Deductively Sound Formal Proofs

Using the sound deductive inference model as the basis of the a notion of a formal system defines [Deductively Sound Formal Proofs]. Within (DSFP) closed Well-formed formula that were undecidable in other formal systems are excluded on the basis that they do not belong to deductively sound inference.

The sound deductive inference model specifies:
[a connected sequence of valid deductions from true premises to a true conclusion]

> A wf C is said to be a consequence in S of a set Γ of wfs if and only if there is a sequence B1, …, Bk of wfs such that C is Bk and, for each i, either Bi is an axiom or Bi is in Γ, or Bi is a direct consequence by some rule of inference of some of the preceding wfs in the sequence. Such a sequence is called a proof (or deduction) of C from Γ. The members of Γ are called the hypotheses or premisses of the proof. We use Γ ⊢ C as an abbreviation for "C is a consequence of Γ"... (Mendelson 2015:28)

Within the sound deductive inference model True(x) is defined as:
(1) An expression of language having the semantic value of Boolean True.
(2) The consequence of any formal mathematical proof where every element of its set of premises is True.

> Let *T* be such a theory. Then the elementary statements which belong to *T* we shall call the elementary theorems of *T*; we also say that these elementary statements are true for *T*. Thus, given *T*, **an elementary theorem is an elementary statement which is true.** A theory is thus a way of picking out from the statements of *F* a certain subclass of true statements. (Curry 1977:45)

To define formal systems having true premises we adopt the (Curry 1977:45) convention stipulating that axioms (Curry elementary theorems) are true.
This stipulation derives a corresponding pair of True(x) and False(x) predicates:
(1) True(x) := (⊢x) ":=" is defined below **
(2) False(x) := (⊢¬x)

With True and False formalized we specify a semantic criterion of Well-formedness:
(3) Deductively_Sound_Consequent(x) := (True(x) ∨ False(x))
Sound deduction excludes conclusions/consequences not based on sound deduction, as deductively unsound conclusions. Thus a semantic criteria of well-formedness is provided in addition to the conventional syntactic notion of WFF.

Mendelson, Elliott 2015. Introduction to Mathematical Logic (sixth edition). Boca Raton: Taylor & Francis Group LLC.

Curry, Harkell B. 1977. Foundations of Mathematical Logic. New York: Dover Publications, Inc.

** LHS := RHS (The LHS is defined as an alias for the RHS)