

# The Genesis of General Relativity: Interaction between Einstein's, Abraham's and Nordström's Research Programmes

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**Abstract** The arguments are exhibited in favour of the necessity to modify the history of the genesis and advancement of general relativity (GR). I demonstrate that the dynamic creation of GR had been continually governed by internal tensions between two research traditions, that of special relativity and Newton's gravity. The encounter of the traditions and their interpenetration entailed construction of the hybrid domain at first with an irregular set of theoretical models. Step by step, on eliminating the contradictions between the models contrived, the hybrid set was put into order. It is contended that the main reason of the GR victory over the rival programmes of Abraham and Nordström was a synthetic character of Einstein's programme. Einstein had put forward as a basic synthetic principle the principle of equivalence that radically differed from that of rival approaches by its open, flexible and contra-ontological character.

**Key words** Einstein, Nordström, Abraham, general relativity.

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## 1. Introduction

It is proverbial that Albert Einstein's strenuous efforts to create the General Relativity (GR) were accompanied by its rival flat versions conjured up by Gunnar Nordström, Max Abraham, Gustav Mie, et al. In particular, in 1912–1914 a Finnish mathematician Gunnar Nordström advanced a sca-

lar Lorentz covariant gravitation theory and developed it so that it incorporated the equality of inertial and gravitation mass. Likewise, in 1912 a Göttingen master of classical electrodynamics Max Abraham proposed a vector Lorentz covariant gravitation theory where light and gravitation had the same speed of propagation. Their papers are still considered as peculiar *delusions* that had been capable to stir up problem situations at best and to incite critical discussions around GR highlighting all its splendour.

However, some current history-of-science insights (Norton 2007; Renn and Sauer 2007; Renn 2007b) prompt one to take the Standard View with a considerable grain of salt. For instance, the Einstein-Nordström correspondence underscores that it was Albert Einstein himself who, before November 1915, and even *after* the creation of GR preliminary metric version – the ‘Entwurf’ (1913) – took active part in invention of Nordström’s scalar relativistic theories of gravitation. Einstein was in continued contact with Nordström during the period in which the Nordström theory was advanced. The theory actually evolved through an intensive exchange between Einstein and Nordström, with Einstein often supplying the ideas decisive to the development of the theory. By and large the theory might more accurately be called the “Einstein- Nordström theory”.

The next apparent example is A. Einstein’s and A. Fokker’s paper published in early 1914 that aimed at an “application of new mathematical methods, used in Einstein’s and Grossmann’s paper, to Nordström’s theory” (Einstein and Fokker 1914). Moreover, in the same paper in early 1914 the significant connections between Nordström’s theory and conformally flat spacetimes were revealed. Therefore it comes as no surprise that it was within Nordström’s 1912 theory where the gravitational field equation  $R = \chi T$  ( $\chi = \text{const}$ ) was first derived, with  $R$  being fully contracted Riemann-Christoffel tensor and  $T$  the trace of the stress-energy tensor (in the case of an unstressed, static matter distribution). The field equation is an apparent harbinger of Einstein’s illustrious equations promulgated at Preussische Akademie der Wissenschaften on November 25, 1915 (Einstein 1915). Einstein made clear his preference for Nordström’s theory over other rivals in September 1913 presentation of the ‘Entwurf’ theory (see section 3 for details) to the 85<sup>th</sup> Congress of the German Natural Scientists and Physicians. His single critical remark consisted in that the

theory was incompatible with Mach's principle – a vice that could turn out a virtue to a Naturforscher biased against metaphysical castles in the air. Later none other than Wolfgang Pauli (1921) christened Nordström's theory an 'empirical blunder' since it had not predicted any deflection of a light ray by a gravitational field and had not explained the anomalous motion of Mercury. Yet there had been no eclipse expeditions in 1913 and Einstein's own 'Entwurf' turned out to be completely incapable to explain the anomalous motion of Mercury.

On the contrary, in 1912 G. Pavani calculated the perihelion shift of Mercury according to Abraham's theory, revealing value of  $14''$ , 52, that is approximately one third of the observed value. So, Abraham's vector theory made a more accurate prediction than the 'Entwurf'.

Furthermore, the 'Entwurf' and the GR consequences coincide with the consequences from the theories of Nordström and Abraham for a number of important cases in certain wholesome approximations. For instance, the 'Entwurf' is reduced to a theory with a four-vector field potential that is formally analogous to maxwellian electrodynamics in suitable weak – field approximation. Moreover, special relativity turns out to be an intermediary step in the transition from GR to Newton's theory (see, for instance, Landau and Lifshitz 1987). But the transition is based on the supposition, for weak and stationary gravitational fields, that the gravitational field is described by a scalar in flat (Minkowski) spacetime, i.e. on the reduction to scalar Nordström's theory (Renn and Sauer 2007). Likewise, the so-called "linear approximation" in GR, still in common use to account for gravitational waves (Einstein 1916), presupposes the transition to such a theory of gravitation in which the gravitational wave, in full analogy with classical electrodynamics, is described by a vector in flat spacetime, i.e. the transition to vector theory of Abraham (1915). Abraham's claim that his theory contained Einstein's as a limiting case was rebutted by Einstein, though. But, nevertheless, in both cases the relations between the GR and the theories of Nordström and Abraham strongly resemble the pattern of classical electrodynamics where the general potential is represented by many-component object such as a vector or a tensor, which, in the special case of a static field, reduces to a single-component object.

It is no wonder that the results that Abraham obtained in the course of his research, such as the very possibility and some essential properties of gravitational waves, remain to this day a standard for a relativistic theory of gravity.

“According to our theory, light and gravitation have the same speed of propagation; but whereas light waves are transverse, gravitational waves are longitudinal” (Abraham quoted from Renn 2007, 327).

Moreover, Abraham, in March 1912, was the first to hit upon a singularity in a field theory of gravitation and to calculate what was later called ‘the Schwarzschild radius’ of a black hole (Renn 2007b). So, it was not accidental that only *after* the first publication by Abraham did Einstein also turn to the problem of gravitation after a period of 1908–1911 devoted to quanta.

On the other hand, for a long time it was a commonplace that the GR creation embodies an exceptional case in the history of physics since it was created by a single person – Albert Einstein. Owing to this reason, the theory grounded on extremely narrow empirical base (three critical GR effects) managed to provide an avalanche of empirical data referring to relativistic astrophysics and cosmology. The both peculiarities are explained by a romantic image of a Lone Genius Grasping the Essence of Things in the Bursts of Divine Inspiration. However, in spite of overwhelming importance of Einstein’s impact, this explanation seems untenable in the light of current post-Kuhnian philosophy and sociology of science. A transition from the ‘old’ paradigm to a ‘new’ one can be treated as such only if it involves the majority of the members of the scientific community. And only that explanation for the origins of the paradigm change in science gets the obvious advantage over the other accounts that contemplates the behavior of the majority of the scientists as rational.

As it is elucidated in one of the recent influential studies, “both the peculiar emergence and the *remarkable stability* of Einstein’s theory of gravitation with regard to the further development of physics and astronomy becomes plausible only if the genesis of general relativity is understood, not as a fortunate anticipation of future observational discoveries, but as a *transformation of pre-existing knowledge*” (Janssen, Norton, Renn, Sauer, Stachel 2007, 23; my italics).

All in all, the dynamics of the theory of gravity was predominantly governed by *internal tensions*, contradictions within the knowledge system rather than by new empirical knowledge, which played only a subordinate role at best (Renn 2007). The perihelion advance of Mercury remained a commonly used touchstone for gravitational theories for more than a half century before GR. The bending of light in a gravitational field could simply be inferred from the observation that, in an accelerated frame of reference, light rays must be curved as a consequence of the superposition of the motion of the observer and of the light. Successful, or decisive red shift experiments were performed only in 1960's.

All the above-mentioned conspicuous peculiarities of GR functioning, the common practice of its implementation bolster the following conclusions:

(a) The relations between GR and its ingenious rivals were far more complicated in 1907-1915 than it may seem from the notorious 'truth-falsity' dilemma, so that one can contemplate the *interpenetration* of rival 'paradigms' into each other.

(b) Einstein's GR was better than its inimical rivals if only for the reason that it encompassed them all in significantly modified forms. (Just as the GR encompasses Newton's theory of gravitation and the special theory of relativity, or just as the maxwellian electrodynamics encompasses the partial theoretical schemes of Coulomb, Amperé, Biot&Savare et al.).

(c) Einstein could complete the reconciliation of the knowledge on gravitation and inertia (represented by classical mechanics) and the knowledge on the structure of space and time (embodied by special relativity) via the 'Entwurf - GR' transition only. As a result, he was able to explain the anomalous motion of Mercury.

Hence *the aim of the paper* is to improve the Standard View on the GR genesis and advancement by taking into account the above-mentioned history of science data and philosophical arguments. My *main idea* consists in that the basic reason for the GR victory over the rival programmes of Abraham and Nordström lied in a synthetic character of the Einstein programme. Einstein's programme did supersede the rival ones because it did assimilate some ideas of the Nordström programme as well as some presuppositions of the programme of Abraham. Einsteinian programme's

victory over its rivals became possible because Einstein had put forward as a basic synthetic principle the principle of equivalence that radically differed from that of rival approaches by its open, flexible and contra-ontological character.

In the *second section* of the paper an epistemological model that fits the advancements of current philosophy of science and deals with mature theory dynamics and structure is exhibited; the model is a crux of the present study.

In the *third section* of the paper the initial stage of GR creation (1907–1912) is scrutinized. The crux of the section is an assertion that the invention of relativistic theory of gravity had commenced with the crossbred object construction in Einstein’s 1907 paper, i.e. with the introduction of mass-energy relation into the theory of gravity. The crossbred object entry – the introduction of inertial and simultaneously gravitational mass – led to a penetration of SR methods into Newtonian theory of gravity and to a reverse penetration of Newtonian gravity methods into SR. As a result the both theories were “blown up” from within and the corresponding changes in both of them were set up. The changes were epitomized in the peculiar sequences of crossbred models, the “splinters” of the explosion performed.

(i) On the one hand, an inevitable consequence of the SR penetration into Newtonian theory of gravity turned out to be Nordström’s and Abraham’s scientific research programmes.

(ii) On the other hand, no less inevitable, owing to the equivalence principle, was the Newtonian theory penetration into the SR that led to the sequence of Einstein’s works on the generalization of relativity principle and to spreading the principle not only on inertial systems of reference, but on the various accelerated systems as well.

But the most valuable result of the hybrid theories of Nordström and Abraham consisted in that the both theories maintained some *extremely promising hints* on how the global theory could be created. Hence the climax of the stage was Einstein’s proposal and apprehension of the equivalence principle that became one of the firm GR heuristic foundations.

The *fourth section* of the paper (1912–1913) is dedicated to the ‘Entwurf’ construction. The metric theory sprung out from the synthesis

of Abraham's and Nordström's theoretical schemes, as well as from the preliminary nonmetric theoretical schemes of Einstein. The staple was the metric tensor introduced owing to equivalence principle and Nordström's, Laue's and Planck's startling results. And *it is namely the fact that Entwurf's basic model was constructed due to direct unification of Nordström's, Abraham's and Einstein's (obtained before 1913) theoretical schemes that can explain the reasons for Einstein's programme victory over its rivals*. For any theory unification becomes a 'true' one only in the case when the corresponding research traditions encountered are successfully reconciled, i.e. when their conjunction leads to firm, concrete and empirically verifiable results (similar to perihelion shift of Mercury).

The *fifth section* grapples with 1913–1915 transition from the 'Entwurf' to the GR exposed by Einstein in the lecture to Berlin Academy on November 25, 1915. The last section deals with an interpretation of the results obtained by Michel Janssen, Jürgen Renn and John Stachel. Their basic claim that I wholeheartedly support here is that they were first and foremost *physical*, and not the mathematical arguments that brought Einstein to the GR fundamental equations. What I want to add is that it was via the 'Entwurf – GR' transition that Einstein could complete the reconciliation of the knowledge on gravitation and inertia (represented by classical mechanics) and the knowledge on the structure of space and time (embodied by special relativity) that was able to explain the anomalous motion of Mercury. Riemannian geometry was deployed as a "neutral language" to co-ordinate the theoretical languages encountered.

## 2. A Simple Theory Change Model

As mentioned in the Introduction, the dynamics of the theory of gravity was predominantly governed by *internal tensions*, contradictions within the knowledge system rather than by new empirical knowledge, which played only a subordinate role at best. In this section, a corresponding epistemological model dealing with mature theory dynamics and structure is posited.

The current philosophy of science debates on scientific revolutions allow one to elucidate the views on the structure and functioning of scientific theories, – on the one hand, – and to construct sufficiently sweep-

ing and exact theory change epistemological models, – on the other. In particular, according to one of the models (Nugayev 1999), a scientific revolution is engendered by encounters of some entrenched “old” paradigms, scientific research programmes or research traditions that cannot be reconciled in a common way – by reducing of one of them to another. The way out of the predicament is to work out such a global theory that encompasses all the theories involved in significantly modified forms. The global theory is aimed at “suing” the hiatuses, eliminating the tensions, smoothing away dissensions between different paradigms involved. Just to recapitulate Werner Heisenberg’s “Physik und Philosophie” (Frankfurt am Main, 1959):

“probably, as a kind of general supposition, it can be said that those directions in the history of human thought appeared to be most fruitful, where different ways of thinking had encountered. These ways of thinking are deeply rooted in different spheres of human culture, or in different times, in different cultural milieu, or in different religious traditions. When they really meet with each other, when they correspond to each other so that an *interaction between them takes place*, one hopes that new and interesting discoveries will follow” (my italics).

In the course of global theory invention an indispensable preliminary stage shows itself in the construction of a series of hybrid theories. The latter are persistently set up to the climax when such a hybrid model is constructed that is able to outline the fruitful way of the global model creation through the generalization of models that belong to the lower level of mature theories. According to the aforementioned epistemological model, *radical breakthroughs in science were not due to invention of new paradigms or the creation of new ideas ex nihilo, but rather to the long-term processes of the reconciliation and interpenetration of ‘old’ research traditions preceding such breaks.*

It is a humdrum that no profound epistemological model of scientific revolutions can be established without preliminary elucidating the structure of mature scientific theories. Yet what I want to stress is that a mature theory of XIX and XX centuries physics encompasses not a single model



or a bundle of models. It embraces a *bundle of groups of models* that are related to one another in rather subtle ways. A mature theory is so organized that the host of its models is disseminated over at least three following interconnected levels (Stepin 2005).

(1) The level of the *basic* model (or ‘the Fundamental Theoretical Scheme’).

(2) The level of the *subordinated* models (or ‘the Partial Theoretical Schemes’) constructed from the fundamental one according to certain (tacit) rules.

(3) The level of the ‘*Empirical Schemes*’ that can be approached through the level of partial theoretical schemes. For instance, the relations between the basic objects of Newtonian mechanics are described by Newton’s laws. The derivative objects of Newtonian mechanics are ‘an absolutely rigid body’, ‘central field’, ‘harmonic oscillator’, etc. The relationships between them are described by certain laws of Newtonian mechanics: that is, by the laws of rigid rotation, movement in the central field, etc.

Likewise, the electric field at a point  $E$ , the magnetic field at a point  $H$ , and the current density  $J$  are the basic theoretical objects of Maxwellian electrodynamics. Maxwell’s equations elucidate the relationships between them.

The set of a mature physical theory’s basic objects forms the *basis*, i.e. the definite subsystem of theoretical objects. All the basic theoretical objects are apparent idealizations and cannot exist as real bodies (like tables and chairs). For example, the material point is defined as a body free of dimensions. As for the other basic objects of Newtonian mechanics, it is assumed that an inertial system of reference can be totally isolated from external influence.

The derivative subsystems are *subordinated* (Stepin) to the basic one, but are independent of each other, referring to different fragments of the same domain of validity. Each subsystem is characterized by its own set of notions and mathematical equations that form a special part (section) of the mature theory. For instance, classical mechanics consists of several independent sections: ‘small-oscillations mechanics’, ‘mechanics of rigid body rotations’, ‘mechanics of movement in a central field’, etc. Each of these sections is characterized by its own subsystem of derivative objects.

Each subsystem is a model of a particular type of mechanical motion (the small oscillations model, the rigid rotations model, etc.). Relations between the elements of the subsystem are described by particular laws of classical mechanics. In general, the relations between a subsystem of basic objects and a subsystem of derivative ones can be described in the following way. Any derivative system is obtained from the basis by a process of reduction. It means that any mature theory is developed not by formal (logical, mathematical) means only, but also through *gedankenexperiments* with abstract theoretical objects. The reduction is put into effect by analyzing the character of the empirically fixed domain of validity. This domain can be “seen through” a cognitive lens of an ideal model, formed by correlations of basic objects. According to the peculiarities of each concrete experimental situation, various restrictions may be imposed on the system of basic theoretical objects. This enables one to define the system, transforming it into a subsystem of derivative objects. The fundamental equations are then applied to the subsystems of derivative objects. In accordance with the system features, they are transformed into the partial laws. The informal nature of such manipulations converts such an inference into a special problem solving operation. The solutions of such problems are included in a theory at its origin. To a theoretician bothered by applying a theory, they serve as a pattern for subsequent activity. Each problem is solved in accordance with primary paradigms (in Thomas Kuhn’s sense).

In classical mechanics, the paradigm examples consist of “derivations” from Newton’s laws: the small-oscillations law, the movement in a central field law, the rigid body rotations law, etc. In classical electrodynamics, the paradigm examples are the laws of Biot & Savart, Coulomb, Ampère, Faraday, et al., derived from Maxwell’s equations.

In general relativity, the host of paradigm examples embraces the derivation of Newton’s theory of gravity from Einstein’s equations in the “weak field approximation”. In the case of weak gravitational field such a system of reference is chosen in which all the metric tensor components slightly differ from their Minkowski values:  $g = \eta + h$  (see, for instance, Landau and Lifshitz 1983). The further demand to ignore the squares and the other multiples of  $h$  is necessary for the transition to Newton’s theory of gravity. But it means nothing else than that index rising operation is carried out

by  $\eta$  – the metric tensor of *flat* spacetime. As a result, in the weak field approximation the gravitational field equations take the form of usual wave equation in flat spacetime for (Nordström’s) scalar potential  $\varphi$ . Thus the basic theoretical object of Nordström’s nonmetric scalar theory turns out to be constructed from the GR basis. “This is quite natural since the weak field is considered as a tensor in flat spacetime” (Zeldovich and Novikov, 1973, p. 56) and is described by an equation  $h_{ik} = -\eta_{ik} 2\varphi/c^2$ . The *construction* of derivative objects from the basic ones enables one to compare theoretical knowledge with experience, to explain the results of real experiments. To this end, an empirical equation – an intermediate relation – is derived from the partial law. In this equation the special constructs are introduced. In contrast to abstract objects, the newly born constructs are no longer idealizations and can be compared with real bodies now. These constructs are called *empirical objects*, and their systems – special representations of empirical situations – are called empirical schemes. Empirical objects are not equivalent to real bodies. An empirical object cannot be compared with a single body with which an experimentalist operates, but only with a *class* of such objects. Consequently, an empirical scheme corresponds not to a concrete experimental situation, but to a *type of such situations*. For example, the empirical scheme of the Biot & Savare experiment with a magnetic needle and a conductor refers to any experiment with any current in the conductor and any small magnetic needle. Of course, a mature theory becomes an *established* one when the links between all the three levels of the organization are vigorously installed that makes possible to use the mature theory as an effective instrument for making predictions. All the bonds between all the three levels of an established mature theory should be rigid ones. This rigidity allows one to connect a prediction referring to the upper level with all levels of a mature theory. Hence it allows one to construct an experimental device to check the prediction. A new result, obtained in the advancement of mathematical apparatus, immediately influences all levels of a mature theory. Hence a theory can predict, and the predictions can be verified. A mature theory obtains the status of an established one when at least some of its predictions are shown to be successful. It demonstrates that the system of basic objects is complete, and the links between all the three levels are made robust.

Due to a mature theory complicated structure, the global theory creation appears to be a slow, adamant and consequent ascent from the lower levels up to the top ones. Any transition from lower level to the upper one is impossible until the construction of all the lower-level models is finished. Yet an important remark here is that the lower models (that served at scaffolding the upper ones) are not eliminated; they can be discovered not only in history-of-science archives. They can be transpired in real practice of theories' functioning (in implicit forms, as a rule).

### 3. The hybrid models construction via the equivalence principle

The advent of the special relativity (SR) and the apparent incompatibility between Newton's theory of gravitation and the SR theory presented Einstein and his contemporaries with the task of constructing a relativistic theory of gravitation. Blatant *contradictions* between the theories consisted first and foremost in the fact that according to Newton's theory the velocity of gravitational interaction was infinite. On the other hand, SR prohibits the signals travelling faster than light. Apparent disparity between the concepts of 'action at a distance' and 'instantaneous action' was revealed just after the construction of maxwellian electrodynamics. It was James Clerk Maxwell himself who tried to invent the first vector theory of gravity. Alas, he was forced to leave the efforts soon due to the problem of gravitational wave's negative energy. SR creation only exacerbated the problem (see Corry 2004, Petkov 2015 for details).

It therefore comes as no surprise that it was Einstein's 1907 review "*On the Relativity Principle and the Conclusions Drawn from it*", published in Johannes Stark's "*Jahrbuch der Radioaktivität und Elektronik*", that laid the true conceptual foundations for relativistic theory of gravity.

"The most important result of the fourth part is that concerning the inertial mass of the energy. This result suggests the question whether energy also possesses heavy (gravitational) mass. A further question suggesting itself is whether the principle of relativity is limited to nonaccelerated moving systems. In order not to leave this question totally undiscussed, I added to the present paper a fifth part that contains a novel consideration, based on the principle of relativity, on acceleration and gravitation» (Einstein 1907, 254–255).

In the fifth part of the epoch-making 1907 paper Einstein formulated first his “*principle of equivalence*”. As he later recalled, when he had prepared his 1907 review article for publication, he had tried to modify Newton’s gravitational theory so as to reconcile it with the special theory of relativity. The corresponding attempts had shown that it was possible, but Einstein did not like them since they were based on physically unacceptable hypotheses.

“At this point, there occurred to me the happiest thought in my life [*der glücklichste Gedanke meines Lebens*]. Just as in the case with the electric field produced by electromagnetic induction, the gravitational field has similarly only a relative existence. For if one considers an observer in free fall, e.g. from the roof of a house, there exists for him during this fall no gravitational field – at least not in his immediate vicinity. Indeed, if the observer drops some bodies, then these remain relative to him in a state of rest or in uniform motion, independent of their particular chemical or physical nature” (Pais 1982, 178; my italics).

Because of the importance of the equivalence principle for the GR creation and the uninterrupted discussions on its ‘true content’, we have to resort to all the piece of 1907 paper where the principle had been first formulated.

“We consider two systems  $\Sigma_1$  and  $\Sigma_2$  in motion. Let  $\Sigma_1$  be accelerated in the direction of its X-axis, and let  $\gamma$  be the (temporally constant) magnitude of that acceleration.  $\Sigma_2$  shall be at rest, but it shall be located in a homogeneous gravitational field that imparts to all objects an acceleration  $-\gamma$  in the direction of the X-axis.

As far as we know, the physical laws with respect to  $\Sigma_1$  do not differ from those with respect to  $\Sigma_2$ ; this is based on the fact that all bodies are equally accelerated in the gravitational field. At our present state of experience we have thus no reason to assume that the systems  $\Sigma_1$  and  $\Sigma_2$  differ from each other in any respect, and in the discussion that follows, we shall therefore *assume* the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system.

This assumption extends the principle of relativity to the uniformly accelerated translational motion of the reference system. The *heuristic value* of this assumption rests on the fact that it permits the replacement

of a homogeneous gravitational field by a uniformly accelerated reference system, the latter case being to some extent accessible to theoretical treatment” (Einstein 2007, 302; my italics).

Note that Einstein was first and foremost interested not in the ontological, metaphysical content of his principle that could enable him to elevate the tenet up to the status of some Ultimate Law of Nature. The latter would be valid everywhere with any degree of validity being contemplated by a Super Reason trying to grasp the essences of the things and events. (For it is well-known, according to Norton 2007, that in 1907 Einstein was unaware of Eotvös’s exact experimental results regarding the equality of inertial and gravitational mass. Moreover, Papapetrou in 1951 disclosed that in the GR a rotating body falls differently, in general, from a non-rotating body).

Furthermore, in his reminiscences on the equivalence principle invention Einstein appeals not to, say, metaphysical systems of Aristotle or Hegel that encouraged grasping the “essences of things” but to his own experience of SR creating (Pais 1982). It seems evidently rational for 1907 Einstein to repeat the same process that had led to success just in 1905.

Thus, both in SR and GR cases he was looking for the *heuristic* components of the principle (see Ryckman 2005 for details). In gravity purview he tried to comprehend gravitational and inertial phenomena from a single point of view. As Michele Janssen puts it,

“While the slide into general covariance turns the relativity of non-uniform motion of space-time coordinate systems into a feature general relativity shares with older theories, it does not so trivialize the relativity of the gravitational field. Even in generally covariant reformulations of these older theories, there will be an inertial field and a gravitational field existing side by side. The unification of these two fields into one inertia-gravitational field that splits differently into inertial and gravitational components in different coordinate systems is one of Einstein’s central achievements with general relativity” (Janssen 2012, 162).

In my view, it was consequent implication of the equivalence principle that promised to invent a consequence of hybrid models unifying SR and Newton's theory of gravity. For Einstein the principle of equivalence was not so much a Law of Nature as a *pattern*, a 'paradigm' for gravitation theories construction.

In particular, it enabled the investigation of special cases of the gravitational field by means of the study of accelerated motion. So, until 1911 Einstein had committed himself mainly to exploring, by means of the equivalence principle, the effects and conceptual changes characterizing a new theory of gravitation, evidently without seriously attempting its construction. Only in early 1912 was he challenged by the publication of Max Abraham to elaborate such a theory, at least for the special case of a static gravitational field (see Norton 1986).

Furthermore, in September 1913 Einstein presented a lecture at the 85<sup>th</sup> Congress of the German Natural Scientists and Physicians in Vienna that was published in December issue of *Physikalische Zeitschrift* under the heading "*On the present state of the problem of gravitation*". In the lecture Einstein made clear his preference for Nordström's theory over other gravitation theories, stating that Nordström's later version of his gravitation theory was the only competitor to the 'Entwurf' theory satisfying four requirements that could be asked of any reasonable theory of gravitation.

1. Satisfaction of the laws of energy and momentum conservation.
2. The equivalence principle.
3. Validity of SR.
4. The observable laws of nature do not depend on the absolute magnitude of the gravitational potentials.

Note that Einstein stressed the heuristic value of almost all the requirements admitting "the postulates 2-4 resemble a *scientific profession of faith more than a firm foundation*" (Einstein 1913).

On the other hand, the second important component of Einstein's heuristic – "the Lorentz model of a field theory" (Renn, Sauer) – enabled Einstein to conceive Newtonian gravitation and inertia as special cases of a more general interaction. For the case of uniform acceleration he could

directly identify inertial effects with a scalar Newtonian gravitational field and he expected that he would be able to do the same for more general cases by generalizing the notion of gravitational field. A model for the generalizations was provided by Maxwellian electrodynamics. It was Maxwell who “unified” electricity and magnetism through treating electric field  $\mathbf{E}$  and magnetic field  $\mathbf{B}$  as different facets of one and the same electromagnetic field tensor  $F_{\mu\nu}$ . Accordingly, for Einstein the most important achievement of GR was not ‘geometrization of gravity’ but “unification of gravity and inertia” via the metric tensor  $g_{\mu\nu}$ .

In one of his papers Einstein (1912) even wrote on the “equality of essence” [Wesengleichheit] of inertial and gravitational mass. Between 1907 and 1911 he used the equivalence principle to derive several consequences of his yet to be formulated relativistic theory of gravitation.

It is important that in the case considered Einstein follows the paths of SR. Indeed, the new theory invention begins with the crossbred object construction, i.e. with the mass-energy introduction into theory of gravity. One of the important SR consequences is the equivalence of mass and energy tenet. But, according to Einstein, “this result suggests the question whether energy also possesses heavy (gravitational) mass. A further question suggesting itself is whether the principle of relativity is limited to non-accelerated moving systems” (Einstein 1907, 254).

From the very beginning Einstein was aiming at such a theory of gravitation that was to comprise both the knowledge on gravitation and inertia represented by the classical mechanics and the knowledge on the structure of space and time embodied by the SR. However, the crossbred object introduction – the introduction of inertial and simultaneously gravitational mass – leads to penetration of SR methods into Newtonian theory of gravity and to reverse penetration of Newtonian gravity methods into the SR. As a result the both theories were “blown up” from within and the corresponding changes in both of them were set up. The changes were epitomized in the peculiar sequences of crossbred models, the “splinters” of the explosion performed.

On the one hand, an inevitable consequence of SR penetration into Newtonian theory of gravity turned out to be Nordström’s and Abraham’s scientific research programmes. On the other hand, no less inevitable,



owing to the equivalence principle, was Newtonian theory penetration into the SR that led to the sequence of Einstein's works on the relativity principle generalization and to spreading the principle not only on inertial systems of reference, but on the various accelerated systems as well. Einstein used the principle of equivalence in order to transform the knowledge not of classical mechanics only but the knowledge embodied in *both*, classical mechanics and SR. His theory of the static gravitational field, as well as his early attempts to generalize it, were nothing but a reinterpretation of the SR with the help of the introduction of accelerated frames of reference. His systematic treatment of such accelerated frames induced him to apply generalized Gaussian coordinates in order to describe the coordinate systems adapted to these frames. It was then a natural step for him to consider the metric tensor. And with the introduction of the metric tensor Einstein constructed the theoretical object that was capable of representing gravitational and inertial theoretical objects on the same footing.

By the beginning of 1912, Einstein realized that he would ultimately have to proceed beyond a scalar theory of gravitation. His strategy was to move carefully in a step-by-step manner towards a full dynamical theory. The first step in the programme was to treat the "gravito-static" case, the gravitational analogue of electrostatics. However, he was already thinking about the second step, the "gravito-stationary" case, the gravitational analogue of magnetostatics. His ultimate goal was to advance a theory for time-dependent gravitational fields.

In March 1912 he was able to write to Paul Ehrenfest:

"The investigations of gravitational statics (point mechanics, electromagnetism, gravitostatics) are complete and satisfy me very much. I really believe that I have found a *part of the truth*. Now I am considering the dynamical case, again also proceeding from the more special to the more general case" (quoted from Renn 2007, 98).

As is well known, in 1908–1911 Einstein had neglected gravitation, possibly because of his preoccupation with the problem of quanta. But this, however, is only the part of the explanation. The remaining part consists

in that he realized how much work had to be done to arrive at an ultimate global theory able to embrace all the particular results obtained, the “parts of the truth” as Einstein called them, transforming them into the details of a great edifice. And, since Einstein himself was delved into the peculiarities of the quanta, the problem of creating the gravitation global theory scaffolds had fallen on Abraham’s and Nordström’s broad shoulders.

However, one has to keep in mind that even the pathways of their theories’ creation were outlined by Einstein himself, especially in his ground-breaking 1907 paper. Indeed, one of the important SR consequences states that  $E = mc^2$ . Since, in a gravitational field, the energy of a particle depends on the value of the gravitational potential at the position of the particle, the equivalence of energy and mass suggests that:

- (1) either the particle’s mass  $m$ ;
- (2) or the speed of light  $c$  (or both) must also be a function of the potential.

In 1907 Einstein explored the both possibilities. The possibilities considered, a dependence of the gravitational potential either of the speed of light  $c$  or of the inertial mass  $m$ , were later explored by Max Abraham (1912a, 1912b, 1915) and Gunnar Nordström (1912, 1913a, 1913b) respectively. And first of all it became clear that one can easily construct such a Lorentz-invariant theory of gravitation in which the inertial and gravitational masses are equal (Nordström, 1912–1914). Nordström’s 1912 paper “*The principle of relativity and gravitation*” starts as follows:

“Einstein’s hypothesis that the speed of light  $c$  depends upon gravitational potential leads to considerable difficulties for the principle of relativity, as the discussion between Einstein and Abraham shows us. Hence, one is lead to ask if it would not be possible to replace Einstein’s hypothesis with a different one, which leaves  $c$  constant and still adapts the theory of gravitation to the principle of relativity in such a way that gravitational and inertial mass are equal. I believe that I have found such a hypothesis, and I will present it in the following” (Nordstrom [1912], 2007, p. 488).

On the other hand, Einstein's static gravitational theory did not offer even a hint at how the global theory should be constructed. On the contrary, a Göttingen theoretician, a master of classical electrodynamics Max Abraham was one of the first scholars (along with Gustav Herglotz and Max Born) to propose that the four-dimensional line element, defining the infinitesimal distance between points in Minkowski space in terms of a constant metric, has to be replaced by a variable line element whose functional dependence of the coordinates is determined by a gravitational potential associated with the variable speed of light.

In a lecture presented in October 1912 and published the following year Abraham was one the first (along with Henri Poincaré) to discuss the possibility of gravitational waves in relativistic theories of gravitation. Moreover, in 1912 G. Pavani had calculated the perihelion shift of Mercury according to Abraham's theory, finding a value that was approximately one third of the observed one (Renn 2007). Abraham's theory thus made a more accurate prediction than even the 'Entwurf' theory.

It was not accidental that Einstein turned to the global gravitational theory construction only *after* the publication of Abraham's first vector gravitational theory. It should be noted that for static fields Abraham's theory coincides with Einstein's. But the most valuable result of the hybrid theories of Nordström and Abraham consisted in that the both theories maintained *extremely promising hints* on how the global theory could be created (Renn 2007).

At first, by letting the geometry of Minkowski space depend on the gravitational potential (Abraham). At second, by representing the gravitational potential not by a single function but by a ten-component theoretical object on a par with the stress-energy tensor and having this tensor as its source (Laue and Nordström). At third, by including an effect of the gravitational potential on the measurement of space and time (Nordström).

#### 4. The genesis of Einstein's and Grossmann's 'Entwurf'

Let me start from Nordström's startling result obtained with a help of M. Laue's achievements. The result draws on the fundamental problem of classical electrodynamics – the problem of electron's electromagnetic mass that owes so much to Abraham's works (see, for instance,

Abraham 1909). If one computed total momentum and energy of the electromagnetic field of an electron, the result universally accepted at that time was: (Total field momentum) =  $4/3c^2$  (Total field energy) (Electron velocity).

Hence, as Poincaré and Einstein elucidated, there must be also stresses of a non-electromagnetic character within the electron ('Poincaré's stresses'). So, the puzzle Max von Laue addressed in 1911 was to find very general circumstances under which the dynamic of such an electron would agree with the relativistic dynamics of point masses. While Hermann Minkowski had introduced the four-dimensional stress-energy tensor at the birth of four-dimensional methods in SR, his use of the tensor was restricted to the special case of the electromagnetic field. Laue's work concentrated on extending the use of this tensor to the most general domain (Laue 1911a, 1911b, 1911c). The properties of this tensor and its behavior under Lorentz transformations summarized a great deal of the then current knowledge of the behavior of stressed bodies.

As a result, Laue arrived at the expression for the stress-energy tensor  $T_{\mu\nu}$  ( $\mu, \nu = 1, 2, 3, 4$ ) that embraced three main blocs.

(1) The first bloc represents the familiar three dimensional tensor  $p_{ik}$  ( $i, k = 1, 2, 3$ ).

(2) The second bloc represents the momentum density  $\mathbf{g}$  ( $g_x, g_y, g_z$ ).

(3) The third bloc represents the energy flux  $\boldsymbol{\theta}$  ( $\theta_x, \theta_y, \theta_z$ ).

And surely the ( $T_{44}$ ) component of the energy-momentum tensor represents energy. Einstein's equivalence principle prompted that each stress-tensor bloc should give *its own impact* into the gravitational field potentials, i.e. each bloc is related to the gravitational potentials of its own. Hence there should be many potentials – scalar ones, vector potentials, etc. and not a single one. That is why the overall gravitational field potential should be a group of several potentials and should in the most general case be described by a matrix, a tensor, since its components are transformed in the coordinate transformations like scalars, vectors, etc. The most pertinent analogy that played an important heuristic part was, of course, maxwellian electrodynamics with 4-dimensional electromagnetic field potential  $A^\mu = (\mathbf{A}, \varphi)$ . The latter, in particular static electromagnetic field case, is reduced to static potential  $\varphi$ . It's no wonder that in the Zurich

notebook, before the 'Entwurf' theory, Einstein had freely worked with tensors. The traces of the work can be easily found in his unpublished review on the SR, probably written between 1912 and 1914 for "Handbuch der Radiologie". The heading of the section 3, dealing with vectors, tensors, etc., is: "Some Concepts and Theorems of the Four-Dimensional Vector and Tensor Theories that Are Necessary for the Comprehension of Minkowski's Presentation of the Theory of Relativity" (CPAE, vol. 4, Doc. 1). Moreover, one of the subsections is entitled "The Stress-Energy Tensor of Electromagnetic Processes".

This peculiarity was later thoroughly described by Göttingen master of electrodynamics Max Abraham in his 1915 thought-provoking paper "*Recent Theories of Gravitation*". The paper contained a special important passage critically analyzing Einstein's and Grossmann's 'Entwurf' that is worth quoting in full.

"The basic idea of the tensor theory of the gravitational field can be understood as follows. The energy density, which in a static field is determined by the divergence of the gradient of the gravitational potential, plays in the theory of relativity merely the role of one component of the resulting world tensor  $T$ ; it is joined by nine other tensor components which characterize the energy flux and the stresses. The tensor theory assumes that, like the energy density ( $T_{44}$ ), the remaining nine components  $T_{\mu\nu}$  ( $\mu, \nu = 1, 2, 3, 4$ ) *generate gravitational fields* whose potentials  $g_{\mu\nu}$  form a ten-tensor themselves" (Abraham 1915, 499).

The physical sense of the components is explained by Abraham below when he remarks that the integration of 'Entwurf's field equations:

"is extraordinary difficult. Only the method of successive approximations promises success. In this one will usually take as a first approximation the solution that treats the field as static. Here, *Einstein's theory becomes identical with Abraham's theory* (...). In his Vienna lecture A. Einstein takes the normal values of the  $g_{\mu\nu}$  as the first approximation:  $g_{11} = g_{22} = g_{33} = 1$ ;  $g_{44} = -c^2$ ,  $g_{\mu\nu} = 0$  for  $\mu\nu$ ; he considers

the deviations  $g^*_{\mu\nu}$  from these normal values as quantities of first order, and arrives, by neglecting quantities of second order, at the following differential equations:  $\square g^*_{\mu\nu} = T^m_{\mu\nu}$ . For incoherent motions of masses, the last ( $T^m_{44}$ ) among the components of the material tensor  $T^m$  is the most important; it determines the potential  $g^*_{44} = \Phi^g$ . Then follow the components  $T^m_{14}$ ,  $T^m_{24}$ ,  $T^m_{34}$ , which are of first order in  $v/c$ ; these determine the potentials  $g^*_{14}$ ,  $g^*_{24}$ ,  $g^*_{34}$ , which can be viewed as the components of a space vector –  $(1/c) U^g$ . The remaining components of  $T^m$  are of second order in  $v/c$ . If one neglects quantities of this order, then one only needs to consider those four potentials, and obtains for them the differential equations

$$\square \Phi^g = c^2 \mu \quad (60a)$$

$$\square U^g = c^2 \mu (v/c) \quad (60b)$$

where  $\mu$  is the mass density.

Here the analogy with electrodynamics catches one's eye. Except for the sign, the field equations (60 a, b) agree with those that must be satisfied in the theory of electrons by the 'electromagnetic potentials', the scalar one ( $\Phi$ ) and the vector one ( $A$ ). In this approximation, *the Einstein-Grossmann tensor theory of the gravitational fields leads to the same results as the vector theory sketched in (IA) [i.e. the theory of Abraham]*" (Abraham 1915, 500–501; my italics).

As the correspondence and the papers indicate, Einstein agreed with Nordström's assessment of the importance of Laue's work for gravitation theory. Moreover, some pieces of his 1912 and 1913 papers (his proposal to call  $T$  'Laue's scalar', for instance) indicate that he had personal contacts with Laue and discussed the stress-tensor problems with him. Such personal communication is compatible with the fact that both Einstein and Laue were teaching in Zurich, with Einstein at the ETH and Laue at the University of Zurich (Norton 2007). It should be added that the same year

Nordström also came to Zurich where supposedly he had communicated with the both researchers.

Yet what the connection between  $T_{\mu\nu}$  and  $g_{\mu\nu}$  should be? – An important hint is contained in Nordström’s first 1912 paper:  $R = (k/2) T$ , where  $R$  is the fully contracted Riemann – Christoffel tensor and  $T$  the trace of the stress-energy tensor. But if the expression is characteristic of scalar theory, the expression that generalizes it in a most natural and simple way should look like:  $k T_{\mu\nu} = \Gamma_{\mu\nu}$  (where  $\Gamma_{\mu\nu}$  is a “contravariant second rank tensor formed by the derivatives of the fundamental tensor  $g_{\mu\nu}$ ”). But these are exactly the gravitational field equations of the first metric theory – the theory of Einstein and Grossmann, published in 1913!

It was immediately understood that in general the ‘Entwurf’ equations are not covariant; they “remain covariant only with respect to linear orthogonal substitutions”. Yet for a long time this peculiarity did not bother the authors. It indicates once more that the ‘Entwurf’ field equations were born *not* from the covariance principle but represented a direct generalization of hybrid theories of Nordström and Abraham with a help of Laue’s results.

Just as Einstein put it in his November 1913 letter to Paul Ehrenfest,

“The gravitational affair has been clarified to my *complete satisfaction* (namely the circumstance that the equations of the gr. field are covariant only with respect to linear transformations). For it can be proved that generally covariant equations that determine the field completely from the matter tensor cannot exist at all. Can there be anything more beautiful than this, that the necessary specialization follows from the conservation laws” (Einstein 1993, Doc. 481; my italics).

However, on the other hand, according to later reminiscences,

“The equivalence principle allows us...to introduce non-linear coordinate transformations in such a [4-dimensional] space [with pseudo-Euclidean metric]; that is, non-Cartesian (“curvilinear”) coordinates. The pseudo-Euclidean metric then takes the general form:  $ds^2 = \Sigma g_{ik} dx_i dx_k$  summed over

the indices  $i$  and  $k$  (from 1 to 4). These  $g_{ik}$  are then functions of the four coordinates, which according to the equivalence principle describe not only the metric but also the ‘gravitational field’” (Einstein, quoted from Seelig 1955, 55).

Certainly, the question was raised on getting the mathematical apparatus dealing with such mathematical objects. In particular, from the mathematical point of view, the task was to find a differential operator of second order for the metric tensor covariant with respect to the largest possible class of coordinate transformations. In August 1912 Einstein left Prague, where he had taught for a year and a half, to become a full professor at the Eidgenössische Technische Hochschule (ETH). With Einstein’s return to Zurich, he began a collaboration with his old friend Marcel Grossmann. The collaboration ceased in 1914, when Einstein moved to Berlin to become a salaried member of the *Preussische Akademie der Wissenschaften*.

Grossmann’s help was needed to solve the problem. Grossmann found that the exquisite mathematical apparatus was contrived at the end of the XIX-beginning of the XX-th century by Riemann, Levi – Civita, Ricci, Christoffel et al. That is why the first part of the ‘Entwurf’ containing the gravitational field equations was written by A. Einstein, and only the second one – by Grossmann. Continuing the quotation from the 1955 reminiscences:

“I was made aware of these [works by Ricci, Levi – Civita et al.] by my friend Grossmann in Zurich, when I put to him the problem to investigate generally covariant tensors, whose components depend only on the derivative of the coefficients of the quadratic fundamental invariant. He at once caught fire, although as a mathematician he had a somewhat skeptical stance towards physics... He went through the literature and soon discovered that the indicated mathematical problem had already been solved, in particular by Riemann, Ricci and Levi – Civita. This entire development was connected to the Gaussian theory of curved surfaces, in which for the first time systematic use was made of generalized coordinates” (Seelig 1955, 15–16).



Thus, the metric programme advancement commenced from the ‘Entwurf’ paper (Einstein and Grossmann 1913). In section 1 of the physical part, written by Einstein alone, it was contended, with reference to Planck’s 1906 important result, that in special relativity the equation of motion of a point particle not subject to forces follows from extremising the line element

$$\delta \int ds = \delta \left\{ \int \sqrt{(-dx^2 - dy^2 - dz^2 + c^2 dt^2)} \right\} = 0$$

(1) After that, Einstein appeals to his ‘equivalence principle’, stating that as a consequence of it he found that in his scalar 1912 theory of gravitation (with  $c$  representing both the gravitational potential and the local speed of light) equation of motion for force-free particles (1) also applies to point particles moving in a static gravitational field. The only difference consists in that in this case  $c = c(x, y, z)$  varies with the spatial coordinates.

The pertinent question is what happens when one considers the motion of point particles in the presence of general, i.e. non-static gravitational fields? – In that case the abovementioned Laue’s results amended by the ‘equivalence principle’ appeared to be of special importance. And in section 2 of the ‘Entwurf’ Einstein takes the abovementioned variation principle as a starting point to argue that for non-static gravitational fields, too, one should expect equation (1) to describe point-particles’ motions. The only difference is that now the line element on the left-hand side of the equation has to be that defined by a general metric tensor  $g_{\mu\nu}$ .

This was the first time the metric tensor was introduced in a published paper. Three months after submitting the ‘Entwurf’, Einstein submitted a paper to the 85th conference of the German Society for Scientists and Physicians. In the paper he explicitly stated that “A free mass point moves in a straight and uniform line according to Eq. (1), where  $ds^2 = \sum g_{ik} dx_i dx_k$ ”. And finally on September 1913 in Vienna Einstein presented a lecture exposing the physical foundations of the ‘Entwurf’ and those aforementioned general conditions (1) – (4) which any relativistic theory of gravity should satisfy (Einstein 1913). It is important that from the host of the gravitational theories at hand Einstein marked out Nordström’s scalar

theory that satisfied all the requirements for a theory of gravitation that could be imposed on the basis of current experience.

As a result, the main achievement of the second streak consisted in the metric tensor invention; the latter appeared to be a crossbred object that unified two different research traditions – a physical tradition (scalar and vector theories of Nordström and Abraham) and a mathematical one (geometrical results of Riemann, Christoffel, Levi – Civita et al.). Now  $g_{ij}$ 's had a *dual* function: on the one hand, they represented the physical gravitational potentials and on the other they were coefficients in the expression of  $ds^2 = \sum g_{ij} dx^i dx^j$ . By dint of the crossbred object  $g_{ij}$  contrivance the interpenetration of geometry and physics began: physics became geometrized, and geometry was made empirical (Zahar 1989, 267).

Note that *Einstein himself did not consider the geometrization of the gravitational field as a major achievement of his research programme stressing that GR was no more and no less geometrical than Maxwell's theory of electromagnetism*. For instance, in his 8 April 1926 letter to Reichenbach, he famously confessed that

“It is wrong to think that ‘geometrization’ is something essential. It is only a kind of crutch [Eselsbrücke] for the finding numerical results” (quoted from Lehmkuhl 2014, 317).

Or, likewise, in Einstein's review of Emile Meyerson's book “*La deduction relativiste*”, written in German by Einstein in 1927, it is maintained that

“The fact that the metric tensor is denoted as ‘geometrical’ is simply connected to the fact that this formal structure first appeared in the area of study denoted as ‘geometry’. However, this is by no means a justification for denoting as ‘geometry’ every area of study in which this formal structure plays a role, not even for the sake of illustration one makes use of notions which one knows from geometry. Using a similar reasoning Maxwell and Hertz could have denoted the electromagnetic equations of the vacuum as ‘geometrical’ because the geometrical concept of a vector occurs in these equations” (quoted from Lehmkuhl 2014, 318).

However, one more function of ‘geometrization’ should be taken into account. I think that it helped to construct a “neutral theoretical language” necessary to bring the substantially different theoretical traditions – that of Newtonian gravity and Special Relativity – under the same cover to compare and to reconcile them. In this regard the following analogy with Maxwellian electrodynamics genesis seems to be appropriate.

In the course of electricity and magnetism unification, the first Maxwell’s paper “*On Faraday’s Lines of Force*” [1856–1858] was dedicated to elaboration of the “analogies” method borrowed from Kantian epistemology. The method rejects the “ontological” approaches looking for the “essences” of electrical and magnetic phenomena and proclaiming that “in reality” electricity and magnetism are “fields” and not “action at a distance” phenomena, or vice versa. Maxwell’s proposal was to consider Faraday’s lines of force as a kind of tubes filled with *ideal* incompressible fluid.

“I propose then, (...); and lastly to show how by an extension of these methods, and the introduction of another idea due to Faraday, the laws of the attractions and inductive actions of magnets and currents may be clearly conceived, without making assumptions as to the *physical nature* of electricity, or adding anything to that which has been already proved by experiment. By referring everything to the purely geometrical idea of the motion of an imaginary fluid, I hope to attain generality and precision, and to avoid the dangers arising from a premature theory professing to explain the cause of the phenomena” (Maxwell [1890], 1952, p. 159; my italics).

It was crucial for a Kantian that the incompressible poison contrived has nothing to do with experimental reality. The constraints on the theory proposed consist in the demand that the mathematical constructs should not contradict each other. In all the other matters the physical analogies method admits an unlimited freedom of imagination. Even the conservation laws could be broken down.

“There is nothing self-contradictory in the conception of these sources where the fluid is created, and sinks where it is annihilated. The properties of the fluid are at our disposal, we have made it incompressible, and now we suppose it produced from nothing at certain points and reduced to nothing at others” (Maxwell [1890], 1952, p. 162).

Maxwell stressed the generality of the lines of force approach, for it could account for any kind of force. For instance, it does not exclude the force of action at a distance which varies inversely as the square of the distance, as force of gravity or as observed electric and magnetic phenomena.

And in the other parts of the paper Maxwell exhibited the ways by which the idea of incompressible fluid motion could be applied to the sciences of statical electricity, permanent magnetism, magnetism of induction, and uniform galvanic currents. The core element of his innovations consisted in constituting a language game with a “neutral language” for description and comparison of the consequences from the rival theories. Maxwell’s “neutral language” was not Carnap’s and Reichenbach’s “observation language” springing out from the “protokolsatze” generalizations. Maxwell was aware of the theory – laidness of the observation data (“experimental laws already established, which have generally been expressed in the *language of other hypotheses*” (Maxwell [1890], 1952, p. 162)). He clearly understood that every observation always carries the footprints of the theoretical language that helps to describe it. (“The daubing of untempered mortar”, as he will call them later in his “Helmholtz” paper). In order to compare and to unite in a theoretical scheme lacking contradictions all the results of the different experiments carrying the footprints of different theoretical languages, it is necessary to construct an *artificial* theoretical language equally distant from the languages of theories under comparison. This language appeared to be the solid state mechanics (with hydrodynamics as its part). Maxwell’s ultimate aim was to rewrite all the known empirical and theoretical laws of electricity and magnetism using the neutral language and then to compare them in order to create a system without contradictions (Nugayev 2015).

In a similar vein Einstein used geometry as a neutral theoretical language in his unification of gravitational and inertial phenomena (see Sauer 2015 for details). Moreover, the interpenetration of geometry and physics led to construction of the GR fundamental theoretical scheme. The first stage of interpenetration resulted in the gravitational field equations of ‘Entwurf’:  $R_{\mu\nu} = \chi T_{\mu\nu}$  with their ultimately simple premises of gravitational potentials being common partial derivatives of metric. However the further penetration of physics into the geometry led to skilful modification of the plain scheme.

### 5. The transition from 1913 Entwurf to the full-blooded November 1915 theory of gravitation

In hindsight, Einstein gave three reasons for his rejection of the ‘Entwurf’ theory:

- it could not explain, due to Michele Besso’s assiduous demonstration, the perihelion shift of Mercury (Renn and Sauer 2007);
- it did not allow the interpretation of a rotating system as being equivalent to the state of rest, and hence did not satisfy Einstein’s Machian fancies;
- the derivation of the ‘Entwurf’ field equations involved an unjustified assumption.

In a series of four communications to the Prussian Academy of Science in November 1915 Einstein replaced the ‘Entwurf’ by a full-blooded metric theory of gravitation, solving incidentally the problem of Mercury’s perihelion.

Thus, how did the transition from the ‘Entwurf’ to the full-blooded GR (exposed on November 25, 1915 in Berlin Academy of Science) take place? How genuine Einstein’s gravitational field equations  $R_{\mu\nu} - (R/2) g_{\mu\nu} = kT_{\mu\nu}$  were obtained?

The modern scholars tentatively discern two basic strategies in Einstein’s creativity – a ‘*physical strategy*’ and a ‘*mathematical*’ one. Following the physical strategy, one tentatively constructs field equations in analogy with Maxwell’s equations, making sure from the start that energy-momentum conservation is satisfied and that the Poisson equation of

Newtonian theory is recovered in the case of weak static fields. This is the approach that led Einstein to the ‘Entwurf’ equations. Following the mathematical strategy, one picks candidate field equations based largely on considerations of mathematical elegance and only after investigates whether they make sense from a physical point of view. In spite of the fact that the question “which strategy dominated in the GR creation” is a subject of strenuous discussions, my own experience of dwelling into Einstein’s works cries for the physical strategy the strong adherents of which are M. Janssen and J. Renn (2007). According to them, what happened in 1915 was that the physical strategy led Einstein back to field equations to which the mathematical strategy had already led him in the Zurich notebook but which he had then been forced to reject since he could not find a satisfactory physical interpretation to them.

In particular, it was not until October 1915 that Einstein had discovered that the ‘Entwurf’ field equations are incompatible with one of the guiding ideas of his research programme – the idea that the inertial forces of rotation can be conceived of as gravitational forces (the Lense – Thirring effect in the GR). And, what is more important, Einstein realized that the gravitational field should be represented not by common partial derivatives but by the so-called Cristoffel symbols. Only then the gravitational field Lagrangian should more resemble the Lagrangian of the electromagnetic field in full accord with Einstein’s research programme heuristic.

Hence one can discern the two stages in Einstein’s gravitational field equations derivation: (1) Einstein’s, Abraham’s and Nordström’s hybrid models’ synthesis that led to the ‘Entwurf’ construction; (2) analogy with Maxwell’s equations pursuance application that provided the transition from the ‘Entwurf’ to a full-blooded metric theory of gravitation.

Yet what I want to stress here is that the analogy with Maxwell’s equations embraced some important methodological points. The key in correct Mercury’s perihelion motion explanation lied in the correspondence principle. Einstein could successfully explain the anomalous perihelion motion only *after* he was able to exhibit that Newton’s law of gravity turns out to be a limiting case of the gravitational field equations in the ‘weak-field approximation’. Thus, he could explain the perihelion motion only when he had comprehended that Garber’s 1898 theoretical model

that fit the perihelion observations quite well should be comprehended as a partial theoretical scheme of the GR. Thus, he had to construct it from the Fundamental Theoretical Scheme via some reasonable presuppositions. It explains why Einstein “actually did not use his GR equation of motion, but used Newton’s equation with a slight modification of the gravitational potential when he calculated Mercury’s anomalous orbit” (W. Engelhardt).

Yet the GR field equations still remained the capstone of the edifice of GR only. Since that time, much efforts were to be done to construct an ultimate edifice.

## 6. Conclusions

To recapitulate, the outline of the GR genesis proposed enables to highlight the following discernible hallmarks of the process that are obfuscated by the other studies and to arrive at a more comprehensive account of the Einsteinian revolution *intertheoretic context*. (One should always keep in mind that real creative science is always messier and more complicated than philosophers of science – and science educators – like to fancy).

(i) Relativistic theory of gravity invention had begun with the crossbred object construction in Einstein’s 1907 paper. The crossbred object introduction – the introduction of inertial and simultaneously gravitational mass ( $E = m$  – led to SR methods penetration into Newtonian theory of gravity and to reverse penetration of Newtonian gravity methods into SR. As a result, the both theories were “blown up” from within and the corresponding changes in both of them were set up. The changes were epitomized in the peculiar sequences of crossbred models, the “splinters” of the explosion performed.

(a) On the one hand, an inevitable consequence of the SR invasion into Newtonian theory of gravity turned out to be Nordström’s and Abraham’s scientific research programmes.

(b) On the other hand, no less inevitable, owing to the equivalence principle, was the Newtonian theory invasion into the SR that led to the sequence of Einstein’s papers on the relativity principle generalization and to spreading the principle not only on inertial systems of reference,

but on the various accelerated systems as well. But the most valuable result of the hybrid theories of Nordström and Abraham consisted in that the both theories maintained *very promising hints* on how the global theory could be created. Hence the climax of the stage was Einstein's proposal and apprehension of the equivalence principle that became one of firm GR heuristic foundations.

(ii) The basic GR model was constructed due to the unification of the hybrid models of Einstein, Nordström and Abraham constructed within *different* research programmes.

(iii) It is this hallmark that helps to comprehend the true reasons for Einstein's victory over the rival programmes of Nordström and Abraham. Einstein's metric theories – the 'Entwurf' and the GR – superseded the theories of Nordström and Abraham because they were more general, i.e. they embraced Nordström's and Abraham's theories in modified form.

(iv) Author's epistemological standpoint enables to look further and to conceive why it was Einstein that could propose the synthetic approach unifying all the positive achievements of the other approaches. It was because his heuristic contained the equivalence principle interpreted in non-ontological, anti-metaphysical, heuristic spirit.

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