

Rebutting the Sipser Halting Problem Proof V2

A simulating halt decider correctly predicts what the behavior of its input would be if this simulated input never had its simulation aborted. It does this by correctly recognizing several non-halting behavior patterns in a finite number of steps of correct simulation.

When simulating halt decider H correctly predicts that the conventional counter-example input to the halting theorem never halts (because it remains stuck in recursive simulation) this input ceases to prove the halting theorem.

MIT Professor Michael Sipser has agreed that the following verbatim paragraph is correct (he has not agreed to anything else in this paper):

(a) If simulating halt decider H correctly simulates its input D until H correctly determines that its simulated D would never stop running unless aborted then (b) H can abort its simulation of D and correctly report that D specifies a non-halting sequence of configurations.

We start with Sipser's definitions of H and D:

On input (M, w) , where M is a TM and w is a string, H halts and accepts if M accepts w. Furthermore, H halts and rejects if M fails to accept w.

In other words, we assume that H is a TM, where

$$H(\langle M, w \rangle) = \begin{cases} \text{accept} & \text{if } M \text{ accepts } w \\ \text{reject} & \text{if } M \text{ does not accept } w \end{cases}$$

Now we construct a new Turing machine D with H as a subroutine. This new TM calls H to determine what M does when the input to M is its own description $\langle M \rangle$. Once D has determined this information, it does the opposite. That is, it rejects if M accepts and accepts if M does not accept.

$$D(\langle M \rangle) = \begin{cases} \text{accept} & \text{if } M \text{ does not accept } \langle M \rangle \\ \text{reject} & \text{if } M \text{ accepts } \langle M \rangle \end{cases} \quad (\text{Sipser 1997:165})$$

We encode the Sipser D and define the behavior of Sipser H as C functions.

```
int Sipser_D(int (*M)())
{
    if ( HH(M, M) )
        return 0;
    return 1;
}

int main()
{
    Sipser_D(Sipser_D);
}
```

H returns 0 to Sipser_D on the basis that Sipser_D correctly simulated by H would remain stuck in recursive simulation unless H aborts its simulation of Sipser_D.

Diagonal proof: Correctly predict what I will say when I will always say the opposite of whatever you predict. If you correctly predict that I will say nothing then this too is correct.

Sipser, Michael 1997. Introduction to the Theory of Computation. Boston: PWS Publishing Company (165-167)

```

 SIPSER_D()
[00001e84] 55          push ebp
[00001e85] 8bec         mov ebp,esp
[00001e87] 8b4508      mov eax,[ebp+08]
[00001e8a] 50          push eax
[00001e8b] 8b4d08      mov ecx,[ebp+08]
[00001e8e] 51          push ecx
[00001e8f] e8a0f5ffff  call 00001434
[00001e94] 83c408      add esp,+08
[00001e97] 85c0        test eax,eax
[00001e99] 7404        jz 00001e9f
[00001e9b] 33c0        xor eax,eax
[00001e9d] eb05        jmp 00001ea4
[00001e9f] b801000000 mov eax,00000001
[00001ea4] 5d          pop ebp
[00001ea5] c3          ret
Size in bytes:(0034) [00001ea5]

```

```

 MAIN()
[00001eb4] 55          push ebp
[00001eb5] 8bec         mov ebp,esp
[00001eb7] 68841e0000  push 00001e84
[00001ebc] e8c3ffffff  call 00001e84
[00001ec1] 83c404      add esp,+04
[00001ec4] 33c0        xor eax,eax
[00001ec6] 5d          pop ebp
[00001ec7] c3          ret
Size in bytes:(0020) [00001ec7]

```

machine address	stack address	stack data	machine code	assembly language
[00001eb4]	[00103244]	[00000000]	55	push ebp
[00001eb5]	[00103244]	[00000000]	8bec	mov ebp,esp
[00001eb7]	[00103240]	[00001e84]	68841e0000	push 00001e84 // push Sipser_D
[00001ebc]	[0010323c]	[00001ec1]	e8c3ffffff	call 00001e84 // call Sipser_D
[00001e84]	[00103238]	[00103244]	55	push ebp
[00001e85]	[00103238]	[00103244]	8bec	mov ebp,esp
[00001e87]	[00103238]	[00103244]	8b4508	mov eax,[ebp+08] // move arg to eax
[00001e8a]	[00103234]	[00001e84]	50	push eax // push Sipser_D
[00001e8b]	[00103234]	[00001e84]	8b4d08	mov ecx,[ebp+08] // move arg to ecx
[00001e8e]	[00103230]	[00001e84]	51	push ecx // push Sipser_D
[00001e8f]	[0010322c]	[00001e94]	e8a0f5ffff	call 00001434 // call H

New slave_stack at:1032e8

```

Begin Local Halt Decider Simulation Execution Trace Stored at:1132f0
[00001e84] [001132dc] [001132e0] 55          push ebp // begin Sipser_D
[00001e85] [001132dc] [001132e0] 8bec         mov ebp,esp
[00001e87] [001132dc] [001132e0] 8b4508      mov eax,[ebp+08] // move arg to eax
[00001e8a] [001132d8] [00001e84] 50          push eax // push Sipser_D
[00001e8b] [001132d8] [00001e84] 8b4d08      mov ecx,[ebp+08] // move arg to ecx
[00001e8e] [001132d4] [00001e84] 51          push ecx // push Sipser_D
[00001e8f] [001132d0] [00001e94] e8a0f5ffff  call 00001434 // call H
New slave_stack at:14dd10
[00001e84] [0015dd04] [0015dd08] 55          push ebp // begin Sipser_D
[00001e85] [0015dd04] [0015dd08] 8bec         mov ebp,esp
[00001e87] [0015dd04] [0015dd08] 8b4508      mov eax,[ebp+08] // move arg to eax
[00001e8a] [0015dd00] [00001e84] 50          push eax // push Sipser_D
[00001e8b] [0015dd00] [00001e84] 8b4d08      mov ecx,[ebp+08] // move arg to ecx
[00001e8e] [0015dcfc] [00001e84] 51          push ecx // push Sipser_D
[00001e8f] [0015dcf8] [00001e94] e8a0f5ffff  call 00001434 // call H
Local Halt Decider: Infinite Recursion Detected Simulation Stopped

```

Sipser_D(Sipser_D) remains stuck in recursive simulation until H recognizes this and aborts its simulation. The above behavior pattern conclusively proves that Sipser_D(Sipser_D) cannot possibly stop running unless H aborts its simulation of Sipser_D.