Proof that Wittgenstein is correct about Gödel

In order to understand that Wittgenstein's refutation of Gödel's 1931 Incompleteness Theorem is correct we must fill in some details of the notion of a formal system that Wittgenstein had in mind. Expressions of language encoding analytical knowledge are only true if they have been defined to be true or valid deductions from true premises prove that they are true. When these relations are formalized as formal proofs to theorem consequences then Provable(x) cannot possibly diverge from True(x).

Some expressions of language are stipulated to be true. ≅AXIOMS
Basic Facts: are stipulated relations between finite strings that are defined to have the semantic property of Boolean true. (see also Curry 1977:45).

Some relations between expressions of language are stipulated to be truth preserving.
≅rules-of-inference. Valid deduction is expressed as relations between finite strings.

Validity and Soundness https://www.iep.utm.edu/val-snd/
A deductive argument is said to be valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false. Otherwise, a deductive argument is said to be invalid.

A deductive argument is sound if and only if it is both valid, and all of its premises are actually true. Otherwise, a deductive argument is unsound.

Analytical knowledge: The set of all knowledge that can be completely expressed using language and totally verified as true entirely based on its meaning thus not requiring any sense data from the sense organs. This specification divides the analytic versus synthetic distinction overcoming Quine's objections. All of mathematics meets this specification.

Modal Logic
◊P ↔ ¬□¬P // Possibly(P) ↔ ¬Necessarily(¬P)
□P ↔ ¬◊¬P // Necessarily(P) ↔ ¬Possibly(¬P)
∀P (P ∈ Analytical_Knowledge(P) ↔ □P)

When it is understood that every element of the set of analytical knowledge is either a semantic tautology (defined to be true) or deduced from semantic tautologies then we see that these semantic tautologies and deductive rules-of-inference can be expressed as relations between finite strings. This unifies sound deduction with formal proofs to theorem consequenes. True(x) ≡ □x ≡ Conclusion_of_Sound_Deduction(x) ≡ Theorem(x).

Now that we can screen out semantic paradoxes as ill-formed truth bearers, we can define the universal truth predicate that Tarski misconstrued to be impossible as:

Wittgenstein definitions of True() and False()
'True in Russell's system' means, as was said: proved in Russell's system; and 'false in Russell's system' means: the opposite has been proved in Russell's system. (Wittgenstein 1983,118-119) Formalized by Olcott as:

LHS := RHS means LHS is defined as the RHS

$\forall x\ (\text{True}(RS, x) := (RS \vdash x))$ **// x is a theorem of RS**

$\forall x\ (\text{False}(RS, x) := (RS \vdash \neg x))$ **// ¬x is a theorem of RS**

Wittgenstein's minimal essence of the 1931 Incompleteness Theorem sentence
"I have constructed a proposition (I will use 'P' to designate it) in Russell's symbolism, and by means of certain definitions and transformations it can be so interpreted that it says 'P is not provable in Russell's system'. (Wittgenstein 1983,118-119)

Formalized by Olcott: $P \leftrightarrow (RS \nvdash P)$

When we sum up the results of Gödel's 1931 Incompleteness Theorem by formalizing Wittgenstein's verbal specification such that this formalization meets Gödel's own sufficiency requirement: "Every epistemological antinomy can likewise be used for a similar undecidability proof." then we can see that Gödel's famous logic sentence is only unprovable in PA because it is untrue in PA because it specifies the logical equivalence to self contradiction in PA.

Since the Wittgenstein-Olcott axiom schema define True(RS, x) as Provable(RS, x) then ¬Provable(RS, x) would be defined as ¬True(RS, x). This means that the Wittgenstein-Olcott minimal essence of the 1931 Incompleteness Theorem <IS> The Liar Paradox.

The Formalized Liar Paradox says that P is materially equivalent to Not True.
The truth table shows that this is self-contradictory.

P	↔	¬True(P)
T	F	F
F	F	T

P	↔	RS⊬P
T	F	F
F	F	T

The truth table of minimal essence of the 1931 Incompleteness theorem is identical to the truth table of the Liar Paradox because the third columns of these truth tables are stipulated by the Wittgenstein-Olcott axiom schema to mean exactly the same thing.

The failure of logical equivalence shows that both P and ¬P are contradicted (false) (in the above formula) thus meeting the [epistemological antinomy] sufficiency condition that Gödel stipulated for proof equivalence: "14 Every epistemological antinomy can likewise be used for a similar undecidability proof." (Gödel 1931:40)

The fact that self-contradictory sentences specified in the language of a formal system cannot be proven in that formal system does not make the formal system itself incomplete or inconsistent as long as unprovable (from axioms) is construed as untrue.

At the most abstract level of analysis:
Conceptual Truth is ONLY semantic relations between concepts that can always be expressed as[1] syntactic relations between finite strings[2] thereby logically entailing that truth cannot possibly ever diverge from provability.

[1] Forming an isomorphism between semantic and syntactic relations:
$\forall x\ (\text{True}(x) \cong \text{Provable}(x))$

[2] Such as words, word phrases or predicate logic expressions.

Examples:
"one" [is a] "Integer"
"cats" [are] "Animals"
"cats" [have] "legs"
"2 + 3" [equals] "5"
"A ∧ B" "↔" "B ∧ A"

To make the above abstraction more concrete we focus on the single relation between concepts of [sound deduction] from the sound deductive inference model. Sound deduction begins with stipulated truth, applies a sequence of truth preserving operations, thus necessarily ends up with truth.

Truth ONLY comes from:
(1) Stipulated truth (the definitions of the meaning of words)
(2) Applying a sequence of truth preserving operations to stipulated truth.

Truth ALWAYS comes from:
(1) Stipulated truth (the definitions of the meaning of words)
(2) Applying a sequence of truth preserving operations to stipulated truth.

When we construe a formal systems axioms to essentially be stipulated truth then this same formal systems theorems would also be true because they were derived by applying truth preserving operations to its axioms. Since this is the way that Truth really works we have proven that true can never diverge from provability.

Godel, Kurt 1931. On Formally Undecidable Propositions of Principia Mathematica And Related Systems I, page 40. Footnote 14.

Wittenstein, Ludwig 1983. Remarks on the Foundations of Mathematics (Appendix III), 118-119. Cambridge, Massachusetts and London, England: The MIT Press (**quoted in full below**).

Tarski, Alfred 1983. “The concept of truth in formalized languages” in Logic Semantics, Metamathematics. Indianapolis: Hacket Publishing Company, 275-276.

Curry, Haskell 1977. Foundations of Mathematical Logic. New York: Dover Publications, 45
We begin by postulating a certain non void, definite class {E} of statements, which we call elementary statements...

The statements of {E} are called elementary statements to distinguish them from other statements which we may form from them or about them in the U language...

Then the elementary statements which belong to {T} we shall call the elementary theorems of {T}; we also say that these elementary statements are true for {T}. Thus, given {T}, an elementary theorem is an elementary statement which is true. A theory is thus a way of picking out from the statements of {E} a certain subclass of true statements…

The terminology which has just been used implies that the elementary statements are not such that their truth and falsity are known to us without reference to {T}.

Wittgenstein, Ludwig 1983. Remarks on the Foundations of Mathematics (Appendix III), 118-119. Cambridge, Massachusetts and London, England: The MIT Press

8. I imagine someone asking my advice; he says: "I have constructed a proposition (I will use 'P' to designate it) in Russell's symbolism, and by means of certain definitions and transformations it can be so interpreted that it says: 'P is not provable in Russell's system'. Must I not say that this proposition on the one hand is true, and on the other hand is unprovable? For suppose it were false; then it is true that it is provable. And that surely cannot be! And if it is proved, then it is proved that it is not provable. Thus it can only be true, but unprovable. "

Just as we ask: " 'provable' in what system?", so we must also ask: " 'true' in what system?" 'True in Russell's system' means, as was said: proved in Russell's system; and 'false in Russell's system' means: the opposite has been proved in Russell's system.-Now what does your "suppose it is false" mean? *In the Russell sense* it means 'suppose the opposite is proved in Russell's system'; *if that is your assumption*, you will now presumably give up the interpretation that it is unprovable.

And by 'this interpretation' I understand the translation into this English sentence.-If you assume that the proposition is provable in Russell's system, that means it' is true *in the Russell sense*, and the interpretation "P is not provable" again has to be given up. If you assume that the proposition is true in the Russell sense, *the same* thing follows. Further: if the proposition is supposed to be false in some other than the Russell sense, then it does not contradict this for it to be proved in Russell's system. (What is called "losing" in chess may constitute winning in another game.)