

Eliminating Undecidability and Incompleteness in Formal Systems

Tarski "proved" that there cannot possibly be any correct formalization of the notion of truth entirely on the basis of an insufficiently expressive formal system that was incapable of recognizing and rejecting semantically incorrect expressions of language.

An alternative that corrects the shortcomings of the conventional notion of formal systems is provided that totally eliminates incompleteness and undecidability from all formal systems capable of directly expressing a provability predicate (thus without diagonalization).

When Closed WFF x of formal system F is considered:

True	x is a theorem of F	$(F \vdash x)$
False	$\neg x$ is a theorem of F	$(F \vdash \neg x)$
\negTrue	x is not a theorem of F	$(F \nvdash x)$
\negBoolean	x is neither True nor False in F	$(F \nvdash x) \wedge (F \nvdash \neg x)$

Ensuring that this new notion of formal system is consistent:

When-so-ever any closed WFF contains a \neg Boolean term the whole WFF evaluates to \neg True thus maintaining consistency within the {axioms of truth} adaptation to the notion of a formal system.

When an expression of language lacks a Boolean property it is still not true in the same way that boxcars and billy goats are also not true. Only things analogous to declarative sentences have a Boolean property.

Truth Predicate Axioms (Tarski notion)

- (1) $x \in \text{Tr} \leftrightarrow x \in \text{Pr}$ // $\text{True}(x) \leftrightarrow (\vdash x)$
- (2) $\neg x \in \text{Tr} \leftrightarrow \neg x \in \text{Pr}$ // $\text{False}(x) \leftrightarrow (\vdash \neg x)$
- (3) $x \notin \text{Tr} \leftrightarrow x \notin \text{Pr}$ // $\neg \text{True}(x) \leftrightarrow \neg(\vdash x)$
- (4) $x \notin \text{Pr} \wedge \neg x \notin \text{Pr}$ // $((\nvdash x) \wedge (\nvdash \neg x)) \leftrightarrow \neg \text{Boolean}(x)$

Anyone truly understanding the Tarski Undefinability proof would know that the whole proof would fail as soon as its third step would be proven false: (3) $x \notin \text{Pr} \leftrightarrow x \in \text{Tr}$

Applying Truth Predicate Axiom(3) decides that Tarski's step(3) is false:

Swap the LHS of Tarski(3) $[x \notin \text{Pr}]$ that matches RHS of Axiom(3) $[x \notin \text{Pr}]$ with the LHS of Axiom(3) and we derive $x \notin \text{Tr} \leftrightarrow x \in \text{Tr}$, which is clearly false, thus decidable.

By making a very slight change to the conventional notion of a formal system [using the specified axioms of truth as the foundational basis of truth] we have a new notion of formal system that is in every way identical to the prior notion except that it correctly decides all of the sentences that were previously undecidable.

The above can only be understood within the context of the Tarski Proof:

http://liarparadox.org/Tarski_Proof_275_276.pdf (Tarski 1936:275-276)

The above adaptations show how this logic sentence is \neg Boolean thus \neg True:

$\exists F \in \text{Formal_Systems} \exists G \in \text{WFF}(F) (G \leftrightarrow ((F \not\models G) \wedge (F \not\models \neg G)))$

Making the following paragraph \neg Boolean, thus \neg True

The first incompleteness theorem states that in any consistent formal system F within which a certain amount of arithmetic can be carried out, there are statements of the language of F which can neither be proved nor disproved in F. (Raatikainen 2018:1)

(Raatikainen 2018)

Raatikainen, Panu, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy (Fall 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2018/entries/goedel-incompleteness/>.

(Tarski 1936)

A. Tarski, tr J.H. Woodger, 1983. "The Concept of Truth in Formalized Languages". English translation of Tarski's 1936 article. In A. Tarski, ed. J. Corcoran, 1983, Logic, Semantics, Metamathematics, Hackett.

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