

Tarski Undeifiability Theorem Reexamined

Tarski proved that the Liar Paradox is true in his meta-theory and not provable in his theory without ever realizing that the only reason it is not provable in his theory is that it is not true in his theory.

Examining his Undeifiability Theorem with the (Curry 2010) notion of a formal system:

A theory T is a conceptual class consisting of certain of these elementary statements. The elementary statements which belong to T are called the elementary theorems of T and said to be true. In this way, a theory is a way of designating a subset of E which consists entirely of true statements.

This general way of designating a theory stipulates that the truth of any of its elementary statements is not known without reference to T. Thus the same elementary statement may be true with respect to one theory, and not true with respect to another. (Curry, Haskell. 2010 Foundations of Mathematical Logic).

We derive these three universal Truth predicate axioms:

- (1) $\forall F \forall x (\text{True}(F, x) \leftrightarrow (F \vdash x))$
- (2) $\forall F \forall x (\text{False}(F, x) \leftrightarrow (F \vdash \sim x))$
- (3) $\forall F \forall x (\sim \text{True}(F, x) \leftrightarrow \sim(F \vdash x))$

We begin by formalizing the Liar Paradox: $G \leftrightarrow \sim(F \vdash G)$
G has the same Truth value as its own unprovability in F.

When the RHS $\sim(F \vdash G)$ is true, by Truth axiom(3) we know that x is not true in F. This contradicts the LHS being true, making the Liar Paradox expression false. Which makes this expression false: $\exists F \exists G (G \leftrightarrow \sim(F \vdash G))$. There are no formal systems having a sentence with the same Truth value as its own unprovability.

When Tarski proved that the Liar Paradox is true in his meta-theory and not provable in his theory (according to Curry 2010) he committed a fallacy of equivocation error. The truth of an expression can only be correctly evaluated within the same formal system. Since the conclusion of his proof was that no consistent and complete truth predicate can exist the above truth predicate axioms directly refute this conclusion of his proof.

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Excerpts from “The concept of truth in formalized languages” Tarski 1936

// page 248 Tarski defines x of his proof

Should we succeed in constructing in the metalanguage a correct definition of truth, then ... It would then be possible to reconstruct the antinomy of the liar in the metalanguage, by forming in the language itself a sentence x such that the sentence of the metalanguage which is correlated with x asserts that x is not a true sentence.

// page 276 The Tarski Undefinability Theorem proof

The formulas (8) and (9) together express the fact that x is an undecidable sentence; moreover from (7) it follows that x is a true sentence.

By establishing the truth of the sentence x we have eo ipso -by reason of (2)-also proved x itself in the metatheory. Since, moreover, the metatheory can be interpreted in the theory enriched by variables of higher order (cf. p. 184) and since in this interpretation the sentence x , which contains no specific term of the metatheory, is its own correlate, the proof of the sentence x given in the metatheory can automatically be carried over into the theory itself: the sentence x which is undecidable in the original theory becomes a decidable sentence in the enriched theory.

Proof on pages 275-276, x defined on page 248

http://www.thatmarcusfamily.org/philosophy/Course_Websites/Readings/Tarski%20-%20The%20Concept%20of%20Truth%20in%20Formalized%20Languages.pdf