Eliminating Undecidability and Incompleteness in Formal Systems

To eliminate incompleteness, undecidability and inconsistency from formal systems we only need to convert the formal proofs to theorem consequences of symbolic logic to conform to the sound deductive inference model.

Sound deductive inference
(a) All dogs are mammals
(b) All mammals breath
(c) Therefore all dogs breath

Unsound deductive inference (false premise)
(a) All dogs are office buildings
(b) All office buildings have windows
(c) Therefore all dogs have windows

Unsound deductive inference (invalid inference)
(a) All dogs are mammals
(b) All fish swim
(c) Therefore all dogs breath

Within the sound deductive inference model there is a *(connected sequence of valid deductions from true premises to a true conclusion)* thus unlike the formal proofs of symbolic logic provability cannot diverge from truth.

Within the Deductive Inference Model (DIM) **Provability**: i*s a connected sequence of valid deductions from premises to a conclusion.*

Within the Sound Deductive Inference Model (SDIM) **Soundness** *is **Provability** from true premises to a true conclusion.*

Thus Within the Sound Deductive Inference Model (SDIM)
(unlike the formal proofs of symbolic logic) provability cannot diverge from truth.

When we consider sound deductive inference to the negation of a conclusion we now also have a definitive specification of falsity.

Within the sound deductive inference model we can be certain that valid inference from true premises derives a true conclusion.

∴ Within the sound deductive inference model any argument that does not evaluate to True or False is unsound, there is no undecidability or incompleteness in this model.

Summing it up:
True(X) is Provable(X) from True Premises.
False(X) is Provable(¬X) from True Premises.
Otherwise Unsound(X).

The key criterion measure that the sound deductive inference model would add to the formal proofs to theorem consequences of symbolic logic would be the semantic notion of soundness. That formal proofs cannot recognize and reject unsound logic sentences is an expressiveness gap of formal proofs relative to the sound deductive inference model.

Formalizing the Sound Deductive Inference Model in Symbolic Logic

*Axiom(0) Stipulates** this definition of Axiom:*
Expressions of language defined to have the semantic value of Boolean True.
Provides the symbolic logic equivalent of true premises.

*Stipulating** this specification of True and False:*
Axiom(1) $\forall F \in \text{Formal\_System}\ \forall x \in \text{Closed\_WFF}(F)\ (\text{True}(F, x) \leftrightarrow (F \vdash x))$
Axiom(2) $\forall F \in \text{Formal\_System}\ \forall x \in \text{Closed\_WFF}(F)\ (\text{False}(F, x) \leftrightarrow (F \vdash \neg x))$
Thus stipulating** that consequences are provable from axioms.

*Stipulating** that formal systems are Boolean:*
Axiom(3) $\forall F \in \text{Formal\_System}\ \forall x \in \text{Closed\_WFF}(F)\ (\text{True}(F,x) \lor \text{False}(F,x))$
Screens out semantically unsound sentences as not belonging to the formal system.

Applying the Formalized Sound Deductive Inference Model

The third step of the Tarski Undefinability Theorem proof:
(3) $x \notin Pr$ if and only if $x \in Tr$
is refuted by contradicting simplified Axiom(1): $x \in Tr \leftrightarrow x \in Pr$

The following logic sentence is refuted on the basis of Axiom(3)
$\exists F \in \text{Formal\_System}\ \exists G \in \text{Closed\_WFF}(F)\ (G \leftrightarrow ((F \nvdash G) \land (F \nvdash \neg G)))$
There is no sentence G of Formal System F that is neither True nor False in F.

Making the following paragraph false:
The first incompleteness theorem states that in any consistent formal system F within which a certain amount of arithmetic can be carried out, there are statements of the language of F which can neither be proved nor disproved in F. (Raatikainen 2018)

https://en.wikipedia.org/wiki/Stipulative_definition
** A stipulative definition is a type of definition in which a new or currently-existing term is given a new specific meaning for the purposes of argument or discussion in a given context. When the term already exists, this definition may, but does not necessarily, contradict the dictionary (lexical) definition of the term.

Raatikainen, Panu, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy (Fall 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/fall2018/entries/goedel-incompleteness/.

Validity and Soundness https://www.iep.utm.edu/val-snd/
A deductive argument is said to be valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false. Otherwise, a deductive argument is said to be invalid. $P \rightarrow C$

A deductive argument is sound if and only if it is both valid, and all of its premises are actually true. Otherwise, a deductive argument is unsound. P

(1) impossible for the premises to be true and
the conclusion nevertheless to be false: $P \vdash C$
(2) all of its premises are actually true: P
(3) Derives $(P \wedge (P \vdash C)) \leftrightarrow \text{Sound}(P, C)$

Within the deductive inference model Provability is: $(P \vdash C)$
[a connected sequence of valid deductions from premises to a conclusion]

Within the sound deductive inference model Soundness is: $(\text{True}(P) \vdash \text{True}(C))$
[a connected sequence of valid deductions from true premises to a true conclusion]

P	C	$(P \wedge (P \vdash C))$	
F	F	F	// Unsound because false premise
F	T	F	// Unsound because false premise
T	F	F	// Unsound because invalid inference
T	T	T	// Sound because true premise and valid inference

∴ Thus in sound deductive inference model provability cannot diverge from Truth.

WFF: https://en.wikipedia.org/wiki/Well-formed_formula
Tarski Undefinability Proof: http://liarparadox.org/Tarski_Proof_275_276.pdf