Tarski Undefinability Theorem Reexamined

Tarski proves that the Liar Paradox is true in his meta-theory and not provable in his theory. By creating three universal truth predicates that Tarski presumed could not possibly exist I prove that the Liar Paradox is false in his theory with no need to reference any meta-theory.

We derive these three universal Truth predicate axioms:

(1) $\forall F \in Formal\_Systems\ \forall x \in WFF(F)\ (\mathbf{True}(F, x) \leftrightarrow (F \vdash x))$ // x is provable in F

(2) $\forall F \in Formal\_Systems\ \forall x \in WFF(F)\ (\mathbf{False}(F, x) \leftrightarrow (F \vdash {\sim}x))$ // ~x is provable in F

(3) $\forall F \in Formal\_Systems\ \forall x \in WFF(F)\ ({\sim}\mathbf{True}(F, x) \leftrightarrow {\sim}(F \vdash x))$

The last truth predicate axiom includes Semantically_Incorrect(x) and False(x).

We begin by formalizing the Liar_Paradox: $True(F, G) \leftrightarrow {\sim}(F \vdash G)$

The Truth Value of G in F is the same as the Truth value of the unprovability of G in F.

By Truth axiom (3) we substitute $\sim True(F, G)$ for ${\sim}(F \vdash G)$

deriving $True(F, G) \leftrightarrow {\sim}True(F, G)$ ∴ the Liar_Paradox is false.

Tarski's conclusion that his x is undecidable in his theory is refuted using three universal truth predicates proving that his x is false in his theory.

Tarski notation for simplified Truth Predicate Axioms (with simple English)

(1) $x \in Tr \leftrightarrow x \in Pr$ // $True(x) \leftrightarrow (\vdash x)$

A set of facts adds up to X being TRUE.

(2) ${\sim}x \in Tr \leftrightarrow {\sim}x \in Pr$ // $False(x) \leftrightarrow (\vdash {\sim}x)$

A set of facts adds up to X being FALSE.

(3) $x \notin Tr \leftrightarrow x \notin Pr$ // ${\sim}True(x) \leftrightarrow {\sim}(\vdash x)$

There is no set of facts that add up to X being TRUE.

Excerpts from "The concept of truth in formalized languages" Tarski 1936

// page 248 Tarski defines x of his proof
Should we succeed in constructing in the metalanguage a correct definition of truth, then ... It would then be possible to reconstruct the antinomy of the liar in the metalanguage, by forming in the language itself a sentence x such that the sentence of the metalanguage which is correlated with x asserts that x is not a true sentence.

// page 276 From the Tarski Undefinability Theorem proof
The formulas (8) and (9) together express the fact that x is an undecidable sentence; moreover from (7) it follows that x is a true sentence.

By establishing the truth of the sentence x we have eo ipso -by reason of (2)-also proved x itself in the metatheory. Since, moreover, the metatheory can be interpreted in the theory enriched by variables of higher order (cf. p. 184) and since in this interpretation the sentence x, which contains no specific term of the metatheory, is its own correlate, the proof of the sentence x given in the metatheory can automatically be carried over into the theory itself: the sentence x which is undecidable in the original theory becomes a decidable sentence in the enriched theory.

Proof on pages 275-276, x defined on page 248
http://www.thatmarcusfamily.org/philosophy/Course_Websites/Readings/Tarski%20-%20The%20Concept%20of%20Truth%20in%20Formalized%20Languages.pdf