

Deductively Sound Formal Proofs

Using the sound deductive inference model as the basis of the a notion of a formal system defines [Deductively Sound Formal Proofs]. Within (DSFP) closed Well-formed formula that were undecidable in other formal systems are excluded on the basis that they do not belong to deductively sound inference.

The sound deductive inference model specifies:

[a connected sequence of valid deductions from true premises to a true conclusion]

Introduction to Mathematical logic Sixth edition Elliott Mendelson (2015)

1.4 An Axiom System for the Propositional Calculus page 28

A wf C is said to be a consequence in S of a set Γ of wfs if and only if there is a sequence B_1, \dots, B_k of wfs such that C is B_k and, for each i , either B_i is an axiom or B_i is in Γ , or B_i is a direct consequence by some rule of inference of some of the preceding wfs in the sequence. Such a sequence is called a proof (or deduction) of C from Γ . The members of Γ are called the hypotheses or premisses of the proof.

We use $\Gamma \vdash C$ as an abbreviation for “ C is a consequence of Γ ”...

When we simply assume that the set of premises: Γ are true we transform conventional formal proofs into [**Deductively Sound Formal Proofs**]. These formal proofs: $(\Gamma \vdash C)$ transmit the truth value of their premises to their consequent making the consequent of these proofs necessarily true.

Haskell Curry Foundations of Mathematical Logic, 1977

Let \mathcal{T} be such a theory. Then the elementary statements which belong to \mathcal{T} we shall call the elementary theorems of \mathcal{T} ; we also say that these elementary statements are true for \mathcal{T} . Thus, given \mathcal{T} , an elementary theorem is an elementary statement which is true. A theory is thus a way of picking out from the statements of F a certain subclass of true statements.

When we assume that Axioms are True we create a corresponding pair of predicates.

(1) $\text{True}(x) \leftrightarrow (\vdash x)$

(2) $\text{False}(x) \leftrightarrow (\vdash \neg x)$

Providing another example of: [**Deductively Sound Formal Proofs**].

With True and False formalized we specify a semantic criterion of Well-formedness:

(3) $\text{Deductively_Sound_Consequent}(x) \leftrightarrow (\text{True}(x) \vee \text{False}(x))$

This predicate excludes all consequences that do not belong to deductively sound inference. This eliminates undecidability in all [**Deductively Sound Formal Systems**] which are any formal system that implements [**Deductively Sound Formal Proofs**].