Tarski Undefinability Theorem Reexamined

Tarski proved that the Liar Paradox is true in his meta-theory and not provable in his theory without ever realizing that the only reason it is not provable in his theory is that it is not true in his theory.

Examining his Undefinability Theorem with the (Curry 2010) notion of a formal system:
A theory T is a conceptual class consisting of certain of these elementary statements. The elementary statements which belong to T are called the elementary theorems of T and said to be true. In this way, a theory is a way of designating a subset of E which consists entirely of true statements.

This general way of designating a theory stipulates that the truth of any of its elementary statements is not known without reference to T. Thus the same elementary statement may be true with respect to one theory, and not true with respect to another. (Curry, Haskell. 2010 Foundations of Mathematical Logic).

We derive these three universal Truth predicate axioms:
(1) ∀F ∈ Formal_Systems ∀x ∈ WFF(F) (**True**(F, x) ↔ (F ⊢ x))
(2) ∀F ∈ Formal_Systems ∀x ∈ WFF(F) (**False**(F, x) ↔ (F ⊢ ~x))

Rejects Semantically ill-formed logic sentences
(3) ∀F ∈ Formal_Systems ∀x ∈ WFF(F) (**~True**(F, x) ↔ ~(F ⊢ x))

We begin by formalizing the Liar Paradox: G ↔ ~(F ⊢ G)
G has the same Truth value as its own unprovability in F.

When the RHS ~(F ⊢ G) is true, by Truth axiom(3) we know that **G is not true in F.** This contradicts the LHS being true, making the whole Liar Paradox expression false. Which makes this expression false: ∃F∃G (G ↔ ~(F ⊢ G)). There are no formal systems having a sentence with the same Truth value as its own unprovability.

Tarski's conclusion that his x is undecidable in his theory on the basis that there are no possible truth predicates that could exist in his theory that would correctly decide x is refuted by my truth predicates that correctly decide his x in his theory.

Tarski notation for simplified Truth Predicate Axioms (with simple English)

(1) $x \in Tr \leftrightarrow x \in Pr$ // $True(x) \leftrightarrow (\vdash x)$

A set of facts adds up to X being TRUE.

(2) $\sim x \in Tr \leftrightarrow \sim x \in Pr$ // $False(x) \leftrightarrow (\vdash \sim x)$

A set of facts adds up to X being FALSE.

(3) $x \notin Tr \leftrightarrow x \notin Pr$ // $\sim True(x) \leftrightarrow \sim(\vdash x)$

There is no set of facts that add up to X being TRUE.

Excerpts from "The concept of truth in formalized languages" Tarski 1936

// page 248 Tarski defines x of his proof

Should we succeed in constructing in the metalanguage
a correct definition of truth, then ... It would
then be possible to reconstruct the antinomy of the liar in the
metalanguage, by forming in the language itself a sentence x
such that the sentence of the metalanguage which is correlated
with x asserts that x is not a true sentence.

// page 276 From the Tarski Undefinability Theorem proof

The formulas (8) and (9) together express the fact that x is an
undecidable sentence; moreover from (7) it follows that x is a
true sentence.

By establishing the truth of the sentence x we have eo ipso
-by reason of (2)-also proved x itself in the metatheory.
Since, moreover, the metatheory can be interpreted in the
theory enriched by variables of higher order (cf. p. 184) and
since in this interpretation the sentence x, which contains no
specific term of the metatheory, is its own correlate, the proof of
the sentence x given in the metatheory can automatically be
carried over into the theory itself: the sentence x which is
undecidable in the original theory becomes a decidable sentence
in the enriched theory.

Proof on pages 275-276, x defined on page 248
http://www.thatmarcusfamily.org/philosophy/Course_Websites/Readings/Tarski%20-%20The%20Concept%20of%20Truth%20in%20Formalized%20Languages.pdf