

Halting problem undecidability and infinitely nested simulation

When halting is defined as any computation that halts without ever having its simulation aborted then it can be understood that partial halt decider H correctly decides not halting on the simplified version of the Linz \hat{H} . When this simplified concrete example is fully understood then the exact same reasoning can be applied to the actual Linz H correctly deciding not halting on its input.

The x86utm operating system was created so that the halting problem could be examined concretely in the high level language of C. x86utm UTM tape elements are 32-bit unsigned integers. H examines the behavior of the x86 emulation of its input. As soon as a non-halting behavior pattern is matched H aborts the simulation of its input and decides not halting.

```
int Simulate(u32 P, u32 I)
{
  ((void (*)(u32))P)(I);
  return 1;
}

// Simplified Linz  $\hat{H}$  (Linz:1990:319)
void H_Hat(u32 P)
{
  // Linz  $H$  as a simulating partial halt decider
  u32 Input_Halts = H(P, P);
  if (Input_Halts)
    HERE: goto HERE;
}

void H_Hat2(u32 P)
{
  u32 Input_Halts = Simulate(P, P);
  if (Input_Halts)
    HERE: goto HERE;
}

int main()
{
  H_Hat2((u32)H_Hat2);
  H_Hat((u32)H_Hat);
}
```

Anyone that knows C programming very well will know that line 1 of main() won't halt and line 2 of main() will only halt if simulating partial halt decider $H()$ stops simulating $H_Hat()$. A simulating halt decider that never stops simulating its input is simply a simulator on this input.

When we know that the UTM simulation of TM Description P on input I would never halt we know that the execution of TM $P(I)$ would never halt.

On this basis we know that any computation that must have its simulation aborted to prevent its otherwise infinite execution is correctly rejected as non-halting.

Peter Linz \hat{H} applied to the Turing machine description of itself: $[\hat{H}]$

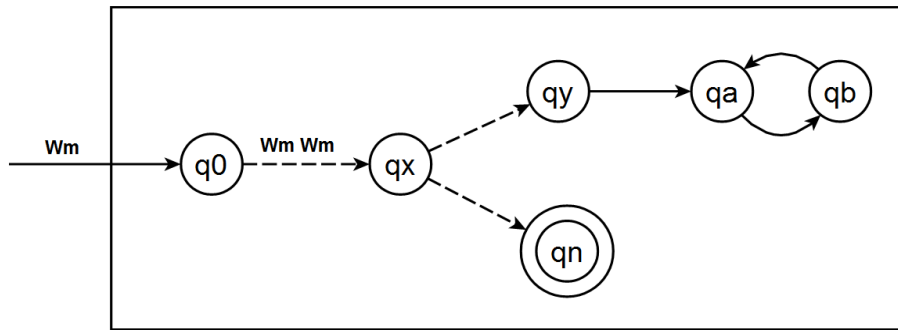


Figure 12.3 Turing Machine \hat{H}

$\hat{H}.q_0 \ wM \vdash^* \hat{H}.q_x \ wM \ wM \vdash^* \hat{H}.q_y \ \infty$
 $\hat{H}.q_0 \ wM \vdash^* \hat{H}.q_x \ wM \ wM \vdash^* \hat{H}.q_n$

The above is adapted from (Linz:1990:319).

It shows that Turing machine \hat{H} copies its input at (q_0) and begins executing an embedded copy of the original halt decider with this input at (q_x).

The (q_y) state indicates that the halt decider has determined that its input would halt. The (q_n) state indicates the input would not halt. The appended (q_a) and (q_b) states cause \hat{H} to infinitely loop if the halt decider decides that its input would halt.

The above definition specifies this execution trace:

It can be understood from the above specification that when the embedded halt decider at $\hat{H}.q_x$ bases its halting decision on simulating its input, and it has $([\hat{H}], [\hat{H}])$ as its input that:

- $\hat{H}.q_0$ would copy its input and then $\hat{H}.q_x$ would simulate its input with this copy then
- $\hat{H}.q_0$ would copy its input and then $\hat{H}.q_x$ would simulate its input with this copy then
- $\hat{H}.q_0$ would copy its input and then $\hat{H}.q_x$ would simulate its input with this copy...

unless and until the halt decider at $\hat{H}.q_x$ stops simulating its input.

When halting is defined as any computation that halts without ever having its simulation aborted by a simulating halt decider then we can see that the halt decider at state $\hat{H}.q_x$ stops simulating its input and correctly transitions to its final $\hat{H}.q_n$ state deciding not halting on its input in the computation: $\hat{H}([\hat{H}])$.

When halting is defined as any computation that halts without ever having its simulation aborted then the fact that $\hat{H}([\hat{H}])$ stops running does not indicate that it is a halting computation. The simulating halt decider would also force an infinite loop to stop running.

Linz, Peter 1990. An Introduction to Formal Languages and Automata. Lexington/Toronto: D. C. Heath and Company.

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Appendix

```
int Factorial(int n)
{
    Output("Factorial(n)",n);
    if (n > 1)
        return n * Factorial(n - 1);
    else
        return 1;
}

void Infinite_Loop()
{
    HERE: goto HERE;
}

int Infinite_Recursion(u32 N)
{
    u32 M = Infinite_Recursion(N);
    return M;
}

void H_Hat(u32 P)
{
    u32 Input_Halts = H(P, P);
    if (Input_Halts)
        HERE: goto HERE;
}

int main()
{
    u32 Input_Would_Halt1 = Halts((u32)Factorial, 3);
    Output("Input_Would_Halt1 = ", Input_Would_Halt1);

    u32 Input_Would_Halt2 = Halts((u32)Infinite_Loop, (u32)Infinite_Loop);
    Output("Input_Would_Halt2 = ", Input_Would_Halt2);

    u32 Input_Would_Halt3 = Halts((u32)Infinite_Recursion, 3);
    Output("Input_Would_Halt3 = ", Input_Would_Halt2);

    u32 Input_Would_Halt4 = Halts((u32)H_Hat, (u32)H_Hat);
    Output("Input_Would_Halt4 = ", Input_Would_Halt4);
}
```

```

_Factorial()
[0000ca0] (01) 55          push ebp
[0000ca1] (02) 8bec         mov ebp,esp
[0000ca3] (03) 8b4508      mov eax,[ebp+08]
[0000ca6] (01) 50          push eax
[0000ca7] (05) 68a3030000  push 000003a3
[0000cac] (05) e87ff7ffff  call 00000430
[0000cb1] (03) 83c408      add esp,+08
[0000cb4] (04) 837d0801    cmp dword [ebp+08],+01
[0000cb8] (02) 7e17         jng 0000cd1
[0000cba] (03) 8b4d08      mov ecx,[ebp+08]
[0000cbd] (03) 83e901      sub ecx,+01
[0000cc0] (01) 51          push ecx
[0000cc1] (05) e8daf7ffff  call 0000ca0
[0000cc6] (03) 83c404      add esp,+04
[0000cc9] (04) 0faf4508    imul eax,[ebp+08]
[0000ccd] (02) eb07         jmp 0000cd6
[0000ccf] (02) eb05         jmp 0000cd6
[0000cd1] (05) b801000000  mov eax,00000001
[0000cd6] (01) 5d          pop ebp
[0000cd7] (01) c3          ret
Size in bytes:(0056) [0000cd7]

```

```

_Infinite_Loop()
[0000ce0] (01) 55          push ebp
[0000ce1] (02) 8bec         mov ebp,esp
[0000ce3] (02) ebfe         jmp 0000ce3
[0000ce5] (01) 5d          pop ebp
[0000ce6] (01) c3          ret
Size in bytes:(0007) [0000ce6]

```

```

_Infinite_Recursion()
[0000cf0] (01) 55          push ebp
[0000cf1] (02) 8bec         mov ebp,esp
[0000cf3] (01) 51          push ecx
[0000cf4] (03) 8b4508      mov eax,[ebp+08]
[0000cf7] (01) 50          push eax
[0000cf8] (05) e8f3ffff  call 0000cf0
[0000cfd] (03) 83c404      add esp,+04
[0000d00] (03) 8945fc      mov [ebp-04],eax
[0000d03] (03) 8b45fc      mov eax,[ebp-04]
[0000d06] (02) 8be5         mov esp,ebp
[0000d08] (01) 5d          pop ebp
[0000d09] (01) c3          ret
Size in bytes:(0026) [0000d09]

```

```

_H_Hat()
[0000d10] (01) 55          push ebp
[0000d11] (02) 8bec         mov ebp,esp
[0000d13] (01) 51          push ecx
[0000d14] (03) 8b4508      mov eax,[ebp+08]
[0000d17] (01) 50          push eax
[0000d18] (03) 8b4d08      mov ecx,[ebp+08]
[0000d1b] (01) 51          push ecx
[0000d1c] (05) e80ffeffff  call 00000b30
[0000d21] (03) 83c408      add esp,+08
[0000d24] (03) 8945fc      mov [ebp-04],eax
[0000d27] (04) 837dfc00    cmp dword [ebp-04],+00
[0000d2b] (02) 7402         jz 0000d2f
[0000d2d] (02) ebfe         jmp 0000d2d
[0000d2f] (02) 8be5         mov esp,ebp
[0000d31] (01) 5d          pop ebp
[0000d32] (01) c3          ret
Size in bytes:(0035) [0000d32]

```

```

main()
[0000d40] (01) 55          push ebp
[0000d41] (02) 8bec         mov ebp,esp
[0000d43] (03) 83ec10      sub esp,+10
[0000d46] (02) 6a03        push +03
[0000d48] (05) 68a00c0000 push 0000ca0
[0000d4d] (05) e8defdffff  call 0000b30
[0000d52] (03) 83c408      add esp,+08
[0000d55] (03) 8945f8      mov [ebp-08],eax
[0000d58] (03) 8b45f8      mov eax,[ebp-08]
[0000d5b] (01) 50          push eax
[0000d5c] (05) 68b3030000 push 000003b3
[0000d61] (05) e8caf6ffff  call 0000430
[0000d66] (03) 83c408      add esp,+08
[0000d69] (05) 68e00c0000 push 0000ce0
[0000d6e] (05) 68e00c0000 push 0000ce0
[0000d73] (05) e8b8fdffff  call 0000b30
[0000d78] (03) 83c408      add esp,+08
[0000d7b] (03) 8945fc      mov [ebp-04],eax
[0000d7e] (03) 8b4dfc      mov ecx,[ebp-04]
[0000d81] (01) 51          push ecx
[0000d82] (05) 68cb030000 push 000003cb
[0000d87] (05) e8a4f6ffff  call 0000430
[0000d8c] (03) 83c408      add esp,+08
[0000d8f] (02) 6a03        push +03
[0000d91] (05) 68f00c0000 push 0000cf0
[0000d96] (05) e895fdffff  call 0000b30
[0000d9b] (03) 83c408      add esp,+08
[0000d9e] (03) 8945f0      mov [ebp-10],eax
[0000da1] (03) 8b55fc      mov edx,[ebp-04]
[0000da4] (01) 52          push edx
[0000da5] (05) 68e3030000 push 000003e3
[0000daa] (05) e881f6ffff  call 0000430
[0000daf] (03) 83c408      add esp,+08
[0000db2] (05) 68100d0000 push 0000d10
[0000db7] (05) 68100d0000 push 0000d10
[0000dbc] (05) e86ffdffff  call 0000b30
[0000dc1] (03) 83c408      add esp,+08
[0000dc4] (03) 8945f4      mov [ebp-0c],eax
[0000dc7] (03) 8b45f4      mov eax,[ebp-0c]
[0000dca] (01) 50          push eax
[0000dcb] (05) 68fb030000 push 000003fb
[0000dd0] (05) e85bf6ffff  call 0000430
[0000dd5] (03) 83c408      add esp,+08
[0000dd8] (02) 33c0        xor eax,eax
[0000dda] (02) 8be5        mov esp,ebp
[0000ddc] (01) 5d          pop ebp
[0000ddd] (01) c3          ret
Size in bytes:(0158) [0000ddd]

```

```

=====
... [0000d40] [001018d4] [00000000] (01) 55          push ebp
... [0000d41] [001018d4] [00000000] (02) 8bec         mov ebp,esp
... [0000d43] [001018c4] [90909090] (03) 83ec10      sub esp,+10
... [0000d46] [001018c0] [00000003] (02) 6a03        push +03
... [0000d48] [001018bc] [00000ca0] (05) 68a00c0000 push 0000ca0
... [0000d4d] [001018b8] [00000d52] (05) e8defdffff  call 0000b30
Begin Local Halt Decider Simulation at Machine Address:ca0
... [0000ca0] [00211974] [00211978] (01) 55          push ebp
... [0000ca1] [00211974] [00211978] (02) 8bec         mov ebp,esp
... [0000ca3] [00211974] [00211978] (03) 8b4508      mov eax,[ebp+08]
... [0000ca6] [00211970] [00000003] (01) 50          push eax
... [0000ca7] [0021196c] [000003a3] (05) 68a3030000 push 00003a3
--- [0000cac] [0021196c] [000003a3] (05) e87ff7ffff  call 0000430
Factorial(n)3
... [0000cb1] [00211974] [00211978] (03) 83c408      add esp,+08

```

```

... [0000cb4] [00211974] [00211978] (04) 837d0801    cmp dword [ebp+08],+01
... [0000cb8] [00211974] [00211978] (02) 7e17       jng 0000cd1
... [0000cba] [00211974] [00211978] (03) 8b4d08    mov ecx,[ebp+08]
... [0000cbd] [00211974] [00211978] (03) 83e901    sub ecx,+01
... [0000cc0] [00211970] [00000002] (01) 51        push ecx
... [0000cc1] [0021196c] [00000006] (05) e8daffffff call 0000ca0
... [0000ca0] [00211968] [00211974] (01) 55        push ebp
... [0000ca1] [00211968] [00211974] (02) 8bec     mov ebp,esp
... [0000ca3] [00211968] [00211974] (03) 8b4508    mov eax,[ebp+08]
... [0000ca6] [00211964] [00000002] (01) 50        push eax
... [0000ca7] [00211960] [000003a3] (05) 68a3030000 push 000003a3
--- [0000cac] [00211960] [000003a3] (05) e87ff7ffff call 00000430
Factorial(n)2
... [0000cb1] [00211968] [00211974] (03) 83c408    add esp,+08
... [0000cb4] [00211968] [00211974] (04) 837d0801    cmp dword [ebp+08],+01
... [0000cb8] [00211968] [00211974] (02) 7e17       jng 0000cd1
... [0000cba] [00211968] [00211974] (03) 8b4d08    mov ecx,[ebp+08]
... [0000cbd] [00211968] [00211974] (03) 83e901    sub ecx,+01
... [0000cc0] [00211964] [00000001] (01) 51        push ecx
... [0000cc1] [00211960] [00000006] (05) e8daffffff call 0000ca0
... [0000ca0] [0021195c] [00211968] (01) 55        push ebp
... [0000ca1] [0021195c] [00211968] (02) 8bec     mov ebp,esp
... [0000ca3] [0021195c] [00211968] (03) 8b4508    mov eax,[ebp+08]
... [0000ca6] [00211958] [00000001] (01) 50        push eax
... [0000ca7] [00211954] [000003a3] (05) 68a3030000 push 000003a3
--- [0000cac] [00211954] [000003a3] (05) e87ff7ffff call 00000430
Factorial(n)1
... [0000cb1] [0021195c] [00211968] (03) 83c408    add esp,+08
... [0000cb4] [0021195c] [00211968] (04) 837d0801    cmp dword [ebp+08],+01
... [0000cb8] [0021195c] [00211968] (02) 7e17       jng 0000cd1
... [0000cd1] [0021195c] [00211968] (05) b801000000 mov eax,00000001
... [0000cd6] [00211960] [00000006] (01) 5d        pop ebp
... [0000cd7] [00211964] [00000001] (01) c3        ret
... [0000d52] [001018c4] [90909090] (03) 83c408    add esp,+08
... [0000d55] [001018c4] [90909090] (03) 8945f8    mov [ebp-08],eax
... [0000d58] [001018c4] [90909090] (03) 8b45f8    mov eax,[ebp-08]
... [0000d5b] [001018c0] [00000001] (01) 50        push eax
... [0000d5c] [001018bc] [000003b3] (05) 68b3030000 push 000003b3
--- [0000d61] [001018bc] [000003b3] (05) e8caf6ffff call 00000430
Input_Would_Halt1 = 1
... [0000d66] [001018c4] [90909090] (03) 83c408    add esp,+08
... [0000d69] [001018c0] [000000ce0] (05) 68e00c0000 push 000000ce0
... [0000d6e] [001018bc] [000000ce0] (05) 68e00c0000 push 000000ce0
... [0000d73] [001018b8] [000000d78] (05) e8b8fdffff call 00000b30
Begin Local Halt Decider Simulation at Machine Address:ce0
... [0000ce0] [0025c39c] [0025c3a0] (01) 55        push ebp
... [0000ce1] [0025c39c] [0025c3a0] (02) 8bec     mov ebp,esp
... [0000ce3] [0025c39c] [0025c3a0] (02) ebfe     jmp 0000ce3
... [0000ce3] [0025c39c] [0025c3a0] (02) ebfe     jmp 0000ce3
Local Halt Decider: Infinite Loop Detected Simulation Stopped
... [0000d78] [001018c4] [90909090] (03) 83c408    add esp,+08
... [0000d7b] [001018c4] [90909090] (03) 8945fc    mov [ebp-04],eax
... [0000d7e] [001018c4] [90909090] (03) 8b4dfc    mov ecx,[ebp-04]
... [0000d81] [001018c0] [00000000] (01) 51        push ecx
... [0000d82] [001018bc] [000003cb] (05) 68cb030000 push 000003cb
--- [0000d87] [001018bc] [000003cb] (05) e8a4f6ffff call 00000430
Input_Would_Halt2 = 0
... [0000d8c] [001018c4] [90909090] (03) 83c408    add esp,+08
... [0000d8f] [001018c0] [00000003] (02) 6a03     push +03
... [0000d91] [001018bc] [000000cf0] (05) 68f00c0000 push 000000cf0
... [0000d96] [001018b8] [000000d9b] (05) e895fdffff call 00000b30
Begin Local Halt Decider Simulation at Machine Address:cf0
... [0000cf0] [002a6dc4] [002a6dc8] (01) 55        push ebp
... [0000cf1] [002a6dc4] [002a6dc8] (02) 8bec     mov ebp,esp
... [0000cf3] [002a6dc0] [00296d94] (01) 51        push ecx
... [0000cf4] [002a6dc0] [00296d94] (03) 8b4508    mov eax,[ebp+08]

```

```

... [0000cf7] [002a6dbc] [00000003] (01) 50          push eax
... [0000cf8] [002a6db8] [0000cfd] (05) e8f3ffffff call 0000cf0
... [0000cf0] [002a6db4] [002a6dc4] (01) 55          push ebp
... [0000cf1] [002a6db4] [002a6dc4] (02) 8bec       mov ebp,esp
... [0000cf3] [002a6db0] [00296d94] (01) 51          push ecx
... [0000cf4] [002a6db0] [00296d94] (03) 8b4508     mov eax,[ebp+08]
... [0000cf7] [002a6dac] [00000003] (01) 50          push eax
... [0000cf8] [002a6da8] [0000cfd] (05) e8f3ffffff call 0000cf0
Local Halt Decider: Infinite Recursion Detected Simulation Stopped
... [0000d9b] [001018c4] [90909090] (03) 83c408     add esp,+08
... [0000d9e] [001018c4] [00000000] (03) 8945f0     mov [ebp-10],eax
... [0000da1] [001018c4] [00000000] (03) 8b55fc     mov edx,[ebp-04]
... [0000da4] [001018c0] [00000000] (01) 52          push edx
... [0000da5] [001018bc] [000003e3] (05) 68e3030000 push 000003e3
--- [0000daa] [001018bc] [000003e3] (05) e881f6ffff call 0000430
Input_Would_Halt3 = 0
... [0000daf] [001018c4] [00000000] (03) 83c408     add esp,+08
... [0000db2] [001018c0] [0000d10] (05) 68100d0000 push 0000d10
... [0000db7] [001018bc] [0000d10] (05) 68100d0000 push 0000d10
... [0000dbc] [001018b8] [0000dc1] (05) e86ffdffff call 0000b30
Begin Local Halt Decider Simulation at Machine Address:d10
... [0000d10] [002f17ec] [002f17f0] (01) 55          push ebp
... [0000d11] [002f17ec] [002f17f0] (02) 8bec       mov ebp,esp
... [0000d13] [002f17e8] [002e17bc] (01) 51          push ecx
... [0000d14] [002f17e8] [002e17bc] (03) 8b4508     mov eax,[ebp+08]
... [0000d17] [002f17e4] [0000d10] (01) 50          push eax
... [0000d18] [002f17e4] [0000d10] (03) 8b4d08     mov ecx,[ebp+08]
... [0000d1b] [002f17e0] [0000d10] (01) 51          push ecx
... [0000d1c] [002f17dc] [0000d21] (05) e80ffeffff call 0000b30
... [0000d10] [0033c214] [0033c218] (01) 55          push ebp
... [0000d11] [0033c214] [0033c218] (02) 8bec       mov ebp,esp
... [0000d13] [0033c210] [0032c1e4] (01) 51          push ecx
... [0000d14] [0033c210] [0032c1e4] (03) 8b4508     mov eax,[ebp+08]
... [0000d17] [0033c20c] [0000d10] (01) 50          push eax
... [0000d18] [0033c20c] [0000d10] (03) 8b4d08     mov ecx,[ebp+08]
... [0000d1b] [0033c208] [0000d10] (01) 51          push ecx
... [0000d1c] [0033c204] [0000d21] (05) e80ffeffff call 0000b30
Local Halt Decider: Infinite Recursion Detected Simulation Stopped
... [0000dc1] [001018c4] [00000000] (03) 83c408     add esp,+08
... [0000dc4] [001018c4] [00000000] (03) 8945f4     mov [ebp-0c],eax
... [0000dc7] [001018c4] [00000000] (03) 8b45f4     mov eax,[ebp-0c]
... [0000dca] [001018c0] [00000000] (01) 50          push eax
... [0000dcb] [001018bc] [000003fb] (05) 68fb030000 push 000003fb
--- [0000dd0] [001018bc] [000003fb] (05) e85bf6ffff call 0000430
Input_Would_Halt4 = 0
... [0000dd5] [001018c4] [00000000] (03) 83c408     add esp,+08
... [0000dd8] [001018c4] [00000000] (02) 33c0       xor eax,eax
... [0000dda] [001018d4] [00000000] (02) 8be5       mov esp,ebp
... [0000ddc] [001018d8] [00100000] (01) 5d          pop ebp
... [0000ddd] [001018dc] [000001d0] (01) c3          ret
Number_of_User_Instructions(117)
Number_of_Instructions_Executed(32231)

```