Eliminating Undecidability and Incompleteness in Formal Systems

If the conclusion of the Tarski Undefinability Theorem was that some artificially constrained limited notions of a formal system necessarily have undecidable sentences, then Tarski made no mistake within his assumptions. When we expand the scope of his investigation to other notions of formal systems we reach an entirely different conclusion showing that **Tarski's assumptions were wrong**.

A very slight augmentation to the conventional notion of a formal system refutes the more narrowly constrained Tarski results. This slightly augmented notion of a formal system is in every way identical to the conventional notion except that it recognizes and rejects semantically incorrect expressions of language.

This refutation applies to the generalized result of the Tarski Undefinability Theorem: **All formal systems of greater expressive power than arithmetic necessarily have undecidable sentences.** and requires that the formal system have its own provability predicate, eliminating the need for diagonalization.

When Closed WFF x of formal system F is considered:

True	x is a theorem of F	$(F \vdash x)$
False	¬x is a theorem of F	$(F \vdash \neg x)$
¬True	x is not a theorem of F	$(F \nvdash x)$
¬Boolean	x is neither True nor False in F	$(F \nvdash x) \land (F \nvdash \neg x)$

When-so-ever any closed WFF contains a ¬Boolean term the whole WFF evaluates to ¬True thus maintaining consistency within the {axioms of truth} adaptation to the notion of a formal system. When an expression of language lacks a Boolean property it is still not true in the same way that boxcars and billy goats are also not true. Only things analogous to declarative sentences have a Boolean property.

Truth Predicate Axioms
(Tarski Notation, Conventional Notation and Simple English)
(1) $x \in Tr \leftrightarrow x \in Pr$ // $True(x) \leftrightarrow (\vdash x)$
A set of facts adds up to X being TRUE.

(2) $\neg x \in Tr \leftrightarrow \neg x \in Pr$ // $False(x) \leftrightarrow (\vdash \neg x)$
A set of facts adds up to X being FALSE.

(3) $x \notin Tr \leftrightarrow x \notin Pr$ // $\neg True(x) \leftrightarrow \neg(\vdash x)$
There are no set of facts that add up to X being TRUE.

(4) $x \notin Pr \land \neg x \notin Pr$ // $((\nvdash x) \land (\nvdash \neg x)) \leftrightarrow \neg Boolean(x)$
x is neither TRUE nor FALSE

Anyone truly understanding the Tarski Undefinability proof would know that the whole proof would fail as soon as its third step would be proven false: (3) x ∉ Pr ↔ x ∈ Tr

Applying Truth Predicate Axiom(3) decides that Tarski's step(3) is false:
Swap the LHS of Tarski(3) [x ∉ Pr] that matches RHS of Axiom(3) [x ∉ Pr] with the LHS of Axiom(3) and we derive x ∉ Tr ↔ x ∈ Tr, which is clearly false, thus decidable.

By making a very slight change to the conventional notion of a formal system [using the specified axioms of truth as the foundational basis of truth] we have a new notion of formal system that is in every way identical to the prior notion except that it correctly decides all of the sentences that were previously undecidable.

The above can only be understood within the context of the Tarski Proof:
http://liarparadox.org/Tarski_Proof_275_276.pdf   (Tarski 1936:275-276)

Because every closed WFF containing a ¬Boolean term evaluates to ¬True this sentence:
((F ⊢ G) ↔ (F ⊬ ¬G)) evaluates to ¬True when G is ¬Boolean.

The above adaptations show how this logic sentence is ¬Boolean thus ¬True:
∃F ∈ Formal_Systems ∃G ∈ WFF(F) (G ↔ ((F ⊬ G) ∧ (F ⊬ ¬G)))

Making the following paragraph ¬Boolean, thus ¬True
The first incompleteness theorem states that in any consistent formal system F within which a certain amount of arithmetic can be carried out, there are statements of the language of F which can neither be proved nor disproved in F. (Raatikainen 2018:1)

(Raatikainen 2018)
Raatikainen, Panu, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy (Fall 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2018/entries/goedel-incompleteness/>.

(Tarski 1936)
A. Tarski, tr J.H. Woodger, 1983. "The Concept of Truth in Formalized Languages". English translation of Tarski's 1936 article. In A. Tarski, ed. J. Corcoran, 1983, Logic, Semantics, Metamathematics, Hackett.