

Deductively Sound Formal Proofs

A semantic tautology is essentially a stipulated definition. When the conventional meaning of the {formal proofs of mathematical logic} is combined with the conventional meaning of {sound deductive inference} the semantic meaning of {deductively sound formal proofs of mathematical logic} is stipulated (semantically entailed).

When we evaluate the undecidable sentences of the conventional {formal proofs of mathematical logic} within {deductively sound formal proofs of mathematical logic} that they are decided to be deductively unsound is also stipulated (semantically entailed).

The sound deductive inference model specifies:

[a connected sequence of valid deductions from true premises to a true conclusion]

Mendelson, Elliott 2015. Introduction to Mathematical Logic (sixth edition) page 28

A wf C is said to be a consequence in S of a set Γ of wfs if and only if there is a sequence B_1, \dots, B_k of wfs such that C is B_k and, for each i , either B_i is an axiom or B_i is in Γ , or B_i is a direct consequence by some rule of inference of some of the preceding wfs in the sequence. Such a sequence is called a proof (or deduction) of C from Γ . The members of Γ are called the hypotheses or premisses of the proof.

We use $\Gamma \vdash C$ as an abbreviation for " C is a consequence of Γ "... (Mendelson 2015:28)

To make the conventional formal proofs of mathematical logic conform to the sound deductive inference model we only have to select the subset of formal proofs having true premises: **this $\Gamma \vdash C$ becomes this $\text{True}(\Gamma) \vdash C$.**

How hard is it to understand the basic model of Sound_Deduction?

True premises combined with valid inference necessitates true consequences.

The argument is deductively unsound when: $\neg(\text{True}(\Gamma) \wedge (\Gamma \vdash C) \vee (\Gamma \vdash \neg C))$

- (a) Not all of the premises specified by Γ are true.
- (b) There is no sequence B_1, \dots, B_k of wfs from Γ to $(C$ or $\neg C)$.
- (c) $\therefore \neg \text{Deductively_Sound}(\Gamma, C)$ if (a) or (b) then (c)

When a man that has never been married is asked:

Have you stopped beating your wife yet?

The lack of a correct yes or no answer does not indicate that all of human reasoning is fundamentally incomplete. When-so-ever a yes or no question lacks a correct yes or no answer the question itself is incorrect.

This same reasoning applies equally to logic sentences: When-so-ever a logic sentence lacks a correct true or false Boolean value the logic sentence itself is incorrect.

When the above is applied to every conventional undecidable sentence of mathematical logic it simply decides that these sentences are deductively unsound thus semantically erroneous.