

Rebutting the Sipser Halting Problem Proof V2

A simulating halt decider correctly predicts what the behavior of its input would be if this simulated input never had its simulation aborted. It does this by correctly recognizing several non-halting behavior patterns in a finite number of steps of correct simulation.

When simulating halt decider H correctly predicts that directly executed D(D) would continue to run forever unless H aborts its simulation of D this directly applies to the halting theorem.

In computability theory, the halting problem is the problem of determining, from a description of an arbitrary computer program and an input, whether the program will finish running, or continue to run forever.

For any program H that might determine whether programs halt, a "pathological" program D, called with some input, can pass its own source and its input to H and then specifically do the opposite of what H predicts D will do. https://en.wikipedia.org/wiki/Halting_problem

That (a) proves (b) is a tautology

(a) If simulating halt decider H correctly simulates its input D until H correctly determines that its simulated D would never stop running unless aborted then

(b) H can abort its simulation of D and correctly report that D specifies a non-halting sequence of configurations.

To make the details 100% concrete Sipser D and H are encoded as C* *functions. The exact same rebuttal equally applies to Turing machines

```
int Sipser_D(int (*M)())
{
    int DoesHalt = H(M, M); // *Rejects when Sipser_D fails to accept*
    if (DoesHalt)
        return 0;
    return 1;
}

int main()
{
    // *never stops running unless H aborts its simulation*
    Sipser_D(Sipser_D);
}
```

When Sipser_D calls H to simulate itself this comparable to calling H to call itself and can result in something like infinite recursion. Because there are no control flow instructions in Sipser_D to stop this the recursive simulation continues until H aborts it.

When the simulation of D is aborted this is comparable to a divide by zero error thus is not construed as D halting.

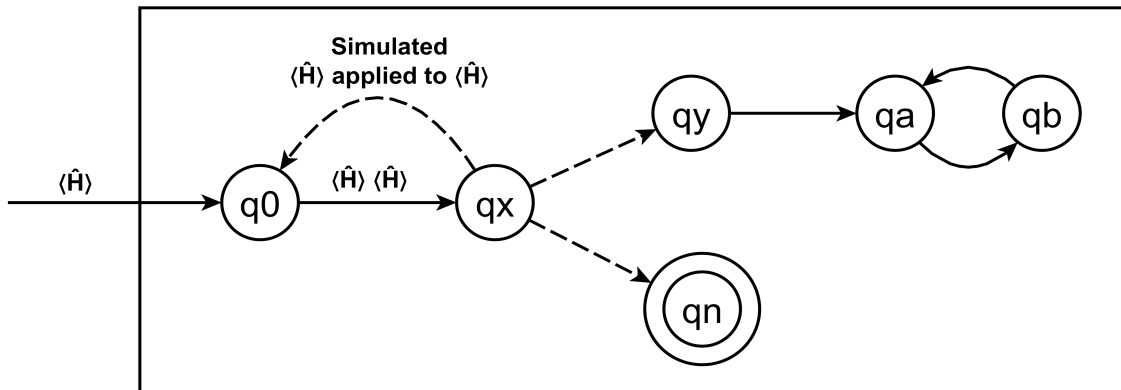
This exact same principle works on all Turing machine based halting theorem proofs

Sipser, Michael 1997. Introduction to the Theory of Computation. Boston: PWS Publishing Company (165-167)

Applying a simulating halt decider to the Linz halting problem proof

The Linz text indicates that \hat{H} is defined on the basis of prepending and appending states to the original Linz H , thus is named `embedded_H`. \vdash^* indicates an arbitrary number of moves such as: `change_state / tape_head_action`. The Peter Linz \hat{H} is applied to its own machine description $\langle \hat{H} \rangle$.

$\hat{H}.q_0 \langle \hat{H} \rangle \vdash^* \text{embedded_H} \langle \hat{H} \rangle \langle \hat{H} \rangle \vdash^* \hat{H}.q_y \infty$
 $\hat{H}.q_0 \langle \hat{H} \rangle \vdash^* \text{embedded_H} \langle \hat{H} \rangle \langle \hat{H} \rangle \vdash^* \hat{H}.q_n$



When \hat{H} is applied to $\langle \hat{H} \rangle$

(q0) The input $\langle \hat{H} \rangle$ is copied then transitions to (qx)

(qx) `embedded_H` is applied to $\langle \hat{H} \rangle \langle \hat{H} \rangle$ (input and copy)

which simulates $\langle \hat{H} \rangle \langle \hat{H} \rangle$ which transitions back to (q0) to repeat the process.

This process continues to repeat until `embedded_H` recognizes the repeating pattern and aborts its simulation of $\langle \hat{H} \rangle \langle \hat{H} \rangle$.

`embedded_H` is correct to abort its simulation and transition to $\hat{H}.q_n$ because it correctly predicts that \hat{H} applied to $\langle \hat{H} \rangle$ would never stop running unless `embedded_H` aborts its simulation of $\langle \hat{H} \rangle \langle \hat{H} \rangle$.

Linz, Peter 1990. An Introduction to Formal Languages and Automata.
 Lexington/Toronto: D. C. Heath and Company. (317-320)