

## Deductively Sound Formal Proofs

Using the sound deductive inference model as the basis of the a notion of a formal system defines [Deductively Sound Formal Proofs]. Within (DSFP) closed Well-formed formula that were undecidable in other formal systems are excluded on the basis that they do not belong to deductively sound inference.

### The sound deductive inference model specifies:

[a connected sequence of valid deductions from true premises to a true conclusion]

#### Introduction to Mathematical logic Sixth edition Elliott Mendelson (2015)

##### 1.4 An Axiom System for the Propositional Calculus page 28

A wf C is said to be a consequence in S of a set  $\Gamma$  of wfs if and only if there is a sequence  $B_1, \dots, B_k$  of wfs such that C is  $B_k$  and, for each  $i$ , either  $B_i$  is an axiom or  $B_i$  is in  $\Gamma$ , or  $B_i$  is a direct consequence by some rule of inference of some of the preceding wfs in the sequence. Such a sequence is called a proof (or deduction) of C from  $\Gamma$ . The members of  $\Gamma$  are called the hypotheses or premisses of the proof.

**We use  $\Gamma \vdash C$  as an abbreviation for "C is a consequence of  $\Gamma$ "...**

When we simply assume that the set of premises:  $\Gamma$  are true we transform conventional formal proofs into [**Deductively Sound Formal Proofs**]. These formal proofs:  $(\Gamma \vdash C)$  transmit the truth value of their premises to their consequent making the consequent of these proofs necessarily true.

#### Haskell Curry Foundations of Mathematical Logic, 1977

Let  $\mathcal{T}$  be such a theory. Then the elementary statements which belong to  $\mathcal{T}$  we shall call the elementary theorems of  $\mathcal{T}$ ; we also say that these elementary statements are true for  $\mathcal{T}$ . Thus, given  $\mathcal{T}$ , an elementary theorem is an elementary statement which is true. A theory is thus a way of picking out from the statements of  $F$  a certain subclass of true statements.

When we assume that Axioms are True we create a corresponding pair of predicates.

(1)  $\text{True}(x) \leftrightarrow (\vdash x)$

(2)  $\text{False}(x) \leftrightarrow (\vdash \neg x)$

Providing another example of: [**Deductively Sound Formal Proofs**].

With True and False formalized we specify a semantic criterion of Well-formedness:

(3)  $\text{Deductively\_Sound\_Consequent}(x) \leftrightarrow (\text{True}(x) \vee \text{False}(x))$

This semantic criterion of Well-formedness works in the same way as syntactic criterion of well-formed formula (WFF) in that every closed WFF that is not a [Deductively Sound Consequent] is excluded from the set of expressions belonging to the formal system. This eliminates undecidability in all of these formal systems.