Proof that Wittgenstein is correct about Gödel

When we sum up the results of Gödel's 1931 Incompleteness Theorem by formalizing Wittgenstein's verbal specification such that this formalization meets Gödel's own sufficiency requirement: "Every epistemological antinomy can likewise be used for a similar undecidability proof." then we can see that Gödel's famous logic sentence is only unprovable in PA because it is untrue in PA because it specifies the logical equivalence to self contradiction in PA.

Wittgenstein's minimal essence of the 1931 Incompleteness Theorem sentence
"I have constructed a proposition (I will use 'P' to designate it) in Russell's symbolism, and by means of certain definitions and transformations it can be so interpreted that it says 'P is not provable in Russell's system'. (Wittgenstein 1956:50e)
Formalized by Olcott: $P \leftrightarrow (RS \nvdash P)$

Wittgenstein definitions of True() and False()
'True in Russell's system' means, as was said: proved in Russell's system; and 'false in Russell's system' means: the opposite has been proved in Russell's system. (Wittgenstein 1956:51e) Formalized by Olcott as:
$\forall x\ (True(RS, x)\ := (RS \vdash x))$
$\forall x\ (False(RS, x) := (RS \vdash \neg x))$ // LHS := RHS means LHS is defined as the RHS.

The first two rows of the truth table are determined by the Wittgenstein definition of True and False specified by their formalized axiom schemata shown in Wittgenstein definitions of True() and False().

The failure of logical equivalence on the last two rows of the truth table shows that both P and ¬P are contradicted (false) (in the above formula) thus meeting the [epistemological antinomy] sufficiency condition that Gödel stipulated for proof equivalence:

"14 Every epistemological antinomy can likewise be used for a similar undecidability proof." (Gödel 1931:40)

In the following truth table ⊬ of "P ↔ (RS ⊬ P)" has been changed to ⊢ to eliminate the cognitive complexity of double negation.

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P ↔ (RS ⊢ P)
T T     T T   // (P is true  ↔  P is  provable) is true
F T     T F   // (P is false ↔ ¬P is  provable) is true
T F     F T   // (P is true  ↔  P is ¬provable) is false
F F     F F   // (P is false ↔ ¬P is ¬provable) is false
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Rows 3 and 4 apply to this formula: $P \leftrightarrow (RS \nvdash P)$. They evaluate to false because (within the Wittgenstein/Olcott True/False predicates) they specify self-contradiction. The fact that self-contradictory sentences cannot be proven true creates no actual incompleteness what-so-ever.

Here is the Liar Paradox: “This sentence is not true” version:

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P ↔ (RS ⊢ ¬P)
T T     T   // (P is true  ↔ ¬P is  provable) is false
F T     T   // (P is false ↔  P is  provable) is false
T F     F   // (P is true  ↔ ¬P is ¬provable) is true
F F     F   // (P is false ↔  P is ¬provable) is true
```

Rows 1 and 2 evaluate to false because they specify self-contradiction.

We are now switching from Wittgenstein’s names:
(1) Russell’s System (RS) becomes Peano Arithmetic (PA)
(2) Wittgenstein’s P becomes Gödel’s G

If G is not provable in PA then G is not true in PA. If G is provable in the Gödelization of PA then G is true in the Gödelization of PA. Diagonalization is an alternative form of provability.

The Gödelization of PA is a distinctly different formal system than PA as shown by the difference of the provability of G.

The key difference between PA and the Gödelization of PA is that G is inside the scope of self-contradiction in PA and G is outside the scope of self-contradiction in the Gödelization of PA.

When we address the comparable proof in the Tarski Undefinability Theorem we also address another aspect of the Incompleteness Theorem.

For formal systems that are not expressive enough to have their own provability operator as in Gödel’s PA and Tarski’s Theory there are indeed expression’s of PA and Tarski’s Theory that can be shown to be true in the Gödelization of PA and Tarski’s meta-theory respectively that are not provable (and therefore untrue) in PA and Tarski’s Theory.

Here is a key brand new insight anchored in the Tarski Undefinability Theorem. Tarski concluded that an infinite hierarchy of Meta-Theories was required to always have provability. This can be easily shown to be untrue.

We simply have two different versions of Tarski’s Theory and two different versions of his Meta-Theory having the exact same relations as the original Theory and Meta-Theory, yet all of these relations are differently named.

Meta-Theory-A is in terms of the relations of Theory-A.
Meta-Theory-B is in terms of the relations of Theory-B.

Now we have two Meta-Theories at the exact same hierarchy level that can each prove the unprovable expressions of the other. They can do this because these expressions are outside of the scope of self-contradiction.

Godel, Kurt 1931 OF PRINCIPIA MATHEMATICA AND RELATED SYSTEMS I, page 40.

Wittgenstein, Ludwig 1956 Remarks on the Foundations of Mathematics Part I, Appendix I Paragraph 8, pages 50e-51e.