

Tarski Undefinability Theorem Succinctly Refuted

If the conclusion of the Tarski Undefinability Theorem was that some artificially constrained limited notions of a formal system necessarily have undecidable sentences, then Tarski made no mistake. When we expand the scope of his conclusion to include every possible notion of a formal system we reach an entirely different conclusion.

A very slight augmentation to the conventional notion of a formal system refutes the much more narrowly constrained Tarski results. This slightly augmented notion of a formal system is in every way identical to the conventional notion except that it recognizes and rejects semantically incorrect expressions of language.

This refutation applies to formal systems having greater expressive power than any language of arithmetic: (The generalized result of the Tarski Undefinability Theorem) and requires that the formal system have its own provability predicate, eliminating the need for diagonalization.

Truth Predicate Axioms

(Tarski Notation, Conventional Notation and Simple English)

(1) $x \in Tr \leftrightarrow x \in Pr$ // $True(x) \leftrightarrow (\vdash x)$

A set of facts adds up to X being TRUE.

(2) $\sim x \in Tr \leftrightarrow \sim x \in Pr$ // $False(x) \leftrightarrow (\vdash \sim x)$

A set of facts adds up to X being FALSE.

(3) $x \notin Tr \leftrightarrow x \notin Pr$ // $\sim True(x) \leftrightarrow \sim(\vdash x)$

There is no set of facts that add up to X being TRUE.

The key aspect of this augmentation to the notion of formal systems is Axiom of Truth (3) that correctly decides that some expressions of language such as the formalized Liar Paradox are semantically incorrect.

Anyone truly understanding the Tarski Undefinability proof would know that the whole proof would fail as soon as its third step would be proven false: (3) $x \notin Pr \leftrightarrow x \in Tr$ // page 275

Applying Truth Predicate Axiom(3) decides that Tarski's step(3) is false:

Swap the LHS of Tarski(3) $[x \notin Pr]$ that matches RHS of Axiom(3) $[x \notin Pr]$ with the LHS of Axiom(3) and we derive $x \notin Tr \leftrightarrow x \in Tr$, which is clearly false, thus decidable.

By making a very slight change to the conventional notion of a formal system we have a new notion of formal system that is in every way identical to the prior notion except that it correctly decides all of the sentences that were previously undecidable.

The above can only be understood within the context of the Tarski Proof:

http://liarparadox.org/Tarski_Proof_275_276.pdf

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