Decomposing the Liar Paradox into its Semantic Atoms

Abstract:
This paper decomposes the Liar Paradox into its semantic atoms using Meaning Postulates (1952) provided by Rudolf Carnap. Formalizing truth values of propositions as Boolean properties of these propositions is a key new insight. This new insight divides the translation of a declarative sentence into its equivalent mathematical proposition into three separate steps. When each of these steps are separately examined the logical error of the Liar Paradox is unequivocally shown.

This paper begins on the basis of key insights provided by Saul Kripke in his famous paper: Outline of a Theory of Truth (1975). Kripke proposed that Kleene's strong three-valued logic value of Undefined applies to the Liar Paradox because the Liar Paradox is semantically ungrounded. Kripke also proposed that the truth form of the Liar Paradox [This sentence is true] is also semantically ungrounded.

Why has the Liar Paradox not had its self-reference error formalized before?
Every other investigation of the Liar Paradox conflated the testing of its Boolean (truth) value and the derivation of this Boolean value as a single atomic operation with no constituent parts. No one had previously ever decomposed the atomic steps of a proposition's Boolean (truth) value transitioning from non-existence into existence. Only after a proposition is formed can it be tested. Only after it has been tested does it derive a Boolean (truth) value.

When we explicitly divide mathematical propositions into their two semantic properties:
(1) Assertion // What it is claiming to be true
(2) Boolean.Value // The {True, False} value based on testing its Assertion
we can decompose every step of the translation of a Declarative Sentence into its equivalent Mathematical Proposition. We no longer conflate the testing of the assertion and the assignment of the Boolean result of this testing as a single atomic step.

Translating the Liar Paradox into a Mathematical Proposition
Now that the truth values of propositions are shown to be a Boolean properties of these propositions the initialization of this property can be explicitly shown rather than implied. This key new insight allows the semantics of the Liar Paradox to be totally explained.

When we translate the Declarative-Sentence [This sentence is false] into its equivalent Mathematical Proposition we must do this as a sequence of steps in strict prerequisite order:

Proposition p:

1) **Define the Proposition.Assertion**

Declarative Sentence: The Boolean value of p equals False.

Formalized as: p.Assertion.haveEqualValues(p.Boolean.Value, False)

2) **Test the Proposition.Assertion**

Interrogative Sentence: Does the Boolean value of p equal False?

Formalized as: TestResult = testEquality(p.Boolean.Value, False)

3) **Initialize the Proposition.Boolean.Value on the basis of prior testing**

Formalized as: p.Boolean.Value = TestResult

From the above formalized steps we can see that the Liar Paradox attempts to initialize its Boolean value entirely on the basis of testing this same Boolean value before it has come into existence. Comparing an undefined (semantically empty thus non-existent) Boolean value to False is like measuring the length of your car when you have no car. Because the comparison shown in step (2) above fails the assignment shown in step (3) never occurs.

Conclusion

When Saul Kripke proposed that the Liar Paradox is ungrounded he meant that there is no place where the Liar Paradox is obtaining its Boolean value from. This paper shows the precise reason why the Liar Paradox is semantically ungrounded and therefore unequivocally logically incorrect.

Outline of a Theory of Truth: Saul Kripke (1975).
http://philo.ruc.edu.cn/logic/reading/Kripke_%20Theory%20of%20Truth.pdf

Meaning Postulates: Rudolf Carnap (1952)
https://www.jstor.org/stable/4318143?seq=1#page_scan_tab_contents

The Philosophy of Logical Atomism: Bertrand Russell (1918)
https://sites.ualberta.ca/~francisp/NewPhil448/RussellPhilLogicalAtomismPears.pdf