This is the formal YACC BNF specification for Minimal Type Theory (MTT). MTT was created by augmenting the syntax of First Order Logic (FOL) to specify Higher Order Logic (HOL) expressions using FOL syntax. Syntax is provided to enable quantifiers to specify type. FOL is a subset of MTT. The ASSIGN_ALIAS operator := enables FOL expressions to be chained together to form HOL expressions.

$\forall P \forall x (x \in P \lor x \notin P)$ // This Second Order Logic (SOL) expression is specified as these two FOL expressions:
(1) Z := Element_Of(x, P)
(2) $\forall P \forall x$ (Z $\lor$ ~Z)   // Z is macro expanded to the RHS of := in (1)

```
%left  IDENTIFIER          //  Letter+ (Letter | Digit)* // Letter includes UTF-8
%left  SUBSET_OF           //  ⊆
%left  ELEMENT_OF          //  ∈
%left  FOR_ALL             //  ∀
%left  THERE_EXISTS        //  ∃
%left  IMPLIES             //  →
%left  PROVES              //  ⊢
%left  IFF                 //  ↔
%left  AND                 //  ∧
%left  OR                  //  ∨
%left  NOT                 //  ~
%left  ASSIGN_ALIAS        //  :=  LHS is assigned as an alias name for the RHS (macro substitution)
%%

sentence
    : atomic_sentence
    | '~' sentence %prec NOT
    | '(' sentence ')'
    | sentence    IMPLIES       sentence
    | sentence    IFF           sentence
    | sentence    AND           sentence
    | sentence    OR            sentence
    | quantifier IDENTIFIER     sentence
    | quantifier IDENTIFIER     type_of IDENTIFIER sentence  // Enhancement to FOL
    | sentence    PROVES        sentence                     // Enhancement to FOL
    | IDENTIFIER ASSIGN_ALIAS sentence                       // Enhancement to FOL
    ;

atomic_sentence
    : IDENTIFIER '(' term_list ')' // ATOMIC PREDICATE
    | IDENTIFIER                   // SENTENTIAL VARIABLE  // Enhancement to FOL
    ;

term  : IDENTIFIER '(' term_list ')' // FUNCTION
    | IDENTIFIER                   // CONSTANT or VARIABLE
    ;

term_list
    : term_list ',' term
    | term
    ;

type_of
    : ELEMENT_OF                                            // Enhancement to FOL
    | SUBSET_OF                                             // Enhancement to FOL
    ;

quantifier
    : THERE_EXISTS
    | FOR_ALL
    ;
```