

# Defining Gödel Incompleteness Away

We can simply define Gödel 1931 Incompleteness away by redefining the meaning of the standard definition of Incompleteness: A theory T is incomplete if and only if there is some sentence  $\varphi$  such that  $(T \not\vdash \varphi)$  and  $(T \not\vdash \neg\varphi)$ . This definition construes the existence of self-contradictory expressions in a formal system as proof that this formal system is incomplete because self-contradictory expressions are neither provable nor disprovable in this formal system. Since self-contradictory expressions are neither provable nor disprovable only because they are self-contradictory we could define them as unsound instead of defining the formal system as incomplete.

## According to Wittgenstein:

'True in Russell's system' means, as was said: proved in Russell's system; and 'false in Russell's system' means: the opposite has been proved in Russell's system. (Wittgenstein 1983,118-119)

## Formalized by Olcott as:

$\forall F \in \text{Formal\_Systems } \forall C \in \text{WFF}(F) ((F \vdash C) \leftrightarrow \text{True}(F, C))$   
 $\forall F \in \text{Formal\_Systems } \forall C \in \text{WFF}(F) ((F \vdash \neg C) \leftrightarrow \text{False}(F, C))$

We had to add that the proofs referred to by Wittgenstein must be to theorem consequences thus requiring the axioms of formal proofs to act as a proxy for the true premises of sound deduction.

We simply construe a formal proof to theorem consequences as isomorphic to deduction from a sound argument to a true conclusion. This requires the theorems of formal systems to be construed as the true premises of sound deduction.

Within the isomorphism between formal proofs and valid deduction  
Formal-Proof- $\rightarrow$ Unprovable(F, X)  $\cong$  Valid-Deduction- $\rightarrow$ Invalid-Argument(F, X).

## Isomorphism

In mathematics, an isomorphism is a mapping between two structures of the same type that can be reversed by an inverse mapping. Two mathematical structures are isomorphic if an isomorphism exists between them. The word isomorphism is derived from the Ancient Greek: ἴσος isos "equal", and μορφή morphe "form" or "shape". <https://en.wikipedia.org/wiki/Isomorphism>

## Simplified Gödel Sentence

<https://plato.stanford.edu/entries/goedel-incompleteness/#FirlncTheCom>

(G)  $F \vdash G_F \leftrightarrow \neg \text{Prov}_F(\ulcorner G_F \urcorner)$ . // Original  
(G)  $F \vdash G_F \leftrightarrow \neg \text{Prov}_F(G_F)$ . // Remove arithmetization

## // Adapt syntax and quantify:

$\exists F \in \text{Formal\_Systems } \exists G \in \text{WFF}(F) (G \leftrightarrow (F \not\vdash G))$

When a WFF expresses that it is logically equivalent to its own unprovability:  $G \leftrightarrow (F \nmid G)$  this expression is self-contradictory thus unsatisfiable. If we could prove that a sentence that asserts it is logically equivalent to its own unprovability is true this contradicts its assertion therefore we cannot prove that it is true.

Likewise with its negation:  $G \leftrightarrow (F \nmid \neg G)$ . If we could prove that a sentence that asserts it is logically equivalent to its own unprovability is false this contradicts its assertion therefore we cannot prove that it is false.

The conventional definition of incompleteness: A theory T is incomplete if and only if there is some sentence  $\varphi$  such that  $(T \nmid \varphi)$  and  $(T \nmid \neg\varphi)$ .

As we can see from the above neither  $(F \nmid G)$  nor  $(F \nmid \neg G)$  can be satisfied only because they are both self-contradictory. Because they are self-contradictory they meet the definition of Incompleteness. Within the sound deductive inference model unprovable expressions of language are simply construed as unsound arguments thus untrue.

### **Satisfiability**

A formula is satisfiable if it is possible to find an interpretation (model) that makes the formula true.

<https://en.wikipedia.org/wiki/Satisfiability>

### **Interpretation (logic)**

An interpretation is an assignment of meaning to the symbols of a formal language. [https://en.wikipedia.org/wiki/Interpretation\\_\(logic\)](https://en.wikipedia.org/wiki/Interpretation_(logic))

### **Model theory**

A model of a theory is a structure (e.g. an interpretation) that satisfies the sentences of that theory. [https://en.wikipedia.org/wiki/Model\\_theory](https://en.wikipedia.org/wiki/Model_theory)

Wittgenstein, Ludwig 1983. Remarks on the Foundations of Mathematics (Appendix III), 118-119. Cambridge, Massachusetts and London, England: The MIT Press  
<http://www.liarparadox.org/Wittgenstein.pdf>

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