

Deductively Sound Formal Proofs

Using the sound deductive inference model as the basis of the a notion of a formal system defines [Deductively Sound Formal Proofs]. Within (DSFP) closed Well-formed formula that were undecidable in other formal systems are excluded on the basis that they do not belong to deductively sound inference.

The sound deductive inference model specifies:

[a connected sequence of valid deductions from true premises to a true conclusion]

---The following is applied to the formal proofs of mathematical logic---

When we augment this definition of Axiom (1) becomes (2)

(1) A proposition regarded as self-evidently true without proof [1]

(2) An expression of language (or WFF) having the semantic value of Boolean true. [2]

Then every formal proof to theorem consequences: $(\vdash x)$ transmits the truth value of the Axioms to these consequences thus making the consequence necessarily true.

This provides: [Deductively Sound Formal Proofs] (DSFP) -- $\text{True}(x) \leftrightarrow (\vdash x)$

True Premises(P) Necessarily derive a True Consequence(C): $\Box(\text{True}(P) \vdash \text{True}(C))$

Now we have a formal specification of the notion of True that cannot possibly fail. From this specification we derive formal specification of the notion of False: $(\vdash \neg x)$.

With True and False formalized we specify a semantic criterion of Well-formedness:

$\text{Deductively_Sound_Consequence}(x) \leftrightarrow (\text{True}(x) \vee \text{False}(x))$

The above derives three meta-mathematical axiom schemata:

(1) $\text{True}(x) \leftrightarrow (\vdash x)$

(2) $\text{False}(x) \leftrightarrow (\vdash \neg x)$

(3) $\text{Deductively_Sound_Consequence}(x) \leftrightarrow (\text{True}(x) \vee \text{False}(x))$

Defining [Deductively Sound Consequences of Formal Proofs].

$\text{Deductively_Sound_Consequence}(x)$ excludes consequences that do not belong to deductively sound inference. This eliminates undecidability in formal systems.

[1] <http://mathworld.wolfram.com/Axiom.html>

[2] Embedding the semantics of Boolean values directly in the syntax of formal proofs is a precedent already established by Haskell Curry Foundations of Mathematical Logic, 1977 and Rudolf Carnap Meaning Postulates, 1952.