Continuity as vagueness: The mathematical antecedents of Peirce's semiotics*

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In the course of his philosophic career, Charles Peirce made repeated attempts to construct mathematical definitions of the commonsense or experimental notion of 'continuity'. In what I will label his Final Definition of Continuity, however, Peirce abandoned the attempt to achieve mathematical definition and assigned the analysis of continuity to an otherwise unnamed extra-mathematical science. In this paper, I identify the Final Definition, attempt to define its terms, and suggest that it belongs to Peirce's emergent semiotics of vagueness. I argue, further, that it marks the transformation of Peirce's synechism. Before the time of his Final Definition, Peirce adopted a theory of continuity as a foundational principle of metaphysics and assumed this principle might be formalized in a mathematics of continuity. After the Final Definition, Peirce abandoned his foundationalism in favor of what he called a critical common-sensism. This is the claim that philosophy (and with it, logic) derives its norms from the observation of actual cognitive practices and that continuity is a distinguishing mark of actual as opposed to merely possible or imagined practices.

In this study, I define *mathematics* according to Peirce's 1902 discussion. There, he cited with favor his father's definition of mathematics as 'the science which draws necessary conclusions' (*CP* 4. 229).[1] Since 'it is impossible to reason necessarily concerning anything else than a pure hypothesis', mathematics is thus 'the study of what is true of hypothetical states of things' (*CP* 4. 233). But this means drawing conclusions from premises which are strictly hypothetical — or not given through any mode of reasoning other than the mathematical: the premises are constructions, and mathematics is reasoning 'with specially constructed schemata' (*CP* 4. 233). Given Peirce's normative theory of logic, it is thus necessary that mathematics be altogether independent of logic — whether or not elements of logic employ mathematical reasoning toward the end of understanding how we ought to think (*CP* 4. 239). Mathematics is therefore the science of the purely possible, which is the general. After 1905–6, Peirce would

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distinguish the generality of the purely possible from that of the indefinitely general, or vague, thereby introducing the terms of his Final Definition.

To explain the relationship between Peirce's final and previous definitions of continuity, I examine Peirce's inquiry into the character of continuity from two different standpoints. I first trace the development of Peirce's inquiry as a search for rational foundations. Here, I define Peirce's inquiry from the standpoint of his initial attempt to identify the principle of continuity which is shared by both the mathematical and experiential sciences. From this standpoint, conflicts emerge within Peirce's inquiry because it attempts to accomplish the impossible. Here, Peirce's Final Definition of Continuity represents the *reductio ad absurdum* of an errant line of argumentation. Examining Peirce's inquiry from a second standpoint, I suppose from the outset that his study of continuity is guided by conflicting tendencies of interpretation, one adopting mathematical, the other adopting extra-mathematical or commonsense criteria for identifying continuity. I suggest that Peirce got himself into trouble when he confused categories and adopted mathematical criteria for defining the continuity of the 'things' of commonsense. His Final Definition got him out of trouble by offering him a method of distinguishing the 'true' continuity of commonsense from what he called the 'pseudo' continuity of mathematics. As we will see, this is the method of semiotic analysis.

Examined from either standpoint, Peirce's inquiry diverged from the line of investigation, from Cantor and Russell to Grünbaum, that defines contemporary discussion of continuity theory. In his middle years, Peirce was attracted to Cantor's First Definition of Continuity, without, however, respecting Cantor's injunction against applying that definition to the interpretation of commonsense phenomena, such as the perceived continuity of feeling or of time. Dissatisfied with the consequences of adopting Cantor's definition, Peirce broke with the Cantorean approach after 1903–4. At the same time, he made this break without making a serious appraisal of Cantor's 1895 memoir introducing the postulate of linearity (Potter and Shields 1977: 25, n13). We can only conjecture about whether or not such an appraisal may have led Peirce to consider alternatives to his notion of infinitesimals, and thereby to contribute to developments in the alternative approach of point-set theory.

Peirce's inquiry as a search for rational foundations

For Peirce, as for Leibniz, 'continuity' represented the leading principle of a search for rational foundations, a synonym for 'affinity' (Cassirer 1902) or 'ultimate regularity'. From the standpoint of this search, Peirce's mathematical investigations may be characterized as an attempt to define more clearly the vague idea of continuity that is displayed in such commonsense notions as 'infinite divisibility' or 'contiguity'.[2] What remains problematic is precisely how the vague notions may stand as criteria against which to judge, or at least from which to generate, the mathematical definitions. This is the problem that underlies each stage of Peirce's attempt to define continuity.

In his 1867–8 series in the *Journal of Speculative Philosophy*, Peirce proposed an objective idealism as an alternative to Cartesian dualism. He argued that the continuum has no limiting points, because it is 'precisely that, every part of which has parts, in the same sense' (*CP* 5. 335). Claiming that cognitions are continua, Peirce therefore argued, against Descartes, that there are no first, or limiting cognitions: a purportedly first cognition is merely an ideal limit of a progression or regression. The progression or regression comes first, which means that the privileged entities in Peirce's early idealism are continua of reasoning, or logoi, rather than primary intuitions. In response to a challenge from W. T. Harris, editor of the *Journal of Speculative Philosophy*, Peirce was prepared to reduce the laws of cognition to logical laws and to offer a rationale for their objective validity.

In his 1878 'Illustrations of the logic of science', Peirce defined continuity as infinite divisibility (*CP* 5. 395; cf. 3. 256), apparently replaying what he later called Kant's erroneous notion.[3] In his 'Logic of number', from this period, Peirce wrote that 'A continuous system is one in which every quantity greater than another is also greater than some intermediate quantity greater than that other' (*CP* 3. 256). This mathematical definition appears beholden to Peirce's understanding of the continuity of mind. He wrote that, existing only over a lapse of time, consciousness represents the continuity of sensory instants. Thought itself is

> a thread of melody running through the succession of our sensations. ... Just as a piece of music may be written in parts, each part having its own air, so various systems of relationship of succession subsist together between the same sensations. These different systems are distinguished by having different motives, ideas or functions. Thought is only one such system. ... (*CP* 5. 395–6)

Peirce did not yet entertain the notion that the percept may generalize, abductively, from the sensory data.

There is one reference in 'Description of a notation for the logic of relatives' (1870) that transitive relatives 'whose products by themselves are equal to themselves' are continuous, with a marginal note that the latter should be 'concatenated'. The marginal note, however, may be a later addition (1889?), since Peirce otherwise referred here to examples of compactness (through 1880 — *CP* 3. 214). Again, 'the individual and the simple... are ideal limits' or 'fictitious limits' (*CP* 3. 216).

Between 1884 and 1894,[4] Peirce's definitions of continuity were influenced by his study, in 1884, of Georg Cantor's *Grundlagen einer allgemeinen Mannigfaltigkeitslehre*.[5] Separating his inquiry from considerations of spatio-temporal continuity, Cantor defined continuity as the character of a 'perfectly concatenated (*zusammenhängende*) collection of points. In such a collection, "P", if t and t' are any two of its points, and e a given arbitrarily small positive number, a finite number of points, t1, t2, ..., tv, of P exist such that the distances tt1, t1t2, ..., tvt are all less than e'.[6] Cantor observed that such a collection of points is perfect when it is both 'closed' (*abgeschlossen*)[7] and 'condensed-in-itself' (*insichdicht*).[8]

From the first, Peirce was not fully satisfied with Cantor's definition, complaining that it employed metrical considerations, 'while the distinction between a continuous and a discontinuous series is manifestly non-metrical' (*CP* 6. 121), and that it is vague and redundant (*CP* 6. 121). Nevertheless, for the time being moderating his 1868 position, he raised no objections to Cantor's employment of collections and, evidently, their contained points. In an 1889 entry in the *Century Dictionary*, he wrote that Cantor's is 'the less unsatisfactory' definition of continuity, as compared with what he called Aristotle's and Kant's definitions. The former referred to 'the fact that adjacent parts have their limits in common'; the latter to 'the fact that between any two points there is a third' (*CP* 6. 164). In 'The law of mind' of 1892, Peirce 'slightly modified' Cantor's definition of perfect concatenation by claiming that continuity is to be defined by combining two properties he now called 'Aristolicity' and 'Kanticity'. He defined the former as 'closure', or *Abgeschlossenkeit*. This is a property of maximal and minimal successorship: that there is a next-superior to all the members of a fundamental sequence, or that 'a continuum contains the end point belonging to every endless series of points which it contains, [thus ...] that every continuum contains its limits' (*CP* 6. 123). He defined the alternative, 'Kanticity', as 'compactness' or *insichdichkeit*. This is a property of immediate successorship, or infinite divisibility.

Unlike Cantor, however, Peirce assumed that these definitions provided an adequate model for explaining the continuity of such phenomena as time (*CP* 6. 127) and feeling (*CP* 6. 123ff). This assumption introduced contradictory tendencies into Peirce's analysis of the Law of Mind, and therefore, into his projected objective idealism.

In 'The law of mind', Peirce introduced his synechism, or doctrine of continuity, as applied to the phenomena of mind. He wrote that logical analysis applied to mental phenomena shows that there is but one law of mind, namely, that ideas tend to spread continuously and to affect certain

others which stand to them in a peculiar relation of affectibility. In this spreading, they lose intensity, and especially the power of affecting others, but gain generality and become welded with other ideas (*CP* 6. 104). Through the law of mind, Peirce attempted both to account for the phenomenon of generalization and to argue that it represents a fundamental tendency of mind and, according to his objective idealism, therefore also of matter. His argument served, secondarily, to provide a metaphysical warrant for the generalizing procedures he employed in articulating his idealism — as if to say that his own tendency to assimilate phenomena that share one or more characteristics represented his fidelity to the generalizing character of mentality itself.

Peirce's procedure in 'The law of mind' epitomized the leading tendencies of his objective idealism. Identifying matter with effete mind, he claimed that the material universe itself displays a tendency to generalization, as for example in the spread of feelings, and that this tendency may be described as a reification of the fundamentally mathematical laws of mental generalization, which are laws of continuity. He believed that a mathematics of continuity would provide him an appropriate model for interpreting the phenomena of experiential continuity — by which I mean both the feeling of continuity and an empirical account of the continuity of feeling, which latter Peirce identified with the phenomenon of generalization. In the search for such a unified theory of synechism, Peirce's inquiry divided into two sub-inquiries whose isomorphism is merely apparent. On the one hand, he identified mind with feeling and described experiential continuity as the flowing together of instantaneous feelings into 'continua of feeling' which are 'general ideas' (*CP* 6. 151). On the other hand, he identified mind with the subject-matter of mathematics and the law of mind with a mathematics of continuity — at this point in his work, slightly modifying Cantor's First Definition of Continuity. Rather than clarifying his phenomenological observations, however, this mathematical model led Peirce into contradiction.

Peirce described 'intensity of feeling' as a physiological mode of hypothesis-making, where 'a number of reactions called for by one occasion get united in a general idea which is called out by the same occasion' (*CP* 6. 146 [1892]).[9] In the context of Peirce's theories of modes of inference at the time, this was equivalent to treating feeling as an 'induction from qualities' (cf. *CP* 2. 706 [1883]; 6. 145 [1892]), for which the following syllogistic model would apply:

i) Some R is q′q′ ...
ii) M is q′q′ ... q^n
iii) R is probably M.[10]

That is, where R represents the occasion which elicits the reaction-qualities (or preconscious sense-qualities) q'q' ..., and M represents the resultant feeling, general idea, or physiological hypothesis. Thus, whatever multitude be assigned to the series of marks (or reaction-qualities) q'q' ..., the predicate of (i) constitutes a sampling of marks attributed to M (ii). This model implies that the feeling, M, is the character of a discrete collection of reaction-qualities.[11] Moreover, the multitude of this collection must be maximal, or of the continuum, since, for Peirce as for Kant, feeling had an 'intensive continuity' (*CP* 6. 102ff). Only the Cantorean model of a compact-and-closed series fulfills these criteria of both maximal multitude and discreteness: i.e., that each member of the series has some distinguishing mark.[12]

At the same time, this syllogistic model denies feeling any creative function. The second premise represents an innate disposition of mind or habit — what Peirce called a physiological induction, or association by resemblance (*CP* 7. 388 [1893]; 4. 157 [1897]), displayed in this case in the previously established clustering of ideas (q'q' ...) into a general idea (M). The sole function of hypothesis therefore becomes merely one of 'suggestion':[13] the mere *selection* of a rule (ii) to account for a new case (i), rather than the construction of a new idea to function in a hypothetical rule (cf. Fann 1970: 20–27, 41–44). This means that, as product of induction, the second premise must itself presuppose a hypothesis concerning M (such as 'some R is an M') and so on, *ad infinitum*. To paraphrase K. T. Fann, we can't account here for the construction of any new ideas.

It is evident that, at this time, Peirce did seek to credit feeling with a creative function. He wrote that 'wherever chance-spontaneity is found, there in the same proportion feeling exists. In fact, chance is but the outward aspect of that which within itself is feeling' ('Man's glassy essence', *CP* 6. 265). The purpose of his Law of Mind is not to reduce feeling to judgment on the model of a Kantian synthesis, but rather to account for the 'tychistic' function of mind: the welding of sense-qualities into feeling and the spread of feeling into general ideas.

> Thus the various sounds made by the instruments of an orchestra strike upon the ear, and the result is a peculiar musical emotion, quite distinct from the sounds themselves. (*CP* 2. 643 [1878])

The only syllogistic model at all adequate to account for this phenomenon of welding is the following modification of the one given earlier:

i) Some R is q'q' ...
ii) q'q' ... is M.
iii) R is M.

According to this model, feeling is analogous not to a hypothesis through sampling, or qualitative induction, but rather to what he later called 'hypostatic abstraction',[14] or thinking the predicate as a subject. Here the series of qualities (q'q' ...) itself becomes the index, or pure denotation, of an occasion of which the general idea, M, is the elicited 'reaction'. In this version, M is the novel relation resulting from that 'spreading of ideas' which epitomized Peirce's 1892 notion of the 'Law of Mind', in which 'a finite interval of time generally contains an innumerable series of feelings; and when these become welded together in association, the result is a general idea' (*CP* 6. 137). This model, furthermore, requires no necessary reference to an antecedent process of inference.

The notion of 'welding' is, however, simply incompatible with the Cantorean model of continuity. In the new model, M is no longer the character of an indeterminate collection of reactive elements, on whose discreteness the possibility of inductive sampling was based. Rather, M appears to be the character of a whole resulting from the mutual interpenetration — or loss of distinctness — of some (supposedly?) innumerable series of elements. At this stage of Peirce's work, it remained unclear precisely how those elements are constitutive of the whole as continuum, and through what kind of parts that whole may be reidentified.

Peirce's writings in 1903 displayed his emergent tendency to resolve tensions between his phenomenological and mathematical models of continuity in favor of the former. In a marginal note written in his copy of the *Century Dictionary*, he wrote that 'further study of the subject has proved that [Cantor's First] definition is wrong' (*CP* 6. 168). He argued that Cantor, like Kant and Peirce after him, misunderstood Kant's commonsense definition of a continuum as 'that all of whose parts have parts of the same kind' (Kant 1799: A169/B211):

> He himself, and I after him, understood that to mean infinite divisibility, which plainly is not what constitutes continuity since the series of rational fractional values is infinitely divisible but is not by anybody regarded as continuous. Kant's real definition implies that *a continuous line contains no points*. (*CP* 6. 168, emphasis mine)

This means that Peirce had to reject his revised Cantorean definition in terms of Kanticity and Aristolicity. According to the property of Aristolicity, a continuum has a definite arrangement — we may not know which individual occupies a certain place in a continuous series, if any, but we have no questions about the ordinal characters predicated of its place. In Peirce's terms, this property therefore entails the notion of a neighborhood, which he identified with an infinitesimal:[15]

> Let us now consider an aspect of the Aristotelical principle which is particularly important in philosophy. Suppose a surface to be part red and part blue; so that every point on it is either red or blue, and, of course, no part can be both red and blue. What, then, is the color of the boundary line [i.e., limit] between the red and the blue? The answer is that red or blue, to exist at all, must be spread over a surface; and the color of the surface is the color of the surface in the immediate neighborhood of the point. I purposely use a vague form of expression. Now, as the parts of the surface in the immediate neighborhood of any ordinary point upon a curved boundary are half of them red and half blue, it follows that the boundary is half red and half blue. ... Just so my immediate feeling is my feeling through an infinitesimal duration containing the present instant. (*CP* 6. 126)

Otherwise put, the principle of the excluded middle does not hold for the limit points of a continuum. According to Peirce's new definition, however, the continuum contains no points at all! In order to resolve the contradiction, Peirce had to introduce a distinction between two kinds of point and two kinds of generality. To consider this distinction, I offer an excursus from the present history.

Excursus: Generality vs. vagueness

As noted by Jarrett Brock (1979), 'in the beginning [in 1867], Peirce spoke only of the indeterminacy and determinacy of terms', where, in Brock's terms,

> a term S is general and/or indeterminate iff (E P)–(S is P or S is –P). ...
>
> A Term S is determinate in at least one respect iff (E P) (S is P or S is–P). (Brock 1979: 41)

Peirce argued that no term is absolutely determinate or indeterminate. By 1896, however, Peirce suggested that there is a distinction between two sorts of indeterminacy. In 'The logic of mathematics' (*CP* 1. 417–520), he wrote that the general must be excluded from the category of fact, where

> generality is either of that negative sort which belongs to the merely potential, as such, and this is peculiar to the category of quality; or it is of that positive kind which belongs to conditional necessity, and this is peculiar to the category of law. (*CP* 1. 427)

In the 1900 'Notes on metaphysics', Peirce called 'definiteness and determinateness' 'the two poles of settledness' (*CP* 6. 348). By 1902–3, he referred to the two complementary poles of unsettledness in this way:

> A subject need not be singular. If it is not so, then when the proposition is expressed in the canonical form used by logicians, this subject will present one or other of two imperfections.
>
> On the one hand, it may be *indesignative*, so that the proposition means that a singular of the universe might replace this subject while the truth was preserved, while failing to designate what singular that is; as when we say '*Some* calf has five legs'.
>
> Or, on the other hand, the subject may be *hypothetical*, that is may allow any singular to be substituted for it that fulfills certain conditions, without guaranteeing that there is any singular which fulfills these conditions; as when we say, 'Any salamander could live in fire. ...' (*CP* 5. 154)

In 1905, Peirce redefined this distinction within the terms of his mature semiotics. He wrote that *indeterminacy* is, in general, one of two characters of a sign. A sign is determinate if its 'meaning would leave "no latitude of interpretation"' (*CP* 5. 448 n1) or 'in respect to any character which inheres in it or is (universally and affirmatively) predicated of it, as well as in respect to the negative of such characters' (*CP* 5. 447), or if it 'indicates an otherwise known individual'. Otherwise, the sign is indeterminate — describing in some way, but not completely, 'how an individual intended is to be selected' (*CP* 5. 505). Of indeterminate signs, there are two kinds: the *vague* or indefinite, and the *general*, which term I will now use to refer strictly to the non-indefinite indeterminate.

For Peirce, the *general* 'turns over to the interpreter the right to complete the determination as he pleases' (*CP* 5. 448 nl [1906]). If the first subject of a proposition of two or more subjects is general, the proposition is called universal (*CP* 5. 155 [1903]): the general indicates the character of a merely possible individual, representing the synthesis of a multitude of subjects (see below for more detailed treatment).

On the other hand, the *vague* 'reserves for some other possible sign or experience the function of completing the determination' (*CP* 5. 505). If the first subject of a complex proposition is vague, the proposition is called particular (*CP* 5. 155): the vague denotes some of the characters of an *existent* individual, representing the synthesis of a multitude of predicates (see below).

I will note here one additional kind of generality — the hypostatically abstract — which appears to remain ill-defined in Peirce's work and may at times take on characteristics of the other two. More briefly put, hypostatic (or subjectal) abstraction is for Peirce the process by which we transform a relative predicate into a subject — or a quality into a thing (e.g., *CP* 4. 235 [1902]). The abstract term is thus deceptively like the general, but with respect to the predicate and not the subject, and like the vague, but with respect to identical rather than (continuously) related predicates.

Returning to Peirce's study of continuity, we can see where he was moving in 1902–3. He wanted to say that Aristolicity is a property of a collection of points whose limits are indeterminately general, while his new definition is predicated of a system without indeterminate points. This is a system whose parts are indefinitely general, or vague. In 1903, however, this new definition was merely emergent and not yet fully explicit nor fully separated from the Cantorean definition. In Peirce's Cantorean view, continuity is a species of indeterminate generality or pure possibility. In his non-Cantorean view, continuity is a species of vagueness or indefinite actuality. In the Cantorean view, continua are mathematical entities; in the non-Cantorean view, continua are phenomenological or experiential entities. Potter and Shields label this penultimate stage of Peirce's inquiry his 'Kantistic period', 'because Peirce discovers one of its important ingredients in Kant's definition of a continuum as "that all of whose parts have parts of the same kind"' (*CP* 6. 168 [1903]). They suggest that, in this period, Peirce sought both to retain and to reject the Cantorean approach to multitude. 'By the term "multitude" Peirce means essentially what Cantor had called the "power" (*Mächtigkeit*) of a collection. Today these are called "cardinal numbers"' (Potter and Shields 1977: 26).

In 1897, Peirce wrote that 'by a *collection*, I mean anything which is *u*'d by whatever else has a certain quality, or general description, and by nothing else. ... It will be perceived, therefore, that there is a collection corresponding to every common noun or general description' (*CP* 4. 171).[16] He added that 'A *part* of a collection called its *whole* is a collection such that whatever is *u* of the part is *u* of the whole, but something that is *u* of the whole is not *u* of the part', (*CP* 4. 173). In these terms, he then redefined *multitude*:

> I shall use the word *multitude* to denote that character of a collection by virtue of which it is greater than some collections and less than others, provided the collection is *discrete*, that is, provided the constituent elements of the collection are or may be discrete. But when the units lose their individual identity because the collection exceeds every possible existence of the universe, the word multitude ceases to be applicable. I will take the word *multiplicity* to mean the greatness of any collection discrete or continuous. (*CP* 4. 175)

He then defined *enumerable multitudes* as

> those multitudes every one of which posess(es) any character whatsoever which is, in the first place, possessed by zero and, in the second place, if it is possessed by any multitude, M, whatsoever, is likewise possessed by the multitude next greater than M. (*CP* 4. 182)
>
> ... . A remarkable and important property of enumerable collections is that every finite part is less than a whole. (*CP* 4. 186)

Peirce claimed that the post-numeral, or *abnumerable*, multitudes are obtained only by constructing the power-collection of a given collection: 2^n for a collection of multitude n. According to Cantor's theorem, which Peirce apparently anticipated: $2^n > n$ for all n. Peirce then developed a series of transfinite multitudes 'solely by repeated application of Cantor's theorem. ... Thus, Peirce's entire series of multitudes, using contemporary notation, would look like this: 0, 1, 2, ... X_0, 2^{X0}, 2^{2}' (Potter and Shields 1977: 26).

Asking, finally, 'is there any multitude larger than all of these?' (*CP* 4. 218), Peirce concluded that an aggregate or enumerable collection of all unequal abnumerable multitudes

> is no longer a discrete multitude, for the formula $2^n > n$ which I have proved holds for all discrete collections cannot hold for this. In fact writing Exp. *n* for 2^n, (Exp.) X^x is evidently so great than this formula ceases to hold and it represents a collection no longer discrete. (*CP* 4. 218)

Peirce therefore described 'true continuity as coming at the end of the series of postnumeral multitudes' (Potter and Shields 1977: 27), arguing, by 1900, that 'the possibility of determining more than any given multitude of points, or in other words, the fact that there is room for any multitude at every part of the line, makes it *continuous*' (*CP* 3. 568). Peirce was attempting both to deny that continuity can be defined at all in terms of multitude and to express his intuition within the terms of the Cantorean project.[17]

Between 1906 and 1908 this tension within Peirce's approach expended itself in his explicit break with the Cantorean approach. Assuming now that a continuum could not contain discrete parts, Peirce argued that 'whatever can be arranged in a block of any finite number of dimensions can be arranged in a linear succession' (*CP* 4. 639). Only the members of a purely abnumerable collection cannot be so arranged, where a pure abnumerable collection is 'a collection of all collections of members of a denumeral collection each of which includes a denumeral collection of those members and excludes a denumeral collection of them' (*CP* 4. 639). But the members of all collections of less than pure abnumerability may be put into one-to-one correspondence with the members of denumerable series. All such collections, therefore, have distinct members. This means, Peirce concluded, that the Cantorean continuum, defined in terms of such collections, contains distinct members. The Cantorean continuum is thus a '*pseudo-continuum*'.[18]

In his 'Law of mind' papers of 1892, Peirce had complained that Cantor's definition 'turns upon metrical considerations; while the distinction between a continuous and a discontinuous series is manifestly non-metrical' (*CP* 6. 121). 'In 1893, Peirce had asked himself the question: How can continua be colored if their proper parts, points, are not colored? (4. 126)' (Potter and Shields 1977: 27). Until 1906, however, Peirce was not prepared to accept the full implications of this question: that, because it cannot be defined with respect to metricality and to points, continuity cannot be defined in terms of multitude at all; in other words, that a continuum is not a collection. Continuity cannot, therefore, be defined within a mathematics of collections. A definition must be found elsewhere or not at all.

Beyond objective idealism

When he abandoned the Cantorean approach to defining continuity, it would appear that Peirce also abandoned the premises of his objective idealism. From the days of his early critique of Cartesianism, Peirce assumed that he could demonstrate the objective validity of the laws of logic: that we encounter these laws directly as the fundamental processes of everyday reasoning, that we encounter them as we encounter the processes of time, and that we may define any such processes formally, identifying them with laws of indeterminate generality or pure possibility. In particular, Peirce attempted for many years to identify these processes with Cantorean continua, ignoring Cantor's own insistence that his theory of continuity does not apply to continua of common sense. When, in 1906, Peirce claimed to break with Cantor's approach, he was in reality breaking with his own metaphysical use of Cantor's theory. He allowed for a Cantorean analysis of what he called pseudo-continua, while reserving the term 'true continua' for the continua of common sense excluded from the mathematics of collections. In this sense, Peirce's break with Cantor may be retermed Peirce's reclassification of his own life-long attempt to define experiential continuity.

Peirce had previously placed this attempt within mathematics, as a science of the purely possible. Now he treated his former effort as a confusion of two different sorts of inquiry: the mathematician's activity of constructing hypothetical systems of possible language, and the philosopher's activity of identifying the normative characters of commonsense practice, or of actual languages. It is as if Peirce's previous work had been guided by conflicting tendencies. On the one hand, he had assumed, against what he calls the 'Cartesians', that philosophic norms are to be discovered only within actual practices and that 'continuity' is the distinguishing mark of actual as opposed to merely imagined or merely possible practices. On the other hand, he had also assumed, with the Cartesians, that this distinguishing mark had to be defined before he could claim to recognize it and that formal definition remained the task of a mathematical science. The result was an inquiry in conflict with itself, serving what I will call both common-sensist and foundational tendencies.

Peirce's 1867–8 papers display the conflict in his early work. Already anticipating his 1903 judgment that a continuum of points is a pseudo-continuum, Peirce wrote that a continuous line has no limiting points. Within the context of his early epistemology, this meant that there are no first or limiting cognitions — a common-sensist claim. At the same time, this claim did not discourage Peirce from his foundational attempt to formulate the law of cognition as such, or what he would later call 'the law of mind'. In his 1878 'Illustrations of the logic of science', Peirce argued, as common-sensist, that continuity is exemplified in the continuity of experienced time and of experienced cognition: he described thought as the 'thread of melody running through the succession of our sensations'. As foundationalist, however, he attempted to formalize this experiential property, identifying it with infinite divisibility, or the continuity of sensory instants. Overall, from 1867 to 1878, Peirce argued as commonsense critic but as foundational advocate. He criticized the Cartesian attempt to identify the fundamental intuitions on which all reasoning shall be based, but then replaced it with his own attempt to identify the fundamental rules of cognition on which all reasoning shall be based. As pragmatist, he had developed the requisite tools for criticizing both of these attempts to substitute the generality of pure possibility for the indefiniteness of actual regularities. He was, however, not yet prepared to use these tools consistently.

Peirce's interest in Cantor's First Definition of continuity was consistent with the foundationalist tendency in his thinking. Cantor's 'perfectly concatenated' collection of points represented a decided improvement over Peirce's 1878 notion of infinite divisibility, and, for the time being, Peirce repressed his common-sensist objections to the notion of points. Ignoring Cantor's objections to confusing mathematical and experiential notions of continuity (above, note 2), Peirce adopted the Cantorean approach as a way of formalizing the law of cognition he had sought to describe since 1868. The result was Peirce's 'Law of mind'. The Law's contradictions, as previously mentioned (see above) display clearly the contradictory tendencies which underlay Peirce's metaphysical employment of the Cantorean approach. Peirce's 1903 criticisms of Cantor indicate his having begun to acknowledge these contradictions. Reiterating the commonsense claims of his initial critique of Descartes, Peirce declared once again that a continuum contains no points. This, he added, means that the parts of a continuum are indefinitely, rather than indeterminately, general.

By 'breaking' with Cantor in 1906, Peirce declared, in effect, that metaphysics deals with empirical rather than with mathematical entities. By attempting, still, to define continuity, Peirce was attempting, still, to identify the distinguishing mark of these entities. It is time, then, to examine Peirce's 1906 definition of continuity and to infer from it what Peirce took to be the characteristic language and characteristic entities of metaphysical science.

The final definition of continuity

I have reconstructed Peirce's Final Definition out of four texts, from 'The bedrock beneath pragmaticism' (*CP* 6.174ff and 4.561n [1906]) and from 'Some amazing mazes' (*CP* 4. 642 and 7. 535 n7 [1908]):

i) Continuity is a species of generality; it is predicated of a whole and refers to the relationships among the parts of a whole.
ii) A continuous whole is one 'whose parts without any exception whatsoever conform to one general law to which same law conform likewise all the parts of each single part' (*CP* 7. 535 n7).
iii) These parts are called material parts (*CP* 6.174ff), where material parts of a whole are:
   a) whatever things are other than W;
   b) all of some one 'internal nature' (character);
   c) form a collection of objects in which no one occurs twice;
   d) 'are such that the Being of each of them together with the modes of connection between all sub-collections of them, constitute the being of W'.

The Final Definition represents Peirce's attempt to offer a quasi-formal definition of the most general character of common sense or experiential continua. In this section, I will describe this definiendum and its definiens in several ways, first within the vocabulary of Peirce's mathematics of generality and then, gradually, introducing alternative vocabularies.

Continua as things. If Peirce's Final Definition of continuity is anything like his earlier definitions, then the definiendum may refer to the class character of some sort of *collection.* We have seen, however, that the Final Definition emerges out of Peirce's claim that continua cannot be collections. The first place to look for an alternative description of continuity, therefore, may be in Peirce's classification of those *wholes* which are not collections. Peirce defined a 'whole' as 'an *ens rationis* whose being consists in the copulate beings of certain other things, either not *entia rationis* or not so much so as the whole'.[19] Among the kinds of whole Peirce listed are 'collective wholes', or aggregates, and 'continuous wholes', or continua regarded as wholes.[20] I find it helpful to call the latter *things.* Making use of Peirce statements about 'things', we may then say that a continuous whole is what Peirce refers to as Kant's *Ding-an-sich* made *conceivable* (*CP* 5. 452 [1905]): we have direct experience of things in themselves (*CP* 6. 96 [1903]). Furthermore,

> What we call a Thing is a cluster of habits of reactions, or, to use a more familiar phrase, is a centre of forces. (*CP* 4. 157 [1898]).

> (a thing) is the sum of all of its characters, or consequences. (*CP* 1. 436 [1896])

As definiendum of the Final Definition, a *thing* would display generality, as an *ens rationis.* Its character would be identified in terms of the relations among its parts. To specify how this character differs from the class character of a *collection*, it is necessary to specify the kind of generalizing procedures through which collections and things may be thought.

Continua as products of abductive inferences. Peirce employed the term *induction* to characterize not only a mode of inference, but also a phenomenon of all material processes. In general, induction may be defined as the replacement of 'large number of propositions having the same novel consequent but different antecedent ... by one proposition which brings in the novel element' (*CP* 3. 516) as the consequent of one synthesized antecedent. From 1866 on, Peirce consistently drew analogies between such an inference and the physiological phenomenon of habit-formation (e.g. *CP* 2. 641ff; 5. 298ff). In a subject-predicate logic, the inference may be schematized as an Aristotelian syllogism, proceeding 'from Case (minor premise) and Result (conclusion) to Rule (major premise)' (*CP* 2. 712). But, as seen in the simplest illustration (schematizing Peirce's example from *CP* 2. 625), the Case of an induction presupposes an abductive inference:

Case:	This man is a Turkish governor.
Result:	This man rides on a canopy-covered horseback, etc.
Rule:	Turkish governors ride on a canopy-covered horseback, etc.

Indeed, Peirce wrote in 1892,

> Habit is that specialization of the law of mind whereby a general idea gains the power of exciting reactions. But in order that the general idea should attain all its

functionality, it is necessary, also, that it should become suggestible by sensations. That is accomplished by a physical process having the form of hypothetic inference. (*CP* 6. 145)

That is, while the physiological requirement for habit-formation may be the mere repetition of response, the recognitive aspect of habit is a function of indefinite predication.

Peirce defined *abduction* most generally as the replacement of 'a large number of novel propositions with one subject or antecedent ... by a single novel proposition' (*CP* 3. 516), which synthesizes the multitude of antecedents in a single indefinite predicate. Drawing a physiological analogy, Peirce likened abduction to the 'sensuous element of thought' (*CP* 2. 643) and, through 1892, termed it 'an induction from qualities' (*CP* 6. 145). Re-enlisting the simple syllogistic scheme employed above, abduction 'proceeds from Rule and Result to Case' (*CP* 2. 712), exemplified in the illustration from *CP* 2.625 as:

Rule:	Turkish governors ride on canopy-covered horseback, etc.
Result:	This man rides on canopy-covered horseback, etc.
Case:	This man is a Turkish governor.

The fundamental move in abduction is to observe a regularity among certain qualities and to suppose that these qualities represent the parts of a certain whole. Thus 'Turkish governor' is, in this example, merely the hypostatic abstraction[21] of what remains the undesignated predicate 'Turkish-governorish' (however expressed). The same applies to induction, for which the repetition of instances can be said to supply only the 'matter' of the inference. At the same time, the product of induction is indifferent to its instances, while the product of abduction is the subjectified continuity among its elements. When prescinded from abduction, however, the hypostatically abstract is, like the indeterminate, indifferent to its instances, which means that it is possible to treat vague entities (such as symbols) as if they were indeterminate abstractions.

For Peirce, it therefore appears that we may characterize *collections* as the subjectifications or hypostastic abstractions of the products of inductive inferences, if the latter are considered independently of the abductive element with which they are associated in practice. These collections are logical individuals representing the syntheses of a potential multitude of (a) subjects with a single predicate (for example, all people), or (b) instances of a single predicate character or quality (e.g., sweet things). In each case, the terms represent wholes logically posterior to their parts, since the terms represent merely the general extension of the characters of some observed individuals of like description. There need be no such individuals.

In these terms, we would characterize '*things*', in contrast with collections, as hypostatic abstractions of the products of abductive inferences (including the abductive aspect of inductive inferences). In terms of the Final Definition, we would then describe the 'Being' of a continuous whole as a hypostatic abstraction of the abductively inferred 'general law' to which all the parts of a continuous whole conform. However, this description would still leave us with an ambiguity. We might conclude that a thing is an entity in the world, independent of its qualities, and that the continuity of its qualities is the subject of our abductive generalization. *Or* we might conclude that a thing is itself nothing but the continuity we attribute to some system of qualities. Whichever option we chose would contradict one of Peirce's two claims: that a thing is knowable in itself and is therefore not to be described as a *Ding-an-sich* independent of its qualities, or that a thing has reality as a center of forces and is therefore not to be described as a mere form of description.

The way out of this dilemma would be to reintroduce the distinction Peirce made, after 1896, between two forms of generality: indeterminacy and vagueness. For Peirce, as we saw, the indeterminately *general* 'turns over to the interpreter the right to complete the determination as he pleases', while the *vague* 'reserves for some other possible sign or experience the function of completing the determination'. This means that indeterminacy is a character of the relation between signs and objects, while vagueness is a character of the relation between signs and objects *with respect to some condition of determination*, or what Peirce called an interpretant. Indefinitely, rather than merely indeterminately general, abductive inferences are symbols of triadic relationships among signs, objects, and interpretants, while inductions are so only by way of their abductive elements. As abductive products, things are therefore neither mere entities, considered independently of their qualities, nor mere systems of qualities. They are, instead, symbols of real relations in the world, particular in time and space, yet capable of further specification. To distinguish a thing from its character is, for the sake of analysis or cognitive experimentation, to hypostatize features of a thing's relationality: for example, its signifiers or its objects or its interpretants. The inherent vagueness of things is the subject matter of Peirce's pragmatism and his semiotics.

Continuous wholes as purposes. The pragmatic theory of inquiry and meaning, as refined in Peirce's 1903 Lectures on Pragmatism (*CP* 5. 14–212) and the 1905 Monist series (*CP* 5. 411–462), identifies purposes with continuous wholes.

According to Peirce's most useful representation of the Maxim, the intellectual purport of any symbol is

> the truth of certain conditional propositions asserting that if the concept (the symbol) be applicable, and the utterer of the proposition or his fellow have a certain purpose in view, he would act in a certain way. (*CP* 5. 528)

'A purpose is essentially general, and so is a way of acting; and a conditional proposition is a proposition about a universe of possibility' (*CP* 5. 528). Thus, the purport of a symbol is the regularity among an indefinite series of facts, each one of which may be represented as a conditional proposition, is other than the symbol, and has some one character which is common to all the others, all of which may be formed into a collection in which no one occurs twice — that is, each fact of which relates to the purport of the symbol as the material parts of a continuum relate to the continuous whole. Clearly, the generality which is of concern in the pragmatic maxim is the indefiniteness of the symbol, as purpose, with respect to each conditional fact. Such a purpose has no instances, but only definitions.

In terms of the Pragmatic Maxim, then, a thing is a symbol,[22] or a regularity among a series of conditional propositions which represent conditional dispositions to action. The regularity is what we call continuity. The formal science of continuity is thus the science of symbolization, or semiotics.

Continuous wholes as semiotic processes or processes of interpretation. Nadin writes that 'semiotics itself, in its divisions and in the sign operations it defines, is the logic of vagueness, and it is in this sense that Peirce affirmed that he had elaborated such a logic' (Nadin 1983: 156).[23] In 1906, Peirce characterized *semiosis* as 'an action or influence which is, or involves, a cooperation of *three* subjects, such as a sign, its object and its interpretant, this tri-relative influence not being in any way resolvable into actions between pairs' (*CP* 5. 485). He later defined

> a *Sign* as anything which on the one hand is so determined by an Object and on the other hand so determines an idea in a person's mind, that this latter determination, which I term the *Interpretant* of the sign, is thereby mediately determined by that Object. (*CP* 8. 343 — letter to Lady Welby, December, 1908).

He added that it is necessary

> to distinguish the *Immediate Interpretant*, i.e., the Interpretant represented or signified in the Sign, from the *Dynamical Interpretant*, or effect actually produced on the mind by the Sign; and both of these from the *Normal Interpretant*, or effect that would be produced on the mind by the Sign after sufficient development of thought. (*CP* 8. 343)

In terms of these definitions, there are two ways to characterize vagueness. The first is to predicate it of the dual relation between sign and object. This is to define vagueness, semantically, as 'a defect of cogitation' (*CP* 4. 344, cited in Nadin 1983: 157) and to suggest that, once vagueness is dispelled, it is possible to signify an object clearly. In Peirce's semiotic, this is vagueness in only a trivial sense. The non-trivial way of characterizing vagueness is to predicate it of the triadic relation among sign, object, and interpretant. According to Peirce's definition of normal interpretant, any dynamical interpretant is also a sign whose interpretant is any of an indefinitely extended series of interpretants, the limit of which series Peirce called the normal interpretant. What the sign means is given, therefore, only in the process of interpretation which constitutes this series. In this case, we would define vagueness as the character of the relationship among a sign's dynamical and its normal interpretants, which relationship 'reserves for some other possible sign or experience the function of completing the determination' of the normal interpretant.

In the terms of Peirce's Final Definition, the indefinite series of interpretants we are considering here represents a continuous whole. The material parts of this whole are any of an indefinite series of dynamical interpretants. No part is discrete, because no interpretant is determinate. The 'Being' of each interpretant together with the modes of connection among all the interpretants constitutes the 'Being' of the whole, and this is the meaning of a given sign. That sign itself (as, more precisely, what Peirce called a 'symbol') represents the general law to which both the whole and any of its parts conform. Any particular definition of a sign (in the form 'x means ...') is the selection of a particular interpretant out of the whole. It represents what Peirce called a *topical singularity*, or a 'place of lower dimensionality where [a continuum] is interrupted or divides' (*CP* 4. 642 [1908]). Such an interpretant is defined out of the continuum with respect to the empirical conditions which characterize any particular definition. Any attempt to define the normal interpretant itself is simply an attempt to select a possible or generally conceived interpretant as privileged representation of the general law characterizing the continuum. The attempt is an activity of hypostatic generalization, usually the result of hypostatizing an induction from the qualities of a sampling of interpretants selected from the continuum. To offer such a definition is to attempt to treat a vague sign as if it were indeterminately general. The result is a useful misrepresentation of the sign. It is a determinate, or finite representation of an indefinite sign, permitting use of the sign within finite systems of meaning. To describe such use as misrepresentation is to declare that it is fallible and, therefore, subject to indefinite adjustment.

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A sign therefore refers to its meaning vaguely, and this reference constitutes a process of interpretation, or a rule governing the determination of one interpretant by another and so on, indefinitely. To identify a particular interpretant is to define a sign with respect to the particular conditions of its interpretation.

Conclusion: Continuity, vagueness, and fuzzy sets

Peirce's Final Definition of continuity belongs to an empirical science of semiotics, while his earlier definitions belong to the mathematics of infinite collections. Nonetheless, each of Peirce's definitions may have served a single purpose: to identify the distinguishing mark of what he took to be metaphysically privileged or normative practices. Between 1867 and 1903, Peirce identified these practices with the practice of cognition as such, assuming that this latter practice could be identified with three coordinated laws of logic and that these laws, as well as their distinguishing mark, could be formally defined. After 1906, Peirce concluded that there may be an indefinite variety of cognitive practices, that all actual as opposed to merely possible practices are to be considered metaphysically privileged, and that among these, there are good reasons, but no necessary reasons, for favoring some practices over others (the sciences, for example). Since there is no single, normative practice of cognition, Peirce eschewed formal analysis of this practice in favor of an empirical science of various cognitive practices — the empirical science of logic. At the same time, he still attempted to identify the distinguishing mark of all such practices, defining it within a classificatory language of logic he called semiotics ('semeiotic'). 'Continuity' represents the mark which distinguishes actual from merely possible or imagined cognitive practices. Within the language of semiotics, it is synonymous with the terms 'semiosis' and 'vagueness', as well as 'Thirdness', 'mediacy', and so on.

A corollary of the Final Definition is that any such definition is a hypostatic abstraction, or what I have called a useful misrepresentation of some sampling of actual practices. To refer to such practices as 'wholes' is, therefore, to identify their knowability and to do so on some particular occasion or for some particular purpose — for example, to argue against the notion of an unknowable *Ding-an-sich*. To refer to their 'continuity' or 'vagueness' is to specify that this knowability is subject to a series of limitations. To refer to them as 'things' is to identify their actual existence over against us, which is their dynamical character. To refer to them, finally, as 'semiotic processes' is, among other things, to identify our capacity to analyze elements of these processes (such as 'sign, object, interpretant'), and thus to gain some control over them.

Contemporary refinements of Peirce's semiotics are to be classed among the efforts to identify and to contribute to this capacity to control. The limits of such efforts are defined on the one hand by Peirce's earlier vision of a mathematics of infinite collections, and on the other by the radical skepticism he had sought to counter in his 1867–8 attacks on 'Cartesianism'. For the Peirce of 1906, we approach this lower limit when we exaggerate the vagueness of our commonsense notions; we approach the upper limit when we exaggerate its clarity. Thus, in a study of Peirce's 'Logic of vagueness and the category of synechism', Nadin argues that the language of semiotics, as practiced today, is refined by recent developments in the mathematics of 'fuzzy sets'. These developments refine the semiotician's recognition of both the depth and the persistent 'fuzziness' of our knowledge of the actual world:

> To quote Zadeh: 'The fundamental concept in mathematics is that of a set — a collection of objects'. We have been slow in coming to the realization that much, perhaps most, of human cognition and interaction with the outside world involves constructs which are not sets in the classical sense, but rather 'fuzzy sets' (or subsets), that is, classes with unsharp boundaries in which the transition from membership to nonmembership is gradual rather than abrupt. Indeed, it may be argued that much of the logic of human reasoning is not the classical two-valued or even multivalued logic but a logic with fuzzy truths, fuzzy connectives, and fuzzy rules of inference. The semiotic and dialogic nature of thought in Peirce's conception and the model of multi-valued logic demonstrated by Zadeh in his definition of fuzzy sets seem to be outright complementary components. ... The exact treatment of the inexact, which many modern tendencies have programmatically assumed, thus becomes semiotically not only possible but also necessary. (Nadin 1983: 163)[24]

Notes

* This paper may be read as a sequel to Potter and Shields (1977). I consider their paper definitional, and I offer these reflections to explore further their references to the 'Post-Cantorial Period' of Peirce's studies of continuity. In preparing the present draft of this paper, I have made use of helpful comments from Rulon Wells of Yale University, Dan Nesher of The Hebrew University, Alexander Nakhimovsky of Colgate University and Robert Corrington, John Copeland and John Knox of Drew University. Regarding 'semiotic', Peirce's preferred spelling is *semeiotic*, and some Peirce scholars use only this spelling for the sake of precision and to distinguish Peirce's science from the Continental variety, with which it is often at odds. Transgressing Peirce's own ethics of terminology, I prefer to use what is now the more common spelling—in part, with the hope that Peirce's approach may influence the other one.

1. References to this collection will be directly to volume and paragraph number: for example (*CP* 4. 229).

2. Peirce's approach is consistent with Kant's, but otherwise stands opposed to the dominant students of continuity. Poincaré (1929) argued that the concept of continuity has no ground in sense-experience and is a purely rational construct to account for apparent contradictions in that experience: for example, that the perceived items A and B, and B and C may appear as pairs of equal dimension, while A appears larger than C. For Poincaré, the apparent contradiction is explained away by the generalizing function of perception — i.e., the effect of the threshold level in sensation, according to which differences of less than a given dimension will not be detected. While maintaining a theory of mathematics opposed to that of Poincaré, Adolf Grünbaum reiterates the latter's meta-mathematical comment and insists on separating definitions derived from sense-experience from those of pure mathematics (or even metrical applications) (Grünbaum 1951: 138ff). Russell rejected the dichotomization and maintained an atomistic theory applied to both mathematical and epistemological investigation. According to Cassirer, the mathematical definition is wholly divorced from observation of sense experience: 'So bleibt die Kontinuitat im echten wissenschaftlichen Sinne immer ein Idealbegriff, den wir der Beobachtung als Regel vorhalten, nicht ein Ergebnis, das wir unmittelbar aus irh ziehen Konnen' (Cassirer 1907: 20).
3. In *CP* 6. 112ff (1892) and 4. 121ff (1893) Peirce ascribed to Kant's *Critique of Pure Reason* (A169/B211 and A659/B607) an erroneous interpretation of Kant's own dictum that a continuum is 'that all of whose parts have parts of the same kind' (*CP* 6. 168): namely, that this is synonymous with 'infinite divisibility', or the continuum's defining 'a point between any two points'. Peirce demonstrates that the latter is merely a quality of compactness, and would permit a linear series of points (-Ai, ii, iii, ..., Ci, ii, iii, ..., D) — i.e., with all points from B to C elided.
4. This corresponds to what Potter and Shields call the second period in the development of Peirce's definitions of continuity: 'Cantorean: 1884–1894'.
5. Peirce read portions of this, in French translation, in *Acta Mathematica.* Potter and Shields note that 'this was originally the fifth of a series of papers entitled "Über unendliche lineare Punktmannigfaltigkeiten', written in 1882 and published in *Mathematische Annalen*, vol. XXI, in 1883. It was reprinted with an added preface [and the subtitle,] ... *Ein Mathematisch-philosophischer Versuch in der Lehre des Unendlichen*, Leipzig, 1883. Portions of this latter were translated into French in *Acta Mathematica*, vol. II, in 1884. See Georg Cantor (1955: Intro., 54n) (Potter and Shields 1977: 21, n2).
6. Potter and Shields (1977: 23, n6). In Peirce's terms, a concatenated series is 'such a one that if any two points be given on it, and any finite distance however small, it is possible to proceed from the first point to the second through a succession of points of the series each at a distance, from the preceding one, less than the given distance' (*CP* 6. 121); Russell calls such a series 'cohesive' (Russell 1903: Principle #272 [all future references to this volume will be to Principle No.]).
7. Meaning a series in which every fundamental sequence has a limit in the series (see Huntington 1917: *passim*, citing from Cantor's 1895 definitions; see Cantor 1895, 1897).
8. Meaning that every element or point in the series is the limit of a fundamental sequence (*Fundamentalreihe*) — which latter is a progression (a discrete series which has a first element but not a last) or a regression (a discrete series which has a last but no first element) (Huntington 1917: *passim*).
9. Peirce offered a complementary definition of habit as a physiological mode of induction, where 'a number of sensations followed by one reaction become united under one general idea followed by the same reaction' (*CP* 6. 146).
10. The form of this syllogism is similar to that of the simplest one employed by Peirce, in 1867 (*CP* 2. 511). While Peirce added terminology from probability theory to his 1883 model of hypothesis, the force of the model is the same as that of 1867 (cf. Fann 1970: 25).

11. Because the model of qualitative induction implies that the series of qualities of R (i) be treated as samples of the whole, M (ii). If M were not a discrete collection, then either the identification of the predicate of (i) with that of (ii) would itself require a feeling (as hypothesis) and so on, or the model of 'induction from qualities' would be inapplicable (see below).
12. As is presupposed in the property of 'Aristolicity' (as discussed earlier).
13. As the term is employed in *CP* 7. 495 to refer to the mental event governed by a previously established association. In *CP* 6. 142, Peirce wrote, 'when a feeling emerges into immediate consciousness, it always appears as a modification of a more or less general object already in the mind. The word suggestion is well adapted to expressing this relation' (*CP* 6. 142).
14. He called this 'hypostatization of relations' in 1890 (*CP* 1. 383), 'subjectification' in 1893 (*CP* 2. 428), and hypostatic abstraction in 1902.
15. See *CP* 4. 125 (1893). Cf. Grünbaum (1952: 291): modern topological studies reject the infinitesimal; a neighborhood is defined in terms of point-sets.
16. The most consistent general definition of *collection* to be drawn out of Peirce's writing is that a collection is an abstraction or 'an ens rationis whose being consists in the truth of an ordinary predication' (*CP* 3. 642 [1901]). In this way, 'we may ... define a collection as a fictitious (thought) individual, whose being consists in the being of certain less fictitious individuals' (*CP* 6. 382 [1902]).

    Murphey (1961: 247–249) argues that there is an inconsistency between two of what he considers the 'eight characteristics' of Peirce's concept of collection: the 'second characteristic' (that 'a collection is an abstraction which depends for its being upon the existence of its members') and the 'eighth characteristic', from *CP* 4. 171 (that 'there is a collection corresponding to every common noun or general description'). Murphey suggests that 'what is required here is some limitation of principle eight which would admit to collectionhood only aggregates which meet the independence requirement' (Murphey 1961: 249). Murphey seems to ignore the context of Peirce's remark, with respect to which I find the inconsistency is at most trivial. In the context of a theory of multitude, Peirce consistently defined a collection *extensionally* (to employ Russell's terminology, in Russell 1903: #71) — that is, 'through enumeration of its parts'. 'Principle eight' from *CP* 4. 171 is merely an illustration of what can be construed only as an extensional definition. To emphasize its extensionality, Peirce notes, 'Hence, if anything is u'd by anything not itself, it is not itself u of anything, and consequently nothing that is u'd by anything but itself is u'd by itself' (*CP* 4. 170; this appears to be an inadvertently proposed solution to Russell's Paradox, as Murphey notes of a comparable remark from Peirce's article, 'Map colouring', n.d.). In sum, the remark cited for 'principle eight' does not mean that every common noun is a 'real general' whose members are nonindependent from the collection corresponding to it, but rather that every common noun corresponds to a collection of individuals sharing some quality of which the noun is a 'subjectal abstraction' (see below). It is for this reason that there is a *null* collection, corresponding to a character possessed by no individual: such as the collection of fairies (*CP* 4. 171) or the collection of Jean Dare's children (4. 649–650).
17. Potter and Shields note, 'on an intuitive level, it must have been extremely disquieting for Peirce, the synechist, to discover that the putative power of the continuum was *only* $2^{\aleph_0}$. If continuity can be distinguished from compactness by greatness of multitude, Peirce must have reasoned, why should not true continuity refer to the very upper limit toward which greatness of multitude can tend' (Potter and Shields 1977: 27).

18. Peirce forced the issue against Cantor in two ways. He argued that 'it is a principle continually employed in the reasoning of the universally accepted "doctrine of limits" that two values, that differ at all, differ by a *finite value*'; that Cantor's continuum stands in one-to-one correspondence to the totality of real values, including unlimited decimal fractions; that the latter do not differ by a finite value; and that Cantor's continuum is therefore a pseudo-continuum (*CP* 6. 176 [1906]). However, from the perspective of modern point-set theory, Peirce had not considered the alternative that, for example, the values may not differ with respect to the point-set to which they belong, while retaining distinctness as members of a collection potentially in correspondence with a denumerable series (cf. Grünbaum 1952). Furthermore, in arguing that 'absolutely' independent members of a collection are those which possess some one definite non-relative character that the others do not possess, Peirce made it easier to claim that Cantor's continuum contains independent members, while holding elsewhere that an individual is absolutely determined.
19. From Peirce's entry on 'Whole and parts' in Baldwin's *Dictionary of Philosophy and Psychology* (*CP* 6. 381f [1901]). For additional definitions of collections, see e.g. 4. 171 and 4. 649 on null collection; 3.537, nl on the class character governing a collection.
20. Russell also drew a dichotomy between kinds of whole — in some respects similar, in other respects radically opposed to the dichotomy drawn here. The two kinds are 'aggregates' and 'unities': the former are classes and 'consist of units from whose addition they result', and the latter 'seem to be indistinguishable from propositions' and 'are not reconstituted by the addition of their constituents' (Russell 1903: #422). Russell, however, does not employ the term 'collection' to refer just to aggregates, but calls 'aggregate' 'the whole formed of the terms of the collection' (Russell 1903: #136). Furthermore, he denies that things can be spoken of as 'unities', unless no things exist (Russell 1903: #440).
21. See the earlier 'Excursus: Generality and vagueness'.
22. More precisely, what Peirce calls an 'intellectual concept', or a concept 'upon the structure of which arguments concerning objective fact may hinge' (*CP* 5. 467). This excludes symbols which belong to what I am calling merely possible as opposed to actual practices (see below, 'Conclusion').
23. Nadin is referring to Peirce's claim that 'I have worked out the logic of vagueness with something like completeness' and to the editorial question 'where?' in *CP* 5. 506 (1906).
24. In private conversation, A. Nakhimovsky suggests that Peirce's post-1906 work anticipates recent work on finite set based models of continuity, among which are the fuzzy-set models developed by Zadeh and others. In making use of such mathematical languages, semioticians reaffirm Peirce's critique of the attempt to reduce continuity to indeterminate generality, and thus to the terms of a mathematics of infinite collections.

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