

Eliminating Undecidability and Incompleteness in Formal Systems

Tarski "proved" that there cannot possibly be any correct formalization of the notion of truth entirely on the basis of an insufficiently expressive formal system that was incapable of recognizing and rejecting semantically incorrect expressions of language.

An alternative that corrects the shortcomings of the conventional notion of formal systems is provided that totally eliminates incompleteness and undecidability from all formal systems capable of directly expressing a provability predicate (thus without the need for diagonalization).

We assume this specification of the philosophical notion of analytical truth:

An expression of natural language only counts as true if it can be completely verified as totally true entirely on the basis of the meaning of its words.

We stipulate this Haskell Curry notion of axiom:

The elementary statements which belong to T are called the elementary theorems of T and said to be true. In this way, a theory is a way of designating a subset of E which consists entirely of true statements. (Curry 2010)

From the above basis these two axioms formalize the notion of True and False

- (1) True x is a theorem of F $(F \vdash x)$
- (2) False $\neg x$ is a theorem of F $(F \vdash \neg x)$

From the law of the excluded middle ($P \vee \neg P$) we derive:

- (3) Boolean x is True or False in F $\text{True}(F, x) \vee \text{False}(F, x)$

We derive Theorem (1) so that we can refer to \neg True:

$(\text{True}(F, x) \leftrightarrow (F \vdash x)) \rightarrow (\neg \text{True}(F, x) \leftrightarrow (F \nvdash x))$

Whenever an expression of language is \neg Boolean this expression lacks a Boolean property. A closed WFF lacking a Boolean property is just like a non-declarative sentence, neither true or false. Any closed WFF containing a \neg Boolean term evaluates to \neg Boolean and can construed as \neg True or \neg False as needed.

Truth Predicate Axioms (Tarski notation)

- (1) $x \in \text{Tr} \leftrightarrow x \in \text{Pr}$ // $\text{True}(x) \leftrightarrow (\vdash x)$
- (2) $\neg x \in \text{Tr} \leftrightarrow \neg x \in \text{Pr}$ // $\text{False}(x) \leftrightarrow (\vdash \neg x)$
- (3) $x \in \text{Tr} \wedge \neg x \in \text{Tr}$ // $\text{Boolean}(x) \leftrightarrow (\text{True}(x) \vee \text{False}(x))$

Theorem

- (1) $x \notin \text{Tr} \leftrightarrow x \notin \text{Pr}$ // $\neg \text{True}(x) \leftrightarrow (\nvdash x)$

Anyone truly understanding the Tarski Undefinability proof would know that the whole proof would fail as soon as its third step would be proven false: $(3) x \notin \text{Pr} \leftrightarrow x \in \text{Tr}$

Applying Truth Theorem (1) decides that Tarski's step(3) is false:

Swap the LHS of Tarski(3) $[x \notin \text{Pr}]$ that matches RHS of Theorem(1) $[x \notin \text{Pr}]$ with the LHS of Theorem(1) and we derive $x \notin \text{Tr} \leftrightarrow x \in \text{Tr}$, which is clearly false, thus decidable.

The above can only be understood within the context of the Tarski Proof:

http://liarparadox.org/Tarski_Proof_275_276.pdf (Tarski 1936:275-276)

By making a very slight change to the conventional notion of a formal system [using the specified axioms of truth as the foundational basis of truth] we have a new notion of formal system that is in every way identical to the prior notion except that it correctly decides all of the sentences that were previously undecidable.

Truth Axiom(3) decides that G is \neg Boolean thus \neg True:

$\exists F \in \text{Formal_Systems} \exists G \in \text{WFF}(F) (G \leftrightarrow ((F \Vdash G) \wedge (F \Vdash \neg G)))$

Making the following paragraph \neg Boolean, thus \neg True

The first incompleteness theorem states that in any consistent formal system F within which a certain amount of arithmetic can be carried out, there are statements of the language of F which can neither be proved nor disproved in F. (Raatikainen 2018:1)

Truth Predicate Axioms in Simple English

- (1) A set of facts adds up to X being TRUE.
- (2) A set of facts adds up to X being FALSE.
- (3) There are no set of facts that add up to X being TRUE.
- (4) X is not declarative sentence.

References:

Curry (2010) Haskell Curry, *Foundations of Mathematical Logic*, 2010.

(Raatikainen 2018) Raatikainen, Panu, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy (Fall 2018 Edition), Edward N. Zalta (ed.), URL = <<https://plato.stanford.edu/archives/fall2018/entries/goedel-incompleteness/>>.

(Tarski 1936) A. Tarski, tr J.H. Woodger, 1983. "*The Concept of Truth in Formalized Languages*". English translation of Tarski's 1936 article. In A. Tarski, ed. J. Corcoran, 1983, *Logic, Semantics, Metamathematics*, Hackett.

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