

Deductively sound formal proofs of mathematical logic

Could the intersection of [formal proofs of mathematical logic] and [sound deductive inference] specify formal systems having [deductively sound formal proofs of mathematical logic]?

All that we have to do to provide [deductively sound formal proofs of mathematical logic] is select the subset of conventional [formal proofs of mathematical logic] having true premises and now we have [deductively sound formal proofs of mathematical logic].

<https://www.iep.utm.edu/val-snd/> A deductive argument is said to be valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false. Otherwise, a deductive argument is said to be invalid.

A deductive argument is sound if and only if it is both valid, and all of its premises are actually true. Otherwise, a deductive argument is unsound.

In other words in sound deduction there is a:

[connected sequence of valid deductions from true premises to a true conclusion]

College textbook explains exactly what is meant by [formal proofs of mathematical logic]

http://liarparadox.org/Provable_Mendelson.pdf

It is best that you totally understand the Mendelson definition of formal proofs before proceeding. Once you totally understand the concept of formal proofs of mathematical logic and the concept sound deductive inference we can proceed.

To explain the notion of [sound deduction] using the terms of the art and symbols of [formal proofs] requires totally understanding this one aspect of [formal proofs]:

$\Gamma \vdash C$ means that "C is a consequence of premises Γ "

" Γ " ----- Specifies the premises of a formal proof.

" C " ----- Specifies the consequence of a formal proofs.

" \vdash " ----- Specifies valid deduction from premises to consequence of formal proofs.

" $\Gamma \vdash C$ " Specifies that C is provable from Γ , in other words " $\Gamma \vdash C$ " a valid deductive argument.

To convert a valid deductive argument into a sound deductive argument only requires that all the premises are true. We only have to find some way to select the subset of conventional formal proofs having entirely true premises.

<http://mathworld.wolfram.com/Axiom.html>

An axiom is a proposition regarded as self-evidently true without proof.

Curry, Harkell B. 1977. Foundations of Mathematical Logic. Page:45

Thus, given [theory] T, an elementary theorem is an elementary statement which is true.

On the basis of these two sources we simply stipulate that Axioms are true. This means that Axioms are expressions of language having been defined to have the semantic value of Boolean True. Within this definition of Axiom and the conventional notation of formal systems we specify this predicate: $\exists \Gamma \subseteq \text{Axioms}(F) \exists C \in \text{WFF}(F)$
(**Deductively_Sound**($\Gamma \vdash C$))

From the above **Deductively_Sound**($\Gamma \vdash C$) we can derive a universal truth predicate: It is common knowledge in the sound deductive inference model that true premises combined with valid deduction necessitates a true conclusion.

Thus we know that **Deductively_Sound**($\Gamma \vdash C$) \rightarrow **True**(C). It is also common notation convention to not indicate an empty set of premises, (which means the proof is based on axioms), thus this ($\Gamma \vdash C$) becomes this ($\vdash C$) Because of this we can define **True**(C) := ($\vdash C$).

We are not simply specifying that **True**(C) is **Provable**(C). We are specifying the common idea from sound deduction that a true conclusion necessarily follows from true premises and valid inference. Within the stipulation that Axioms are True then any formal proof to theorem consequences necessarily derives a true consequence.

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