

This is the formal YACC BNF specification for Minimal Type Theory (MTT). MTT was created by augmenting the syntax of First Order Logic (FOL) to specify Higher Order Logic (HOL) expressions with types. FOL is a subset of MTT. The ASSIGN\_ALIAS operator := enables FOL expressions to be chained together to form HOL expressions.

$\forall P \forall x(x \in P \vee x \notin P)$  // Second Order Logic (SOL) expression specified as these two MTT expressions:

(1)  $Z := \text{Element\_Of}(x, P)$

(2)  $\forall P \forall x(Z \vee \sim Z)$  // Z is macro expanded to: "Element\_Of(x, P)" thus becoming (3)

(3)  $\forall P \forall x(\text{Element\_Of}(x, P) \vee \sim \text{Element\_Of}(x, P))$

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%left IDENTIFIER          // Letter+ (Letter | Digit)* // Letter includes UTF-8
%left SUBSET_OF           // ⊆
%left ELEMENT_OF         // ∈
%left FOR_ALL             // ∀
%left THERE_EXISTS       // ∃
%left IMPLIES             // →
%left PROVES              // ⊢
%left IFF                 // ↔
%left AND                 // ∧
%left OR                  // ∨
%left NOT                 // ~
%left ASSIGN_ALIAS        // := LHS is assigned as an alias name for the RHS (macro substitution)
%%

sentence
: atomic_sentence
| '~' sentence %prec NOT
| '(' sentence ')'
| sentence IMPLIES sentence
| sentence IFF sentence
| sentence AND sentence
| sentence OR sentence
| quantifier IDENTIFIER sentence
| quantifier IDENTIFIER type_of IDENTIFIER sentence // Enhancement to FOL
| sentence PROVES sentence // Enhancement to FOL
| IDENTIFIER ASSIGN_ALIAS sentence // Enhancement to FOL
;

atomic_sentence
: IDENTIFIER '(' term_list ')' // ATOMIC PREDICATE
| IDENTIFIER // SENTENTIAL VARIABLE // Enhancement to FOL
;

term
: IDENTIFIER '(' term_list ')' // FUNCTION
| IDENTIFIER // CONSTANT or VARIABLE
;

term_list
: term_list ',' term
| term
;

type_of
: ELEMENT_OF // Enhancement to FOL
| SUBSET_OF // Enhancement to FOL
;

quantifier
: THERE_EXISTS
| FOR_ALL
;

```