Deductively Sound Formal Proofs

Using the sound deductive inference model as the basis of the a notion of a formal system defines [Deductively Sound Formal Proofs]. Within (DSFP) closed Well-formed formula that were undecidable in other formal systems are excluded on the basis that they do not belong to deductively sound inference.

The sound deductive inference model specifies:
[a connected sequence of valid deductions from true premises to a true conclusion]

> A wf C is said to be a consequence in S of a set Γ of wfs if and only if there is a sequence B1, …, Bk of wfs such that C is Bk and, for each i, either Bi is an axiom or Bi is in Γ, or Bi is a direct consequence by some rule of inference of some of the preceding wfs in the sequence. Such a sequence is called a proof (or deduction) of C from Γ. The members of Γ are called the hypotheses or premisses of the proof. We use Γ ⊢ C as an abbreviation for "C is a consequence of Γ"... (Mendelson 2015:28)

Within the sound deductive inference model True(x) is defined as:
(a) Axioms which are stipulated as expressions of language having the semantic value of Boolean true.
(b) Theorems which are stipulated as the consequence of any formal mathematical proof where every element of its set of premises is True(x).

> Let *T* be such a theory. Then the elementary statements which belong to *T* we shall call the elementary theorems of *T*; we also say that these elementary statements are true for *T*. Thus, given *T*, **an elementary theorem is an elementary statement which is true.** A theory is thus a way of picking out from the statements of *F* a certain subclass of true statements. (Curry 1977:45)

To define formal systems having true premises we adopt the (Curry 1977:45) convention stipulating that axioms (Curry elementary theorems) are true.
This stipulation derives a corresponding pair of True(x) and False(x) predicates:
(1) True(x) := (⊢x)   ":=" is defined below **
(2) False(x) := (⊢¬x)

With True and False formalized we specify a semantic criterion of Well-formedness:
(3) Deductively_Sound_Consequent(x) := (True(x) ∨ False(x))
Sound deduction excludes conclusions/consequences not based on sound deduction, as deductively unsound conclusions. Thus a semantic criteria of well-formedness is provided in addition to the conventional syntactic notion of WFF.

Mendelson, Elliott 2015. Introduction to Mathematical Logic (sixth edition). Boca Raton: Taylor & Francis Group LLC.

Curry, Harkell B. 1977. Foundations of Mathematical Logic. New York: Dover Publications, Inc.

** LHS := RHS   (The LHS is defined as an alias for the RHS)