

Proof that Wittgenstein is correct about Gödel

When we sum up the results of Gödel's 1931 Incompleteness Theorem by formalizing Wittgenstein's verbal specification such that this formalization meets Gödel's own sufficiency requirement: "Every epistemological antinomy can likewise be used for a similar undecidability proof." then we can see that Gödel's famous logic sentence is only unprovable in PA because it is untrue in PA because it specifies the logical equivalence to self contradiction in PA.

Wittgenstein's minimal essence of the 1931 Incompleteness Theorem sentence

"I have constructed a proposition (I will use 'P' to designate it) in Russell's symbolism, and by means of certain definitions and transformations it can be so interpreted that it says 'P is not provable in Russell's system'. (Wittgenstein 1956:50e)

Formalized by Olcott: $P \leftrightarrow (RS \nvdash P)$

As it turns out to actually be the case the entire body of all conceptual truth is entirely comprised of expressions of language that have been stipulated to have the semantic property of Boolean true (paraphrase of Curry 177:45) and truth preserving operations applied to this set.

When these truth preserving operations are defined as finite string transformation rules we establish a bijection such that truth and provability cannot diverge. When we do this we retain all of the original expressiveness of formal systems and screen out semantic paradoxes as ill-formed truth bearers. This allows the universal truth predicate that Tarski proved to not exist to be defined as:

Wittgenstein definitions of True() and False()

'True in Russell's system' means, as was said: proved in Russell's system; and 'false in Russell's system' means: the opposite has been proved in Russell's system.

(Wittgenstein 1956:51e) Formalized by Olcott as:

$\forall x (\text{True}(RS, x) := (RS \vdash x))$

$\forall x (\text{False}(RS, x) := (RS \vdash \neg x))$ // LHS := RHS means LHS is defined as the RHS.

Since the Wittgenstein-Olcott axiom schema define $\text{True}(RS, x)$ as $\text{Provable}(RS, x)$ then $\neg \text{Provable}(RS, x)$ would be defined as $\neg \text{True}(RS, x)$. This means that the Wittgenstein-Olcott minimal essence of the 1931 Incompleteness Theorem <IS> The Liar Paradox.

The Formalized Liar Paradox says that P is materially equivalent to Not True.

The truth table shows that this is self-contradictory.

$P \leftrightarrow \neg \text{True}(P)$	$P \leftrightarrow RS \nvdash P$
T F F	T F F
F F T	F F T

The truth table of minimal essence of the 1931 Incompleteness theorem is identical to the truth table of the Liar Paradox because the third columns of these truth tables are stipulated by the Wittgenstein-Olcott axiom schema to mean exactly the same thing.

The failure of logical equivalence shows that both P and $\neg P$ are contradicted (false) (in the above formula) thus meeting the [epistemological antinomy] sufficiency condition that Gödel stipulated for proof equivalence:

“14 Every epistemological antinomy can likewise be used for a similar undecidability proof.” (Gödel 1931:40)

The fact that self-contradictory sentences specified in the language of a formal system cannot be proven in that formal system does not make the formal system itself incomplete or inconsistent as long as unprovable (from axioms) is construed as untrue.

Furthermore that the Gödel sentence can be proved to be unsatisfiable because its satisfaction would derive a contradiction proves that this sentence is unprovable.

The Gödel sentence is provably unprovable because this proof is one level of indirection away from the original sentence, thus neither self-referential nor self-contradictory:

(a) “This sentence is not provable” is not true or provable.

(b) Provable(“This sentence is not provable”) is true and provable.

"This sentence is unprovable" because its satisfaction would be self contradictory, thus it is provably unsatisfiable thus provably unprovable.

\sim Provable("This sentence is unprovable") is satisfiable and provable and true because it is not self-referential thus not self-contradictory.

At the most abstract level of analysis:

Conceptual Truth is ONLY semantic relations between concepts that can always be expressed as[1] syntactic relations between finite strings[2] thereby logically entailing that truth cannot possibly ever diverge from provability.

[1] Forming a perfect isomorphism between semantic and syntactic relations

$\forall x (\text{True}(x) \cong \text{Provable}(x))$

[2] Such as words, word phrases or predicate logic expressions.

Examples:

"one" [is a] "Integer"

"cats" [are] "Animals"

"cats" [have] "legs"

"2 + 3" [equals] "5"

"A \wedge B" " \leftrightarrow " "B \wedge A"

To make the above abstraction more concrete we focus on the single relation between concepts of [sound deduction] from the sound deductive inference model.

Sound deduction begins with stipulated truth, applies a sequence of truth preserving operations, thus necessarily ends up with truth.

Truth ONLY comes from:

- (1) Stipulated truth (the definitions of the meaning of words)
- (2) Applying a sequence of truth preserving operations to stipulated truth.

Truth ALWAYS comes from:

- (1) Stipulated truth (the definitions of the meaning of words)
- (2) Applying a sequence of truth preserving operations to stipulated truth.

When we construe a formal systems axioms to essentially be stipulated truth then this same formal systems theorems would also be true because they were derived by applying truth preserving operations to its axioms. Since this is the way that Truth really works we have proven that true can never diverge from provability.

Godel, Kurt 1931. On Formally Undecidable Propositions of Principia Mathematica And Related Systems I, page 40. Footnote 14.

Wittgenstein, Ludwig 1956. Remarks on the Foundations of Mathematics Part I, Appendix I Paragraph 8, pages 50e-51e. (quoted from another edition below).

Curry, Haskell 1977. Foundations of Mathematical Logic. New York: Dover Publications, 45
We begin by postulating a certain non void, definite class $\{E\}$ of statements, which we call elementary statements...

The statements of $\{E\}$ are called elementary statements to distinguish them from other statements which we may form from them or about them in the U language...

Then the elementary statements which belong to $\{T\}$ we shall call the elementary theorems of $\{T\}$; we also say that these elementary statements are true for $\{T\}$. Thus, given $\{T\}$, an elementary theorem is an elementary statement which is true. A theory is thus a way of picking out from the statements of $\{E\}$ a certain subclass of true statements.

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Wittgenstein, Ludwig 1983. Remarks on the Foundations of Mathematics (Appendix III), 118-119. Cambridge, Massachusetts and London, England: The MIT Press

8. I imagine someone asking my advice; he says: "I have constructed a proposition (I will use 'P' to designate it) in Russell's symbolism, and by means of certain definitions and transformations it can be so interpreted that it says: 'P is not provable in Russell's system'. Must I not say that this proposition on the one hand is true, and on the other hand is unprovable? For suppose it were false; then it is true that it is provable. And that surely cannot be! And if it is proved, then it is proved that it is not provable. Thus it can only be true, but unprovable. "

Just as we ask: " 'provable' in what system?", so we must also ask: " 'true' in what system?" 'True in Russell's system' means, as was said: proved in Russell's system; and 'false in Russell's system' means: the opposite has been proved in Russell's system.-Now what does your "suppose it is false" mean? *In the Russell sense* it means 'suppose the opposite is proved in Russell's system'; *if that is your assumption*, you will now presumably give up the interpretation that it is unprovable.

And by 'this interpretation' I understand the translation into this English sentence.-If you assume that the proposition is provable in Russell's system, that means it is true *in the Russell sense*, and the interpretation "P is not provable" again has to be given up. If you assume that the proposition is true in the Russell sense, *the same* thing follows. Further: if the proposition is supposed to be false in some other than the Russell sense, then it does not contradict this for it to be proved in Russell's system. (What is called "losing" in chess may constitute winning in another game.)