

Halting problem proofs refuted on the basis of software engineering

This is an explanation of a key new insight into the halting problem provided in the language of software engineering. Technical computer science terms are explained using software engineering terms. No knowledge of the halting problem is required.

It is based on fully operational software executed in the x86utm operating system. The x86utm operating system (based on an excellent open source x86 emulator) was created to study the details of the halting problem proof counter-examples at the much higher level of abstraction of C/x86.

To fully understand this paper a software engineer must be an expert in:

- (a) The C programming language.
- (b) The x86 programming language.
- (c) Exactly how C translates into x86 (how C function calls are implemented in x86).
- (d) The ability to recognize infinite recursion at the x86 assembly language level.

The computer science term "halting" means that a Turing Machine terminated normally reaching its last instruction known as its "final state". This is the same idea as when a function returns to its caller as opposed to and contrast with getting stuck in an infinite loop or infinite recursion.

In computability theory, the halting problem is the problem of determining, from a description of an arbitrary computer program and an input, whether the program will finish running, or continue to run forever. Alan Turing proved in 1936 that a general algorithm to solve the halting problem for all possible program-input pairs cannot exist.

For any program H that might determine if programs halt, a "pathological" program P, called with some input, can pass its own source and its input to H and then specifically do the opposite of what H predicts P will do. No H can exist that handles this case. https://en.wikipedia.org/wiki/Halting_problem

H and P implement the above specified pathological relationship to each other:

```
#include <stdint.h> // To keep things simple a single data type is used;
#define u32 uint32_t // 32-bit unsigned integer is compiled in 32-bit mode

void P(u32 x)
{
    if (H(x, x))
        HERE: goto HERE;
    return;
}

int main()
{
    output("Input_Halts = ", H((u32)P, (u32)P));
}
```

A halt decider must compute the mapping from its inputs to an accept or reject state on the basis of the actual behavior that is actually specified by these inputs.

This general principle refutes conventional halting problem proofs

Every simulating halt decider that correctly simulates its input until it correctly predicts that this simulated input would never reach its final state, correctly rejects this input as non-halting.

From a purely software engineering perspective H(P,P) is required to correctly predict that its correct and complete x86 emulation of its input would never reach the "ret" instruction of this input and H must do this in a finite number of steps. (see **Example 03**).

Example 01: H0 correctly determines that Infinite_Loop() never halts

```
void Infinite_Loop()
{
    HERE: goto HERE;
}

int main()
{
    Output("Input_Halts = ", H0((u32)Infinite_Loop));
}
```

```
_Infinite_Loop()
[00001102](01) 55      push ebp
[00001103](02) 8bec     mov ebp,esp
[00001105](02) ebfe     jmp 00001105
[00001107](01) 5d      pop ebp
[00001108](01) c3      ret
Size in bytes:(0007) [00001108]
```

```
_main()
[00001192](01) 55      push ebp
[00001193](02) 8bec     mov ebp,esp
[00001195](05) 6802110000 push 00001102
[0000119a](05) e8d3fbffff call 00000d72
[0000119f](03) 83c404   add esp,+04
[000011a2](01) 50      push eax
[000011a3](05) 68a3040000 push 000004a3
[000011a8](05) e845f3ffff call 000004f2
[000011ad](03) 83c408   add esp,+08
[000011b0](02) 33c0     xor eax,eax
[000011b2](01) 5d      pop ebp
[000011b3](01) c3      ret
Size in bytes:(0034) [000011b3]
```

| machine address | stack address | stack data | machine code | assembly language |
|-----------------|---------------|------------|--------------|-------------------|
| [00001192] | [00101ef8] | [00000000] | 55 | push ebp |
| [00001193] | [00101ef8] | [00000000] | 8bec | mov ebp,esp |
| [00001195] | [00101ef4] | [00001102] | 6802110000 | push 00001102 |
| [0000119a] | [00101ef0] | [0000119f] | e8d3fbffff | call 00000d72 |

```
H0: Begin Simulation      Execution Trace Stored at:211fac
[00001102][00211f9c][00211fa0] 55      push ebp
[00001103][00211f9c][00211fa0] 8bec     mov ebp,esp
[00001105][00211f9c][00211fa0] ebfe     jmp 00001105
[00001105][00211f9c][00211fa0] ebfe     jmp 00001105
H0: Infinite Loop Detected Simulation Stopped
```

```
if (current->Simplified_Opcode == JMP) // JMP
    if (current->Decode_Target <= current->Address) // upward
        if (traced->Address == current->Decode_Target) // to this address
            if (Conditional_Branch_Count == 0) // no escape
                return 1;
```

| | | | | |
|------------|------------|------------|------------|---------------|
| [0000119f] | [00101ef8] | [00000000] | 83c404 | add esp,+04 |
| [000011a2] | [00101ef4] | [00000000] | 50 | push eax |
| [000011a3] | [00101ef0] | [000004a3] | 68a3040000 | push 000004a3 |
| [000011a8] | [00101ef0] | [000004a3] | e845f3ffff | call 000004f2 |

```
Input_Halts = 0
[000011ad][00101ef8][00000000] 83c408   add esp,+08
[000011b0][00101ef8][00000000] 33c0     xor eax,eax
[000011b2][00101efc][00100000] 5d      pop ebp
[000011b3][00101f00][00000004] c3      ret
Number of Instructions Executed(554) == 8 Pages
```

Example 02: H correctly determines that Infinite_Recursion() never halts

```
void Infinite_Recursion(int N)
{
    Infinite_Recursion(N);
}

int main()
{
    output("Input_Halts = ", H((u32)Infinite_Recursion, 0x777));
}
```

```
_Infinite_Recursion()
[000010f2] (01) 55          push ebp
[000010f3] (02) 8bec          mov ebp,esp
[000010f5] (03) 8b4508        mov eax,[ebp+08]
[000010f8] (01) 50          push eax
[000010f9] (05) e8f4ffffff    call 000010f2
[000010fe] (03) 83c404        add esp,+04
[00001101] (01) 5d          pop ebp
[00001102] (01) c3          ret
Size in bytes:(0017) [00001102]
```

```
_main()
[000011b2] (01) 55          push ebp
[000011b3] (02) 8bec          mov ebp,esp
[000011b5] (05) 6877070000   push 0000777
[000011ba] (05) 68f2100000   push 000010f2
[000011bf] (05) e8aefdffff    call 00000f72
[000011c4] (03) 83c408        add esp,+08
[000011c7] (01) 50          push eax
[000011c8] (05) 68a3040000   push 00004a3
[000011cd] (05) e820f3ffff    call 00004f2
[000011d2] (03) 83c408        add esp,+08
[000011d5] (02) 33c0          xor eax,eax
[000011d7] (01) 5d          pop ebp
[000011d8] (01) c3          ret
Size in bytes:(0039) [000011d8]
```

| machine address | stack address | stack data | machine code | assembly language |
|-----------------|---------------|------------|--------------|-------------------|
| [000011b2] | [00101f39] | [00000000] | 55 | push ebp |
| [000011b3] | [00101f39] | [00000000] | 8bec | mov ebp,esp |
| [000011b5] | [00101f35] | [00000777] | 6877070000 | push 0000777 |
| [000011ba] | [00101f31] | [000010f2] | 68f2100000 | push 000010f2 |
| [000011bf] | [00101f2d] | [000011c4] | e8aefdffff | call 00000f72 |

```
H: Begin Simulation Execution Trace Stored at:111fe5
[000010f2] [00111fd1] [00111fd5] 55          push ebp
[000010f3] [00111fd1] [00111fd5] 8bec          mov ebp,esp
[000010f5] [00111fd1] [00111fd5] 8b4508        mov eax,[ebp+08]
[000010f8] [00111fd1] [00000777] 50          push eax      // push 0x777
[000010f9] [00111fd1] [000010fe] e8f4ffffff    call 000010f2 // call Infinite_Recursion
[000010f2] [00111fc5] [00111fd1] 55          push ebp
[000010f3] [00111fc5] [00111fd1] 8bec          mov ebp,esp
[000010f5] [00111fc5] [00111fd1] 8b4508        mov eax,[ebp+08]
[000010f8] [00111fc1] [00000777] 50          push eax      // push 0x777
[000010f9] [00111fdb] [000010fe] e8f4ffffff    call 000010f2 // call Infinite_Recursion
H: Infinite Recursion Detected Simulation Stopped
```

```
if (current->Simplified_Opcode == CALL)
    if (current->Simplified_Opcode == traced->Simplified_Opcode) // CALL
        if (current->Address == traced->Address) // from same address
            if (current->Decode_Target == traced->Decode_Target) // to Same Function
                if (Conditional_Branch_Count == 0) // no escape
                    return 2;
```

| | | | | |
|-----------------|------------|------------|------------|--------------|
| [000011c4] | [00101f39] | [00000000] | 83c408 | add esp,+08 |
| [000011c7] | [00101f35] | [00000000] | 50 | push eax |
| [000011c8] | [00101f31] | [000004a3] | 68a3040000 | push 00004a3 |
| [000011cd] | [00101f31] | [000004a3] | e820f3ffff | call 00004f2 |
| Input_Halts = 0 | | | | |
| [000011d2] | [00101f39] | [00000000] | 83c408 | add esp,+08 |
| [000011d5] | [00101f39] | [00000000] | 33c0 | xor eax,eax |
| [000011d7] | [00101f3d] | [00000018] | 5d | pop ebp |
| [000011d8] | [00101f41] | [00000000] | c3 | ret |

Number of Instructions Executed(1118) == 17 Pages

Example 03: H(P,P) correctly determines that its input never halts

```
void P(u32 x)
{
  if (H(x, x))
    HERE: goto HERE;
  return;
}

int main()
{
  output("Input_Halts = ", H((u32)P, (u32)P));
}
```

```
_P()
[00001202] (01) 55          push ebp
[00001203] (02) 8bec         mov ebp,esp
[00001205] (03) 8b4508      mov eax,[ebp+08]
[00001208] (01) 50          push eax
[00001209] (03) 8b4d08      mov ecx,[ebp+08]
[0000120c] (01) 51          push ecx
[0000120d] (05) e820feffff    call 00001032
[00001212] (03) 83c408      add esp,+08
[00001215] (02) 85c0         test eax,eax
[00001217] (02) 7402         jz 0000121b
[00001219] (02) ebfe         jmp 00001219
[0000121b] (01) 5d          pop ebp
[0000121c] (01) c3          ret
Size in bytes:(0027) [0000121c]
```

```
_main()
[00001222] (01) 55          push ebp
[00001223] (02) 8bec         mov ebp,esp
[00001225] (05) 6802120000    push 00001202
[0000122a] (05) 6802120000    push 00001202
[0000122f] (05) e8fefdffff    call 00001032
[00001234] (03) 83c408      add esp,+08
[00001237] (01) 50          push eax
[00001238] (05) 68b3030000    push 000003b3
[0000123d] (05) e8c0f1ffff    call 00000402
[00001242] (03) 83c408      add esp,+08
[00001245] (02) 33c0         xor eax,eax
[00001247] (01) 5d          pop ebp
[00001248] (01) c3          ret
Size in bytes:(0039) [00001248]
```

| machine address | stack address | stack data | machine code | assembly language |
|-----------------|---------------|------------|--------------|----------------------------------|
| [00001222] | [0010200f] | [00000000] | 55 | push ebp |
| [00001223] | [0010200f] | [00000000] | 8bec | mov ebp,esp |
| [00001225] | [0010200b] | [00001202] | 6802120000 | push 00001202 // push P |
| [0000122a] | [00102007] | [00001202] | 6802120000 | push 00001202 // push P |
| [0000122f] | [00102003] | [00001234] | e8fefdffff | call 00001032 // call executed H |

```
Begin Simulation Execution Trace Stored at:2120c3
Address_of_H:1032
[00001202] [002120af] [002120b3] 55          push ebp
[00001203] [002120af] [002120b3] 8bec         mov ebp,esp
[00001205] [002120af] [002120b3] 8b4508      mov eax,[ebp+08]
[00001208] [002120ab] [00001202] 50          push eax // push P
[00001209] [002120ab] [00001202] 8b4d08      mov ecx,[ebp+08]
[0000120c] [002120a7] [00001202] 51          push ecx // push P
[0000120d] [002120a3] [00001212] e820feffff  call 00001032 // call emulated H
Infinitely Recursive Simulation Detected Simulation Stopped
```

H knows its own machine address and on this basis it can easily examine its stored execution_trace of P (see above) to determine:

- (a) P is calling H with the same arguments that H was called with.
- (b) No instructions in P could possibly escape this otherwise infinitely recursive emulation.
- (c) H aborts its emulation of P before its call to H is emulated.

```
[00001234] [0010200f] [00000000] 83c408      add esp,+08
[00001237] [0010200b] [00000000] 50          push eax
[00001238] [00102007] [000003b3] 68b3030000  push 000003b3
[0000123d] [00102007] [000003b3] e8c0f1ffff  call 00000402
Input_Halts = 0
[00001242] [0010200f] [00000000] 83c408      add esp,+08
[00001245] [0010200f] [00000000] 33c0         xor eax,eax
[00001247] [00102013] [00100000] 5d          pop ebp
[00001248] [00102017] [00000004] c3          ret
Number of Instructions Executed(870) / 67 = 13 pages
```

From a purely software engineering perspective (anchored in the semantics of the x86 language) it is proven that H(P,P) correctly predicts that its correct and complete x86 emulation of its input would never reach the "ret" instruction (final state) of this input. **Copyright 2022 PL Olcott**