

Tarski Undefinability Theorem Succinctly Refuted

Tarski proves that the Liar Paradox is true in his meta-theory and not provable in his theory. By creating three universal truth predicate axioms that Tarski presumed could not exist I prove that the Liar Paradox is false in his theory with no need to reference any meta-theory.

We derive these three universal Truth predicate axioms:

- (1) $\forall F \in \text{Formal_Systems } \forall x \in \text{WFF}(F) (\text{True}(F, x) \leftrightarrow (F \vdash x))$ // x is provable in F
- (2) $\forall F \in \text{Formal_Systems } \forall x \in \text{WFF}(F) (\text{False}(F, x) \leftrightarrow (F \vdash \sim x))$ // $\sim x$ is provable in F
- (3) $\forall F \in \text{Formal_Systems } \forall x \in \text{WFF}(F) (\sim \text{True}(F, x) \leftrightarrow \sim(F \vdash x))$

The last truth predicate axiom includes **Semantically_Incorrect(x)** and **False(x)**.

We begin by formalizing the Liar_Paradox: $\text{True}(F, G) \leftrightarrow \sim(F \vdash G)$

The Truth Value of G in F is the same as the Truth value of the unprovability of G in F .

By Truth axiom (3) we substitute $\sim \text{True}(F, G)$ for $\sim(F \vdash G)$ deriving $\text{True}(F, G) \leftrightarrow \sim \text{True}(F, G) \therefore$ the Liar_Paradox is false.

Tarski's conclusion that his x is undecidable in his theory is refuted using three universal truth predicates proving that his x is false in his theory.

Tarski notation for simplified Truth Predicate Axioms (with simple English)

- (1) $x \in \text{Tr} \leftrightarrow x \in \text{Pr}$ // $\text{True}(x) \leftrightarrow (\vdash x)$

A set of facts adds up to X being TRUE.

- (2) $\sim x \in \text{Tr} \leftrightarrow \sim x \in \text{Pr}$ // $\text{False}(x) \leftrightarrow (\vdash \sim x)$

A set of facts adds up to X being FALSE.

- (3) $x \notin \text{Tr} \leftrightarrow x \notin \text{Pr}$ // $\sim \text{True}(x) \leftrightarrow \sim(\vdash x)$

There is no set of facts that add up to X being TRUE.

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Excerpts from “The concept of truth in formalized languages” Tarski 1936

// page 248 **Tarski defines x of his proof**

Should we succeed in constructing in the metalanguage a correct definition of truth, then ... It would then be possible to reconstruct the antinomy of the liar in the metalanguage, by forming in the language itself a sentence x such that the sentence of the metalanguage which is correlated with x asserts that x is not a true sentence.

// page 275 **From the Tarski Undefinability Theorem proof**

We shall show that the sentence x is actually undecidable and at the same time true.

// page 276 **From the Tarski Undefinability Theorem proof**

The formulas (8) and (9) together express the fact that x is an undecidable sentence; moreover from (7) it follows that x is a true sentence.

...

By establishing the truth of the sentence x we have eo ipso -by reason of (2)-also proved x itself in the metatheory. Since, moreover, the metatheory can be interpreted in the theory enriched by variables of higher order (cf. p. 184) and since in this interpretation the sentence x , which contains no specific term of the metatheory, is its own correlate, the proof of the sentence x given in the metatheory can automatically be carried over into the theory itself: the sentence x which is undecidable in the original theory becomes a decidable sentence in the enriched theory.

Proof on pages 275-276, x defined on page 248

http://www.thatmarcusfamily.org/philosophy/Course_Websites/Readings/Tarski%20-%20The%20Concept%20of%20Truth%20in%20Formalized%20Languages.pdf