

## Eliminating Undecidability and Incompleteness in Formal Systems

(By making Formal Systems correspond to the Sound Deductive Inference model)

Tarski "proved" that there cannot possibly be any correct formalization of the notion of truth entirely on the basis of an insufficiently expressive formal system that was incapable of recognizing and rejecting semantically incorrect expressions of language.

An alternative that corrects the shortcomings of the conventional notion of formal systems is provided that totally eliminates incompleteness and undecidability from all formal systems capable of directly expressing a provability predicate (thus without the need for diagonalization).

### Stipulative definition of Axiom

The elementary statements which belong to T are called the elementary theorems of T and said to be true. In this way, a theory is a way of designating a subset of E which consists entirely of true statements. (Curry 2010)

**An expression of language defined to have the semantic value of Boolean True.**

### Making Formal Systems correspond to Sound Deductive Inference

When we stipulate the Haskell Curry notion of axiom then formal proofs to theorem consequences simply correspond to sound deductive inference. By restricting the formal notion of Truth to sound deduction we eliminate incompleteness and inconsistency in formal systems.

**From the above basis these two axioms formalize the notion of True and False**

- (1) True            x is a theorem of F             $(F \vdash x)$
- (2) False            $\neg x$  is a theorem of F         $(F \vdash \neg x)$

**From the law of the excluded middle ( $P \vee \neg P$ ) we derive:**

- (3) Boolean        x is True or False in F         $\text{True}(F, x) \vee \text{False}(F, x)$

**We derive Theorem (1) from Axiom(1) so that we can refer to  $\neg$ True:**

$$\neg(\text{True}(F, x) \leftrightarrow (F \vdash x)) \rightarrow (\neg\text{True}(F, x) \leftrightarrow (F \nvdash x))$$

Whenever an expression of language is  $\neg$ Boolean this expression lacks a Boolean property. A closed WFF lacking a Boolean property is just like a non-declarative sentence, neither true or false. Any closed WFF containing a  $\neg$ Boolean term evaluates to  $\neg$ Boolean.

### Truth Predicate Axioms (Tarski notation)

- (1)  $x \in \text{Tr} \leftrightarrow x \in \text{Pr}$             //  $\text{True}(x) \leftrightarrow (\vdash x)$
- (2)  $\neg x \in \text{Tr} \leftrightarrow \neg x \in \text{Pr}$         //  $\text{False}(x) \leftrightarrow (\vdash \neg x)$
- (3)  $x \in \text{Tr} \vee \neg x \in \text{Tr}$             //  $\text{Boolean}(x) \leftrightarrow (\text{True}(x) \vee \text{False}(x))$

Anyone truly understanding the Tarski Undefinability proof would know that the whole proof would fail as soon as its third step would be proven false: (3)  $x \notin Pr \leftrightarrow x \in Tr$   
**Tarski's step(3) is decided to be false on the basis of contradicting Axiom(1).**

**The above can only be understood within the context of the Tarski Proof:**

[http://liarparadox.org/Tarski\\_Proof\\_275\\_276.pdf](http://liarparadox.org/Tarski_Proof_275_276.pdf) (Tarski 1936:275-276)

By making a very slight change to the conventional notion of a formal system [using the specified axioms of truth as the foundational basis of truth] we have a new notion of formal system that is in every way identical to the prior notion except that it correctly decides all of the sentences that were previously undecidable.

**Truth Axiom(3) decides that G is  $\neg$ Boolean thus neither True nor False.**

$\exists F \in \text{Formal\_Systems} \exists G \in \text{WFF}(F) (G \leftrightarrow ((F \not\vdash G) \wedge (F \not\vdash \neg G)))$

**Making the following paragraph  $\neg$ Boolean, thus neither True nor False.**

The first incompleteness theorem states that in any consistent formal system F within which a certain amount of arithmetic can be carried out, there are statements of the language of F which can neither be proved nor disproved in F. (Raatikainen 2018:1)

### **Truth Predicate Axioms in Simple English**

- (1) A set of facts adds up to X being TRUE.
- (2) A set of facts adds up to X being FALSE.
- (3) Because X is neither true nor False: X is not declarative sentence.

### **References:**

**Curry (2010)** Haskell Curry, *Foundations of Mathematical Logic*, 2010.

**(Raatikainen 2018)** Raatikainen, Panu, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy (Fall 2018 Edition), Edward N. Zalta (ed.), URL = <<https://plato.stanford.edu/archives/fall2018/entries/goedel-incompleteness/>>.

**(Tarski 1936)** A. Tarski, tr J.H. Woodger, 1983. "*The Concept of Truth in Formalized Languages*". English translation of Tarski's 1936 article. In A. Tarski, ed. J. Corcoran, 1983, *Logic, Semantics, Metamathematics*, Hackett.

### **Axioms of Truth**

- (1)  $\text{True}(F, x) \leftrightarrow (F \vdash x)$
- (2)  $\text{False}(F, x) \leftrightarrow (F \vdash \neg x)$
- (3)  $\text{Boolean}(F, x) \leftrightarrow (\text{True}(F, x) \vee \text{False}(F, x))$

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