

Eliminating Undecidability and Incompleteness in Formal Systems

If the conclusion of the Tarski Undefinability Theorem was that some artificially constrained limited notions of a formal system necessarily have undecidable sentences, then Tarski made no mistake within his assumptions. When we expand the scope of his investigation to other notions of formal systems we reach an entirely different conclusion showing that Tarski's assumptions were wrong.

A very slight augmentation to the conventional notion of a formal system refutes the more narrowly constrained Tarski results. This slightly augmented notion of a formal system is in every way identical to the conventional notion except that it recognizes and rejects semantically incorrect expressions of language.

This refutation applies to the generalized result of the Tarski Undefinability Theorem: All formal systems of greater expressive power than arithmetic necessarily have undecidable sentences. and requires that the formal system have its own provability predicate, eliminating the need for diagonalization.

These Truth Predicate Axioms are based on the sound deductive inference model. Within the sound deductive inference model there is a (*connected sequence of valid deductions from true premises to a true conclusion*) thus unlike the formal proofs of symbolic logic provability cannot diverge from truth.

Truth Predicate Axioms

(Tarski Notation, Conventional Notation and Simple English)

(1) $x \in Tr \leftrightarrow x \in Pr$ // $True(x) \leftrightarrow (\vdash x)$

A set of facts adds up to X being TRUE.

(2) $\neg x \in Tr \leftrightarrow \neg x \in Pr$ // $False(x) \leftrightarrow (\vdash \neg x)$

A set of facts adds up to X being FALSE.

(3) $x \in Tr \vee \neg x \in Tr$ // $True(x) \vee False(x)$

There are no set of facts that add up to X being TRUE or FALSE.

Anyone truly understanding the Tarski Undefinability proof would know that the whole proof would fail as soon as its third step would be proven false: (3) $x \notin Pr \leftrightarrow x \in Tr$ Because this third step directly contradicts Axiom(1) it is decided to be false.

By making a very slight change to the conventional notion of a formal system we have a new notion of formal system that is in every way identical to the prior notion except that it correctly decides all of the sentences that were previously undecidable.

The above can only be understood within the context of the Tarski Proof:
http://liarparadox.org/Tarski_Proof_275_276.pdf (Tarski 1936:275-276)

*Stipulating** that formal systems are Boolean:*

Axiom(3) $\forall F \in \text{Formal_System} \forall x \in \text{Closed_WFF}(F) (\text{True}(F,x) \vee \text{False}(F,x))$

Screens out semantically unsound sentences as not belonging to the formal system.

The following logic sentence is refuted on the basis of Axiom(3)

$\exists F \in \text{Formal_System} \exists G \in \text{Closed_WFF}(F) (G \leftrightarrow ((F \nVdash G) \wedge (F \nVdash \neg G)))$

There is no sentence G of Formal System F that is neither True nor False in F.

Making the following paragraph false:

The first incompleteness theorem states that in any consistent formal system F within which a certain amount of arithmetic can be carried out, there are statements of the language of F which can neither be proved nor disproved in F. (Raatikainen 2018)

(Raatikainen 2018)

Raatikainen, Panu, "Gödel's Incompleteness Theorems", The Stanford Encyclopedia of Philosophy (Fall 2018 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2018/entries/goedel-incompleteness/>.

(Tarski 1936)

A. Tarski, tr J.H. Woodger, 1983. "The Concept of Truth in Formalized Languages". English translation of Tarski's 1936 article. In A. Tarski, ed. J. Corcoran, 1983, Logic, Semantics, Metamathematics, Hackett.

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