

Eliminating Undecidability and Incompleteness in Formal Systems (transforming formal proofs to theorem consequences into sound deductive logical inference)

The only thing required to eliminate incompleteness, undecidability and inconsistency from formal systems is transforming the formal proofs to theorem consequences of symbolic logic into the sound deductive inference model.

Within sound deductive inference a conclusion only counts as true when a connected sequence of valid deductions proceeds from a set of true premises all the way to the conclusion. Undecidable sentences would therefore simply count as unsound.

Stipulating this definition of Axiom:

An expression of language defined to have the semantic value of Boolean True.

Stipulating this specification of True and False:

Axiom(1) $\text{True}(F, x) \leftrightarrow (F \vdash x)$.

Axiom(2) $\text{False}(F, x) \leftrightarrow (F \vdash \neg x)$.

Stipulating that formal systems are Boolean:

Axiom(3) $\forall F \in \text{Formal_System} \forall x \in \text{Closed_WFF}(F) (\text{True}(F,x) \vee \text{False}(F,x))$

The stipulated definition of Axiom provides the symbolic logic equivalent of true premises. Axiom(1) and Axiom(2) require a connected sequence of inference steps to proceed all the way to the theorem consequences. Axiom(3) merely formalizes the requirement that all formal systems are Boolean.

The third step of the Tarski Undefinability Theorem proof:

(3) $x \notin \text{Pr}$ if and only if $x \in \text{Tr}$

Refuted on the basis of contradicting simplified Axiom(1): $x \in \text{Tr} \leftrightarrow x \in \text{Pr}$

The following logic sentence is refuted on the basis of Axiom(3)

$\exists F \exists G (G \leftrightarrow ((F \nVdash G) \wedge (F \nVdash \neg G)))$

Because it asserts there are sentences G of formal system F that are not true or false.

Making the following paragraph false:

The first incompleteness theorem states that in any consistent formal system F within which a certain amount of arithmetic can be carried out, there are statements of the language of F which can neither be proved nor disproved in F. (Raatikainen 2018)

Raatikainen, Panu, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy (Fall 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2018/entries/goedel-incompleteness/>.

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