science of correct discussion in regard to discourses conducted by question and answer, so that they also define it as the science of what is true and false and neither [of these]". (Diogenes Laertius 7.42 (= LS 31A)) This might suggest that the Stoics (or some of them) thought that there was a third option for assertoric sentences besides true and false. However, there seems to be a much more pedestrian reading available:

that the Stoics thought dialectic was not just the study of entire sentences, but also the components of those sentences, which are not themselves true or false—for instance, the subject term or the predicate. The view that one study encompassed things that were true or false and also the "parts" of sentences need not be very surprising: Aristotle claims to be covering things that are true or false and also the parts of sentences that are neither, such as nouns and verbs, in *On Interpretation*, for example.

Diogenes Laertius's discussion goes on to make it clear that the Stoics under discussion took the science of dialectic to cover more than complete propositions and sentences: among the topics it includes are "predicates and similar actives and passives, genera and species" and later on topics like "solecisms and barbarisms, poetry, ambiguity, euphony, music" (LS 51A, from Diogenes Laertius 7.44). Genera and species, for example, are not true or false, and neither are generic expressions nor specific expressions. So whether or not we suppose that dialectic treated of generic and specific expressions or genera and species in any metaphysical sense, we should not say that genera or species are true or false, but rather that they are neither. Furthermore, Diogenes Laertius 7.63 and 7.66-8 make it clear that dialectic includes discussion of sentences such as questions, commands, vocatives, "timid suggestions", and other utterances that are neither true nor false. It may also be true that some sentences containing generics are not supposed, by Chrysippus, to express assertibles that are true or false (e.g. "Man is in Athens"): see e.g. Bobzien, 'Vagueness' pp 220-221. But whatever we make of this interpretive proposal, *lekta* corresponding to parts of sentences, or to questions or commands, will be good enough examples of some of the "neither true nor false" things that dialectic is concerned with.

Of course, from the fact that dialectic dealt with some things that were uncontroversially neither true nor false it does not follow that the Stoics thought that it was only these things that were neither true nor false. I will discuss in section 4, below, whether the Stoics, and in particular Chrysippus, did maintain that all assertibles are either true or false. For the time being, though, it is enough to notice that the Stoics need not have held any controversial view about assertibles, or the nature of truth, or excluded middle, or anything else to think that the science of dialectic was concerned with some things that were neither true nor false. I am inclined to think this case, while resembling the other trichotomies, is not a case where Chrysippus, or other Stoics, were even presenting a prima facie puzzling or paradoxical-sounding view. So perhaps it is less useful for detecting a pattern among the more surprising trichotomies.

2. Applying the Pattern to A More Puzzling Case: Cones and Pyramids

In the four cases of the previous section, Chrysippus appears to follow a common pattern: defending the view that apparent contradictories are only contraries, and defending a doctrine that is opposed to both of the two most obvious options. Can this insight shed any light on more puzzling cases of Stoic trichotomies? It is not a universal solvent: obviously there will be many ways of implementing that general strategy, and sometimes the testimonia available are meagre and puzzling. Nevertheless, in this section I will discuss a trichotomy I think is more puzzling, and will argue that recognition of the pattern in Chrysippus's thought gives us partial guidance towards a solution. In the paradox of the cone (or pyramid), when asked whether adjacent surfaces we get by cutting the figure are "equal or unequal", Chrysippus appears to have held

"neither equal nor unequal". There are other puzzling apparent trichotomies in Chrysippus's thought: for example, he may have thought that, as well as corporeal and incorporeal objects there are also some that are "neither corporeal nor incorporeal" (Long and Sedley, LS p 163). He may have also divided objects into the existent, mere somethings, and not-somethings (Brunschwig, 'Supreme Genus') and he may have thought that Liar paradox sentences were not true, and not false, but neither true nor false (Caston, 'Something and Nothing', p 193 n 100). But space precludes discussing every puzzling trichotomy here.

The paradox of the cone, and the related paradox of the pyramid, is somewhat mysterious. One of our main sources for it is the discussion in Plutarch *Comm. not.* 1079-D1080D, though Plutarch attributes the puzzle originally to Democritus. Here is a reconstruction of the puzzle.[10] Consider a cone sitting on its base, and consider a plane through the cone parallel to the base. Imagine separating out the two halves of the cone that lie on either side of the plane, and consider the surfaces revealed by that division. Are they equal in area, or is the lower one larger by some amount than the upper one? If the lower is larger by some amount, then there appears to be a discontinuous "jump" in area: the surface of the cone would not be smooth at that point, but would undergo a small but jagged jump. On the other hand, suppose the two surfaces were

[10] It is a controversial reconstruction: in particular, some alternative reconstructions add several atomistic assumptions, such as that offered in A. Drozdek, 'Democritus, Chrysippus, and the Cone Problem', *Acta Antiqua Scientiarum Hungaricae,* 49 (2009), 117-125.

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exactly equal in size. Then, presumably, if we were to cut the cone anywhere in this fashion the two surfaces would be equal in size. But now consider starting at the bottom and moving up the cone circle by circle. If, at each stage, the circles are equal in size, then circles near the top will be the same size as circles near the bottom, since at no point there is any diminution in size. The cone would then be a cylinder, and not a cone. A very similar paradox arises for pyramids, when we consider the triangles that share the top point of the pyramid but are at different angles between a side and the perpendicular triangle in the middle of the pyramid: allow that the areas of these triangles are unequal when a cut is made, and a smooth pyramid seems to be replaced with a solid with a jagged surface, but insist that all the surfaces are equal and the pyramid seems to be replaced with a triangular prism, with the side surfaces not differing in area from the central perpendicular triangle. (Both puzzles are very good ones, in my view, and an entirely adequate solution to them requires resources not available to ancient geometers—they deserve to be on the standard lists of ancient paradoxes of infinity with Zeno's paradoxes.)

A face value reading of Chrysippus's answer that the surfaces are "neither equal nor unequal" would suggest that Chrysippus postulated a third status for the relationship between areas besides equality or inequality; or perhaps that he thought that there were truth-value gaps in geometry. However, if his method of proceeding here is as above, we should expect that he takes there to be a presupposition in the "equal area or unequal area?" question that he wishes to reject. In fact, I suspect things may be more complicated, since the Stoics believed both in cone-shaped bodies,

and perhaps also three-dimensional *geometric* entities such as cones,[11] but let me focus on the puzzle for conic bodies, since the version of the puzzle for geometric entities brings in additional interpretive uncertainties.

Let us consider a cone-shaped body resting on its base. (Perhaps one could deny there are any perfectly conic bodies, but if any body has a smooth curved surface a variant of the cone paradox can be run.) First consider a plane running through an undivided cone. One way to deny the presuppositions of the question would be to deny that this plane divides the cone into two bodies with surfaces: if there are not two surfaces, the question of whether they are equal or unequal does not arise.[12] Another way the question can be sidestepped is if we think that the plane is the surface of *both* the lower and upper bodies: then there will not be *two* surfaces to compare, and to ask whether they are equal or unequal in area. Yet another is to suppose that the question implicitly assumes mathematical atomism, and is asking about surfaces one minimal unit apart: then Chrysippus could easily reject the existence of surfaces separated by a mathematical atom. This seems to be the suggestion of Hahm 'Solution' p 212-213, 220: though it is hard to see why the question as reported by Plutarch presupposes anything atomistic.

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[11] See D.G. Robertson, 'Chrysippus on Mathematical Objects ['Mathematical Objects'], *Ancient Philosophy*, 24 (2004), 169-191 for discussion.

[12] This is the interpretation offered by H. Cherniss, (translation and notes)., Plutarch, *Moralia Volume XIII Part II,* (Harvard University Press, 1976), pp 820-21 footnote b.

Neither way of denying that there are two surfaces here helps when we think of a cone *in fact* being divided along a plane—cut or snapped into a smaller cone and a frustum. Now the two separated bodies each have a circular surface: are these surfaces equal or unequal? Presumably there should be an answer here, on pain on denying we can measure areas at all. I can only speculate about whether Chrysippus even considered this version of the puzzle separately. But if he did, there is a little more room to manoeuvre. Presumably there are several different ways to break apart a body—a slice can be taken out between them, or the smaller parts can be distorted and torn in various ways. It may well be possible to split a body so that the two pieces have the same surface area along the split, just as it is surely possible to split them so they have different surface areas. But this does not necessarily tell us anything about how the pre-split object is put together. In particular, there is no guarantee that the pre-existing cone is in any sense "made up of" surfaces created by splitting. So while we have much less room to deny that the two bodies will have comparable surfaces, it is harder to generate something like the original paradox from *this* concession. Even if we allowed that after physical separation one side had a larger surface than the other, it would not follow there was a discontinuity before separation: on the other hand, if we allowed that the surfaces were equal after physical separation, that would not imply that a cone was somehow made up of equal circles in the way that a cylinder might be.

There may be other ways to deny an assumption lurking in the "equal area or unequal area?" question: but given Chrysippus's general approach, we should look for some such denial of the principles needed to ensure both that there are two surfaces to ask about, and that the areas of these two surfaces bear on the measurements of the original cone. A third way that may have

appealed to Chrysippus was to reject the claim that there are any planes or plane-segments: if there are no such planes in the cone, then the question of whether they are equal or unequal does not arise. It is hard to tell what Chrysippus's attitude to geometrical objects like lines and planes was. Stobaeus 1.142 (LS 50A) tells us Chrysippus held there were lines and planes. On the other hand, Proclus seems to indicate that Chrysippus held that lines and planes "subsisted in mere thought" (LS 50D from Proclus 1.89) and perhaps were of a piece with Ideas (Proclus 1.395 (= LS 50G), which many people interpret Chrysippus as saying are only in thought and neither exist nor subsist.[13] Of course, whether being "in mere thought" is a way of being something or a way of denying that there is such a thing is also a difficult interpretative question: if there are plane sections but they are only "in thought" it would make sense to require an answer about their areas, and it is only if there are no plane sections at all that it is obvious why we could refuse to say that they are equal and also refuse to say they are unequal.

A final way to reject the presupposition of Democritus's challenge is to allow that there are two surfaces associated with the bodies on either side of the cut, but to deny that these surfaces have any measure. This is the approach taken by Robertson 'Mathematical Objects', who claims Chrysippus 'considers measurability or comparability to be properties unique to physical objects' (p 178), and so the surfaces are neither equal nor unequal, since they are not the sorts of things that stand in those relationships. While this does attribute to Chrysippus an odd view of surfaces,

[13] Though see Caston 'Something and Nothing' for a more nuanced view. See Robertson 'Mathematical Objects' for a summary of the ancient evidence about Stoic attitudes to geometric objects.

and requires a story about geometry where it is not after all about objects like lines and planes, insofar as those are surfaces of objects, it does meet the stricture I am arguing for here, that we understand Chrysippus's response as offering a consistent third alternative, construing what may have seemed to have been contradictories ('equal or unequal?') as only contraries.

There may be still further ways to deny a supposition of the puzzles of the cone and the plane: I cannot rule out that Chrysippus's ingenuity spotted another strategy of this kind to respond to the puzzle, especially since we only Plutarch's report about how exactly the puzzle was framed.

All of the particular ways of blocking the assumptions of the paradoxes of the cone and the pyramid I have suggested here raise further challenges. Whether or not Chrysippus met these challenges raises puzzles about Stoic geometrical theory that I cannot address here. If I am right about Chrysippus's approach to Stoic trichotomies, then, that will not by itself deliver a complete interpretation of Chrysippus's approach to the puzzles of the cone and the pyramid, but it helps us to constrain the options to explore. We can rule out options where there is a truth-value gap about the relative size of the surfaces, for example. If the suggestion here is right, we can also rule out hypotheses on which Chrysippus does, after all, embrace one of the two initial horns (i.e. that the surfaces are equal, or that the surfaces are unequal). One such view is suggested by Michael White[14], which is that one surface is greater than the other by a non-zero infinitesimal

[14] M.J. White, *The Continuous and the Discrete* ['Continuous and Discrete'], (Clarendon, 1992), pp 310-314.

amount: though White himself at one point interprets "not equal" and "unequal" as contraries[15] so that surfaces differing by an infinitesimal magnitude are "not equal" but fail to be "unequal", so with this interpretation of "unequal" and "not equal", his proposal conforms to the constraint suggested here after all.

White's preferred interpretation in his 'Continuous and Discrete', however, seems to be a fuzzy-logical one according to which the value of "surface 1 is greater than surface 2" falls somewhere between 1 and 0 (White, 'Continuous and Discrete' p 292). Interpreted as a claim about truth-values, this conflicts with a full endorsement of bivalence, since both of a pair of contradictories can fail to be fully true in the fuzzy-logical framework White suggests. Interpreted only as a claim about a (fuzzy) set-theoretic representation of degrees of greatness, it is hard to see what difference White thinks will follow from adding a fuzzy layer of interpretation on an infinitesimal interpretation of magnitudes. So determining whether White's final position falls inside or outside the family of positions permitted by the present approach requires further interpretation of White's suggestive but unspecific proposal about how to understand Chrysippus.

3. Why Might Chrysippus Have Put Things This Way?

If my suggestion is right, and that Chrysippus often says that there is a third option when apparently presented with two options that are exhaustive, the question arises of why he would

[15] M.J. White, 'Zeno's Arrow, Divisible Infinitesimals, and Chrysippus', *Phronesis*, 27 (1982), 239-254 at p 244.

do so. After all, his way of putting things *suggests* that he is flirting with violations of excluded middle, or otherwise seeking for a logically non-standard option. Why put things in this way, producing the appearance of paradox, when this does not seem necessary for the point he is making? For example, why say "the ultimate parts are neither finite nor infinite", risking the implication that there are ultimate parts, when instead he could have just said "I claim there are no ultimate parts, so I think there are none of them, not finitely many nor infinitely many"?

One possible explanation is that this was mere showmanship. Many philosophers feel the temptation to present a conclusion in a way that makes it sound crazy or at least far from commonsense, and then try to argue an audience around to thinking it is true (or at least could well be true). Doing things this way can be a tool to grab immediate audience interest; or a way to avoid making the conclusion seem obvious from the beginning, and thus not really worth arguing for; or a way to try to impress an audience with one's philosophical chops. (If he could make *that* seem compelling, he must be good!) The explanation for why Chrysippus proceeded in this fashion could be no deeper than that it was a quirky, but not unheard of, choice of philosophical style.

Another, perhaps more interesting, explanation turns on the context in which these philosophical challenges may have arisen. Philosophical puzzles were sometimes presented as questions: consider almost any Platonic dialogue. If Chrysippus, or his intended audience, was familiar with philosophers being challenged to take a stand by being asked questions of this form, the presentation of his answers makes more sense. If the Megarians, or Peripatetics, or other

opponents presented challenges of the form “Is Socrates’s hand the same as Socrates or different from Socrates?”, or “Is health good or bad for him who has it?” or “Is the number of ultimate parts of a thing finite or infinite?” then we would have quite a reasonable explanation of Chrysippus’s answers “neither the same nor different, but other than same or different”, “neither good nor bad but other than good or bad”, “neither finite nor infinite but other than finite nor infinite”. Chrysippus would be playing a game of question and answer by pre-established rules.[16]. Conventions like this may have already been in place for dialectical disputes at the time of Chrysippus, especially if Aristotle's discussion of dialectic in *Topics* 8 reflects common

[16] See S. Bobzien, 'How to give someone Horns—Paradoxes and Presuppositions in Antiquity', *Logical Analysis and History of Philosophy,* 15 (2012), 159-184 for a discussion of the ancient rules of dialectic, and of asking and answering dichotomous questions. It is clear at least that some later Stoics discussed how to argue in accord with these rules. L. Castagnoli, 'How Dialectical was Stoic Dialectic?' in Nightingale, A. and Sedley, D. *Ancient Models of Mind*, (Cambridge University Press, 2015), 153-179, argues that treating arguments as genuinely dialectical (i.e. as exchanges between disagreeing parties) was important for the Stoics, and in particular illuminates their attitude to self-refuting arguments. Castagnoli does suggest that some of the cases he particularly focuses on may be due to Stoics responding to skeptical challenges raised after Chrysippus's time, however (p 176-77). Castagnoli (p 166) rightly notes that the fact that Chrysippus identified Socrates as the keenest student of dialectic among the ancient dialecticians suggests that the question-and-answer aspect of dialectic was important to Chrysippus. (Chrysippus's opinion of Socrates is reported by Plutarch *Stoic Rep.* 1046A.)

philosophical practice, or what became common philosophical practice by Chrysippus's day, and not merely Aristotle's own recommendations.[17]

There is evidence that Chrysippus thought that an important part of dialectic was the ability to avoid refutation through questioning, and to pose philosophical questions. Diogenes Laertius 7.47, when discussing why the Stoics claimed dialectic was "indispensable", not only claims that "[w]ithout it... the wise man cannot guard himself in argument so as never to fall" but also "without it he cannot methodically put questions and give answers". This suggests that being able to hold onto one's doctrines when answering questions put by opponents is either "indispensable" to the Stoics, or at least very important.

Answering stock philosophical questions may have been particularly important to Chrysippus if he wanted to show that he avoided the paradoxes that seemed to ensnare others. In his discussion of sorites paradoxes, Chrysippus talks of "pulling up" and refusing to answer questions that would lead him down the path to absurd conclusions (as quoted in Cicero *Acad.* 2.94). So other

[17] It is difficult to determine with any certainty what influence Aristotle's views of dialectic had on Chrysippus and his interlocutors. See J. Barnes, 'Aristotle and Stoic Logic', in Barnes, J. *Logical Matters: Essays in Ancient Philosophy II*, (Clarendon, 2012), 382-412. Of course Chrysippus could have been influenced by a practice of posing and answering questions put in the form of a dilemma even if that practice was not nearly as constrained as the one presented in the *Topics*.

paradoxes of the day may have been posed as a series of questions that lead to an unacceptable combination of answers. We might imagine, for example, that just as Plato illustrated in the *Parmenides* that there were traps in wait both for those who said that there was only one thing and those who said that there were many things, there were philosophers prepared to argue that there were problems with saying that health was good and for saying that health was bad. Or to take a case where we know more about the dialectical situation, Democritus was prepared to argue about the cone that saying that the surfaces were equal was unacceptable (because by repeating the argument we could show the cone was really a cylinder) and that if the surfaces were unequal there was no continuous variation in length, but rather jagged "steps" in a supposedly smooth cylinder.

If my interpretation above of Chrysippus is correct, he would have a similar view of the cone as he would of the pyramid: he would want to allow that there are cones that vary continuously in diameter without any jagged "steps". He would thus need to address Democritus's challenge, which may have also been used by Epicurean atomists against their rivals. If the challenge was standardly put by setting up the problem and asking "are the surfaces equal or unequal?" then the Stoics might have wanted a direct response to *that question*. "Neither equal nor unequal" seems to be a reasonable direct response for Chrysippus and his followers to that question. It may risk sounding paradoxical at first, but that is at least as much the fault of the question, which suggests that these are the only two possible answers, as it is of the answer itself.

Both of these suggestions are speculative, but given the very small amount of Chrysippus's own writings that have survived, many hypotheses about his philosophical style must remain speculative.

4. What Other Options Are There?

The fact that "the Stoics" in general, and Chrysippus in particular, keep being reported as putting forward these trichotomies stands in need of a general explanation, as opposed to just a disconnected series of explanations of each individual case. The explanation offered above is that these paradoxical-sounding trichotomies were put forward by Chrysippus, not because he was seeking a "third truth value" answer to various philosophical problems, but because he wished to argue that classifications which might have been thought to exhaust the options were, on closer examination, only contraries, and a third option contrary to the other two could obtain without any compromise of excluded middle or bivalence.

It is worthwhile to compare this proposed explanation with other general explanatory strategies for collectively accounting for these trichotomies. To my knowledge there is little current discussion of this problem as such, beyond the brief remark in Long and Sedley 1987 p 165. Nevertheless, I will discuss what I take to be the most serious competitor explanations, even though many of these have not appeared in the literature. In a number of cases these explanations will be generalisations of explanations writers have put forward in particular cases: obviously those writers should not be held responsible for the general explanations when their targets were considerably narrower.

One of the most obvious interpretive strategies for these trichotomies is to think that Chrysippus believes some claims are neither true nor false. As a general interpretative option (as opposed to just an option for interpreting Chrysippus where he seems to be explicitly saying something is neither true nor false) this option requires reading the "neither p nor q" third option as the claim that, in the relevant case, both *p* and *q* are neither true nor false. So, for example, "my hand is identical to me" and "my hand is distinct from me" will both be neither true nor false, if this is a case where we wish to reject both the claim that my hand is part of me, and that my hand is distinct from me. (And likewise with the other cases: e.g. "the ultimate parts are finite" and "the ultimate parts are infinite" will both be neither true nor false.) This would have the advantage of keeping the appearance that the first two options were contradictory and apparently exhaustive, as well, of course, as vindicating those testimonia that suggest that Chrysippus thought there was a status for assertibles other than true or false.

This interpretation of Chrysippus faces two main problems. The first is obvious: it conflicts with the testimonia that suggest Chrysippus was committed to bivalence for assertibles (e.g. Cicero *On Fate* 21, though there are many others.) A subsidiary part of this problem is that it makes it hard to vindicate the evidence that Chrysippus accepted excluded middle (*p* or not-*p*, for all assertibles). See, for example, Sextus *M*. 11 282. While some logical systems reject bivalence while keeping excluded middle (most famously supervaluationism)[18], this is a sophisticated

[18] see B.C. van Fraassen, 'Singular Terms, Truth-Value Gaps, and Free Logic', *Journal of Philosophy,* 63.17 (1966), 481-495

maneuver, as far as I can tell not otherwise known in ancient logic (though it may, perhaps, fit the puzzling position attributed to Epicureans by Cicero in *On Fate* 37), so it would be surprising if even Chrysippus maintained that position. It would be better, for those who pursue a "neither true nor false" strategy, to claim that the testimonia suggesting Chrysippus accepted excluded middle are misleading.[19] But of course that would be a serious interpretative cost.

The second significant problem, in my view, is that Chrysippus's saying "neither *p* nor *q*" would not have been a very good way for him to indicate that both *p* and *q* lack truth values, even when *p* and *q* are genuinely contradictories. It follows from not-(*p* or *q*) both that not-*p* and that not-*q*,[20] and for that matter (not-*p* and not-*q*). So it follows, for example, from (*neither* the ultimate parts are finite *nor* the ultimate parts are infinite) that *both* not-(the ultimate parts are finite) *and* not-(the ultimate parts are infinite). Now, Chrysippus is clear that when not-*p* is true, *p* is false

[19] One motivation for attributing to Chrysippus the denial of excluded middle and bivalence are the remarks in Sextus *M.* 7 241-246 about generic appearances and judgements. I follow Caston 'Something and Nothing' pp 187-195 in thinking that the best interpretation of Sextus here does not attribute violations of excluded middle or bivalence to Chrysippus. Caston argues against an alternative reading on which Chrysippus accepts "generic objects" that are incomplete in a way that gives rise to truth-value gaps.

[20] Or, when an exclusive sense of "or" is in play, what follows from not-(*p* or *q*) is rather (not-*p* and not-*q*) or (*p* and *q*). But we can safely reject the second disjunct when *p* and *q* are obvious contraries.

(Sextus *M*. 8 103). So it follows from not-(the ultimate parts are finite) that the claim "the ultimate parts are finite" is *false*: and so, in particular, it would follow that this claim is *not* neither-true-nor-false. To hold that Chrysippus maintained claims of the form "neither *p* nor *q*" in some cases where he maintained that *p* was neither true nor false is to attribute to Chrysippus a fairly elementary logical blunder.[21] People do commit logical blunders sometimes, of course, and it can be seductive to say "neither *p* nor *not-p*" to express the thought that neither *p* nor not-*p* have a truth value. Still, we should prefer an interpretation of Chrysippus that does not make him repeatedly confused about elementary consequences of his own utterances if it is available and plausible, so I think we should reject the interpretation of Stoic trichotomies that, in general, Chrysippus was adopting a "neither true nor false" strategy.

A cousin of this approach is to maintain, not that Chrysippus accepted that there were claims that were neither true nor false, but instead thought that in these unusual cases, sentences which appear to express an assertible do not at all. On this approach, some *sentences* will be neither true nor false, but this is only because they are in some sense meaningless or fail to say anything (or at least they fail to express an *axioma* assessable for truth or falsehood, whatever else these sentences might do). This approach to interpreting Chrysippus is similar to the defended by Bobzien 'Vagueness', in the case of sentences about borderline cases in sorites sequences, where she argues that the relevant vague sentences are not associated with *axiomata*. A number of

[21] Though this is perhaps not a logical error that his formal system of logic could rule out—the five indemonstrables say little about negated disjunctions.

authors have also interpreted Chrysippus's response to the Liar paradox in this way.[22] On this approach it is natural to restrict excluded middle to cases of complexes of simple sentences that express assertibles as well, allowing room for apparent counter-examples to excluded middle. While Bobzien and Caston do not extend this kind of treatment to any of the "trichotomy" cases above (except for Bobzien's account of generic sentences, see above p *7*), we can evaluate this generalisation of the "no assertible" approach as offering an interpretive strategy we could try out across the board.

This approach is not very appealing in some of the trichotomies discussed above: why think it is not even *assertible* that my hand is identical to me, rather than just that it is false? Saying that something that is in fact indifferent is good (health, for example), seems to be saying something, albeit something false by Chrysippus's lights: Chrysippus's attempts to argue against the doctrine that health is good would be very hard to explain if he thought it did not even express an assertible that was assessable for truth value. Apart from this initial implausibility as a general interpretation of some of the cases, it also faces the logical problem mentioned for the previous option: when Chrysippus says "neither finitely many ultimate parts nor infinitely many ultimate parts" he is committed to it being *false* that there are finitely many ultimately parts, and so that "there are finitely many ultimate parts" expresses an assertible, since only these (and by extension the sentences that express them) can be true or false.

[22] They include Caston 'Something and Nothing': see p 193 n 100 for a discussion of the literature on this option.

If this "missing assertible" strategy is implausible as a *general* explanation of this pattern of trichotomies across Chrysippus's views, that hurts its plausibility as an explanation of any particular trichotomy, unless it can be integrated in an alternative good theory of the general phenomenon in Chrysippus's thought when he proposes to take a "neither" option. Whether this sheds light on Chrysippus's approach to paradoxes like the sorites and the liar depends on whether we think Chrysippus presented his doctrines about these puzzles in his signature trichotomous way: at the moment I am not sure whether we should think he took this sort of approach to either paradox.

A final option worth brief consideration is that this depiction of Stoic views is an artifact of the reports that have survived of Stoic views. Most of what we have about Stoic doctrine, particularly that of Chrysippus, is second hand and often from sources critical of Stoic doctrines or even hostile to them, so perhaps this way of thinking about the topics is due to some influential commentator or reporter on Stoic doctrines presenting them this way. There are two problems with this suggestion. The first is that we have reports of trichotomies from a range of ancient authors who seem to have had direct access to Stoic texts: trichotomies are attributed to the Stoics by Plutarch (a number in *Comm. not.* 1073E-1074D and twice *in Comm. not.* 1079B-80D (= LS 50C)), Diogenes Laertius 7.42 (= LS 31A5), 7.101 (= LS 58A) and Sextus Empiricus (*M*. 11 22-24 (= LS 60G3)). It would be very surprising if reports of this pattern were not due to this pattern being found in the Stoic texts themselves. The other important problem for this suggestion is that one of the trichotomies appears in Plutarch in text that appears to be a direct quote from Chrysippus. The previously mentioned passage in Plutarch where Plutarch says “I

have, I think, quoted his actual words" contains Chrysippus saying, of ultimate parts "we must say neither… likewise, of how many [there are], either infinite or finite". (*Comm. not*. 1079B-C) It is true that Plutarch's own paraphrase is more explicitly trichotomous than his direct quote ("urging us to think of each body as consisting neither of certain parts nor of some number of them, either infinite or finite"), but the fact that the passage directly quoted so strongly suggests a trichotomous distinction (finite, infinite, neither) does make it very likely that this distinction is in Chrysippus and not just in Plutarch's report.

5. Conclusion

When Chrysippus endorsed an apparently paradoxical third option, it is always a matter of his pointing out that the original options, perhaps despite appearances, were not exhaustive, and rejecting both contrary options in favour of a third way. There are things that are neither good nor bad, parts of speech that are not true or false because they are only parts of sentences, neither finite nor infinite ultimate parts because there are *no* ultimate parts, and so on. While adopting these third options was sometimes philosophically innovative, and always required some further explanation about what the third option could be, it did not require compromising Stoic logical principles such as bivalence of truth value and excluded middle, nor did it require supposing that apparently meaningful sentences failed to be associated with assertibles. Armed with this insight, we can make progress in resolving a range of interpretive puzzles.

This pattern in Chrysippus's thought raises an interpretive puzzle of its own: *why* would Chrysippus present his views in this superficially paradoxical way? I have suggested that the

explanation may lie in a pattern of philosophical dispute involving a "question and answer" format of a particular sort, complete with questions containing presuppositions Chrysippus wanted to reject, though this explanation can only be tentative without more evidence about Chrysippus's philosophical environment.

Even those who reject my diagnosis of these “trichotomy” cases should note that there is a general phenomenon to be explained here. A diagnosis of any one of these cases that cannot fit into a more general explanation of Stoic trichotomies comes at a cost: the cost of making the pattern more puzzling. At the limit, theories that treat each trichotomy in terms that cannot be generalised would in effect be treating this general pattern as a widespread coincidence: and the implausibility of that interpretative hypothesis would go some way to undercut the individual interpretations.

There are at least two avenues of further investigation suggested by the hypothesis put forward in this paper. One is to pin down further puzzling trichotomies using the guidance we have from more straightforward cases. For example, further investigation may settle on a particular interpretation of what Chrysippus was up to with the cone and pyramid, let alone other trichotomies not discussed in any detail here. A second is work out the role in philosophy of disputes presented in question-and-answer mode at the time of Chrysippus, so as to work out what influences this may have had on the development of philosophical positions at the time: this may enable us to better evaluate the suggestion in section 3 about why Chrysippus presented his

responses to a range of philosophical problems in a way that can appear, at least on the surface, as paradoxical.[23]

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[23] Thanks to Alex Sandgren and an anonymous referee for suggestions, and especial thanks to Victor Caston for very constructive editorial comments. Work on this paper was supported by the National Humanities Center.

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