Deductively Sound Formal Proofs

Using the sound deductive inference model as the basis of the a notion of a formal system defines [Deductively Sound Formal Proofs]. Within (DSFP) closed Well-formed formula that were undecidable in other formal systems are excluded on the basis that they do not belong to deductively sound inference.

The sound deductive inference model specifies:
[a connected sequence of valid deductions from true premises to a true conclusion]

> **Introduction to Mathematical logic Sixth edition Elliott Mendelson (2015)**
> 1.4 An Axiom System for the Propositional Calculus page 28
> A wf C is said to be a consequence in S of a set Γ of wfs if and only if there is a sequence B1, …, Bk of wfs such that C is Bk and, for each i, either Bi is an axiom or Bi is in Γ, or Bi is a direct consequence by some rule of inference of some of the preceding wfs in the sequence. Such a sequence is called a proof (or deduction) of C from Γ. The members of Γ are called the hypotheses or premises of the proof.
> **We use Γ ⊢ C as an abbreviation for "C is a consequence of Γ"...**

When we simply assume that the set of premises: **Γ** are true we transform conventional formal proofs into **[Deductively Sound Formal Proofs].** These formal proofs: **(Γ ⊢ C)** transmit the truth value of their premises to their consequent making the consequent of these proofs necessarily true.

> **Haskell Curry Foundations of Mathematical Logic, 1977**
> Let *T* be such a theory. Then the elementary statements which belong to *T* we shall call the elementary theorems of *T*; we also say that these elementary statements are true for *T*. Thus, given *T*, an elementary theorem is an elementary statement which is true. A theory is thus a way of picking out from the statements of *F* a certain subclass of true statements.

When we assume that Axioms are True we create a corresponding pair of predicates.
(1) True(x) ↔ (⊢x)
(2) False(x) ↔ (⊢¬x)
Providing another example of: [Deductively Sound Formal Proofs].

With True and False formalized we specify a semantic criterion of Well-formedness:
(3) Deductively_Sound_Consequent(x) ↔ (True(x) ∨ False(x))

This semantic criterion of Well-formedness works in the same way as syntactic criterion of well-formed formula (WFF) in that every closed WFF that is not a [Deductively Sound Consequent] is excluded from the set of expressions belonging to the [Deductively Sound Formal System]. This eliminates undecidability in all of these formal systems.