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**Mathematizing Bodies.**

**Leibniz on the Application of Mathematics to Nature, and its Metaphysical Ground[[1]](#footnote-1)**

ABSTRSCT: There are two axes of Leibniz’s philosophy about bodies that are deeply intertwined, as this paper shows: the scientific investigation of bodies due to the application of mathematics to nature – Leibniz’s mixed mathematics – and the issue of matter/bodies idealism. This intertwinement raises an issue: How did Leibniz frame the relationship between mathematics, natural sciences, and metaphysics? Due to the increasing application of mathematics to natural sciences, especially physics, philosophers of the early modern period used the reliability of mathematics to predict phenomena as the basis to infer the metaphysical outlook of nature. I argue that although Leibniz thought metaphysics must be scientifically informed and that mathematics is a valuable instrument to understand nature, metaphysics is more fundamental than mathematics and natural sciences. By highlighting the foundational relation between metaphysics and the sciences, this paper showcases an argument for the reality of bodies: the ideality of bodies, necessary for epistemic purposes, is not proof that they are not real.

Keywords: Metaphysics, Application of Mathematics, Body, Idealism, Fundamentality.

ABSTRSCT: Deux axes de la philosophie de Leibniz sur les corps sont profondément entrelacés, comme le montre cet article: l’enquête scientifique des corps par l'application des mathématiques à la nature – les mathématiques mixtes de Leibniz – et la question de l'idéalisme de la matière et des corps. Cette entrelacement soulève une question: Comment Leibniz a-t-il défini la relation entre les mathématiques, les sciences naturelles et la métaphysique? En raison de l'application croissante des mathématiques aux sciences naturelles, en particulier à la physique, les philosophes de la période moderne ont utilisé la fiabilité des mathématiques pour prédire les phénomènes comme base pour déduire la perspective métaphysique de la nature. Je soutiens que, bien que Leibniz ait pensé que la métaphysique devait être scientifiquement informée et que les mathématiques étaient un instrument précieux pour comprendre la nature, la métaphysique est plus fondamentale que les mathématiques et les sciences naturelles. En soulignant la relation fondamentale entre la métaphysique et les sciences, cet article présente un argument en faveur de la réalité des corps: l'idéalité des corps, nécessaire à des fins épistémiques, n'est pas une preuve que ils ne sont pas réelle.

Mots-clés : Métaphysique, application des mathématiques, corps, idéalisme, fondamentalité.

Zusammenfassung: Es gibt zwei Achsen der Leibniz'schen Philosophie über Körper, die eng miteinander verwoben sind, wie dieser Beitrag zeigt: die wissenschaftliche Untersuchbarkeit von Körpern aufgrund der Anwendung der Mathematik auf die Natur – Leibniz' gemischte Mathematik – und die Frage des Idealismus von Materie und Körper. Diese Verflechtung wirft eine Frage auf: Wie gestaltete Leibniz das Verhältnis von Mathematik, Naturwissenschaften und Metaphysik? Aufgrund der zunehmenden Anwendung der Mathematik auf die Naturwissenschaften, insbesondere die Physik, nutzten Philosophen der frühen Neuzeit die Zuverlässigkeit der Mathematik bei der Vorhersage von Phänomenen als Grundlage für die Ableitung einer metaphysischen Sicht der Natur. Ich argumentiere, dass, obwohl Leibniz der Meinung war, dass die Metaphysik wissenschaftlich fundiert sein muss und dass die Mathematik ein wertvolles Instrument zum Verständnis der Natur ist, die Metaphysik grundlegender ist als Mathematik und Naturwissenschaften. Indem er die grundlegende Beziehung zwischen Metaphysik und Naturwissenschaften hervorhebt, zeigt dieser Aufsatz ein Argument für die Realität von Körpern auf: Die Idealität von Körpern, die für epistemische Zwecke notwendig ist, ist kein Beweis dafür, dass sie nicht real sind.

Schlüsselwörter: Metaphysik, Anwendung der Mathematik, Körper, Idealismus, Fundamentalität.

I. Mathematics and Metaphysics in the Early Modern Period

The book of nature “is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible
to understand a single word of it; without these, one wanders about in a dark labyrinth.”[[2]](#footnote-2) Galilei’s confidence in how, by managing mathematical concepts, we understand the same language used by God to write the natural world, is a metaphor, an analogy that nonetheless points to relevant epistemological and metaphysical issues concerning the (then) increasing application of mathematics to nature, as in physics.[[3]](#footnote-3) These issues can be articulated as follows.[[4]](#footnote-4) Stones and trees, seas and magnetic phenomena are not immediately seen in their mathematical encoding; only once we unwrap their sensory appearances and reduce them to quantifiable properties of *bodies* do we disenchant[[5]](#footnote-5) the world, discover its secret texture, and rule it by anticipating its course. “Knowledge is power”, as some philosophers said at the time,[[6]](#footnote-6) but what justifies the fact that the ruler of nature *must* be a mathematician?

Implied in the Galilean motto is the idea that mathematics is not *one* possible means to understand nature; it is the only correct means to do so, *because* nature *is* written in mathematical characters by a God-Mathematician. That we do not see its texture straightforwardly is contingent on the fact that we are still confined to a body and subject to physiological processes. So, secondary qualities occur in the mind when bodies impact their sense-organs, a God-given power to keep them alive, to find orientation in the permanent mutations of their own body,[[7]](#footnote-7) while in truth nature is “geometry made real.”[[8]](#footnote-8) “The essential properties of matter are whatever mathematically representable properties our best physical theory reveal matter to have.”[[9]](#footnote-9) The world *is* constituted by *res extensa* and *res cogitans*, following Descartes, and *res extensa* is the subject matter of mathematical enquiries.

This exposition highlights that a discourse about the metaphysics of bodies serves justificatory purposes in the use of mathematics applied to nature. If nature *is* extension, then the use of an abstract calculation to predict the movement of, for instance, a bullet, *is* the thing described, just written in its proper form. If the calculation succeeds, then the predicted phenomenon must obtain. Observations and experiments help in unwrapping the veil of secondary qualities; they do not fully justify scientific claims, since justification rests on necessity embedded in mathematics.

Leibniz does not subscribe to such a justificatory programme, as I argue in the following, yet he is one of the strongest supporters of a “mathematization of the natural world”.[[10]](#footnote-10) We can reliably use mathematics to grasp more stable properties of the world, such that metaphysics should be mathematically informed, without being committed to the claim that the world – in its metaphysical constitution – has mathematical structure. This move apparently weakens the justificatory force upon which one bases the application of mathematics to nature. I shall argue that it instead consolidates and guarantees the independence of mathematical discovery from natural ones (IV.), reassessing metaphysical and epistemological issues related to mathematizing nature. It moreover seals the independent development of sciences and metaphysics, determining the relation between metaphysics and mathematics: mathematics cannot settle metaphysical issues (V.). Despite this respective autonomy, natural sciences and metaphysics are relevantly related to the point that the application of mathematics to bodies has a metaphysical ground that makes phenomena and their expressions real (V.-VI.). To unpack this view, I argue, Leibniz re-thinks the category of “body” (II), and this has an impact on his theory of cognition (III.).

II. At the Source of the Problem: Analysing Bodies in Terms of Matter

The body is one of the most controversial philosophical concepts of the early modern period – and beyond. The quest for a robust understanding of what bodies are ranges from metaphysics to politics and constitutes the main object of natural philosophy – broadly construed to include physics and the (then) emerging life sciences, such as biology. In all these disciplines, the tacit assumption is that bodies must be analysed in terms of *matter*.

The term ‘body’ indeed mainly refers to an aggregate of parts, each of which has the property of being extended, to have a degree of impenetrability, and resistance. Parts of a body are hence smaller bodies which can or cannot end in the smallest parts, depending on the theory. Since parts are material, understanding what matter is implies understanding what body is.[[11]](#footnote-11) There was no consensus among early modern philosophers on how to analyse matter. The offer ranges from Hobbes’ materialism of matter as passive to atomist theories of matter as being composed of small imperceptible atoms, to Descartes’ notion of matter as *res extensa*. Leibniz criticized the principal theories of his time. By briefly reviewing Leibniz’s main arguments against those theories, I highlight that they are underlined by the same criticism: assuming as fundamental what justifies our knowledge of the natural world represented through mathematical means. Metaphysics is hence in service of epistemology.

In Hobbes’ account, bodies are divided into material parts that work as parts of a machine.[[12]](#footnote-12) Each part moves because it is moved and transfers motion to a nearby part. Transfer of motion is the cause of various functions that the machine can fulfil. The main assumption of this view is that matter is passive and divisible into parts that are capable of receiving and forwarding motion through *conatuses*. To explain *all* phenomena, it suffices to assume that matter and motion exist. Everything is explicable in mechanical terms, i.e., through reduction of states to motion of bodies taken as causes of other motions.

In the atomist worldview, matter is constituted by small, indivisible (hard and impenetrable) particles moving because they are set free in a vacuum. Each particle differs in size, shape, and motion and constitutes bigger bodies through collision and transfer of motion. The main assumptions of this theory are that matter is divisible to a certain point – hard atoms – and that motion, size, and shape are fundamental properties of bodies.[[13]](#footnote-13) As with Hobbes, any phenomenon can be explained through causes and effects reduced to impacts among particles. Since properties of atoms are mathematizable, bodies are objects of quantifiable sciences.

Descartes disagrees about the existence of atoms: for him, there is no vacuum and the world is a plenum.[[14]](#footnote-14) This plenum is constituted by *res extensa* that is indefinitely divisible (there is no minimum). *Res extensa* divides because it is set into motion. Imagine a fluid in a box and imagine agitating the box. The water moves and there are changes in the liquid, like local waves. So, imagine the world being a plenum and consider that every division one can find in bodies is the result of motion on that part of the extension. There is no limit to the division. Particular bodies are the result of this division and can be quantifiable *because* they are constituted by *res extensa*. Bodies are just smaller or bigger pieces of a plenum whose fundamental property is to be extended; therefore they are explicable in mechanical terms.

Leibniz criticizes these three accounts, saying that Hobbes’ error is to assume matter to be passive. If matter is characterized by passivity, where does motion come from? There must be something active in matter, and in Leibniz’s view, these are forces.

Though flirting with atomism throughout his life,[[15]](#footnote-15) Leibniz never subscribes to this theory. If there is a vacuum and atoms are passive matter, how can we explain the fact that they “stick together” (division-to-dust)?[[16]](#footnote-16) Bodies require true unities to be aggregates. Even if atomism seems to satisfy this necessary assumption of any theory of composition, the reduction of true unities to small hard bodies, Leibniz argues, is a solution that satisfies the imagination, but not reason. In an exchange with Hartsoeker, Leibniz says that the notion of atoms as perfectly-alike constituents of bodies, which differ only in shape, size, and motion, satisfies the imagination, but is inadequate for the intellect: “Atoms are the effect of the weakness of our imagination, that aims at having rest and rushing to an end in the subdivisions or analyses; nature is not like this, since it originates from infinity and tend to infinity. Therefore, atoms satisfy the imagination, but shock superior reasons”.[[17]](#footnote-17) If atoms are extended and extension is infinitely divisible, why should nature stop at some point? The endpoint of division is motivated only by our imagination, which needs an end to understand composition. Another form of unity is required, a substantial one: substantial forms.

The plenum thesis of Cartesian thinkers is not fully robust either, since it is subject to the problem of the labyrinth of the continuum. If bodies are extended, then they are indefinitely divisible. And if they are, there is no minimum, but if there is no minimum, no true unity, how can bodies be aggregates of parts if there are no parts? Leibniz’s diagnosis of Cartesian Dualism of mind and body, *res cogitans* and *res extensa*, can be summarized in the well-known formula scholars have worked with since at least the publication of *Monadology*: *in rigore metaphysico*, bodies cannot exist because, if they did, we would face the labyrinth of the continuum. Ergo, Leibniz veers toward a form of monism[[18]](#footnote-18) that culminates in his theory of monads as only existent substances.[[19]](#footnote-19) But what exactly is the problem with Cartesian extension?

A correct historical answer to this question, one that follows Leibniz’s way of unfolding the argument, should begin with the problem concerning the labyrinth of the continuum and deduce that matter cannot be continuous, therefore it cannot be constituted by *res extensa*.[[20]](#footnote-20) I want to flip the argument and build on its metaphysical, and, as Arthur remarks, nominalist[[21]](#footnote-21) premise: Only concrete things can be metaphysically fundamental. Concrete things are determined to the least of their parts, therefore they are not *abstracta*. If *res extensa* is indefinitely divisible, as Descartes wishes, extension is continuous and not determined. Extension is abstract and ideal, therefore it cannot be a substance. Descartes postulates as a substance (what can subsist *per se*) something that cannot be metaphysically fundamental.

Leibniz concludes that bodies are not composed or constituted by matter, and what we call matter is not analysable in terms of bodies. The error committed by Atomists, Cartesians and Hobbes was to start from appearances of bodies and abstract those qualities that all bodies have, which therefore must constitute a substratum homogeneous to all bodies: matter. What those theories really do is to make an assumption about the nature of bodies to analyse matter. They can analyse bodies in terms of matter because matter is analysed in terms of bodies. Leibniz’s conclusion is that those theories postulate the priority of epistemology over metaphysics: they aim at justifying the experiential behaviours of bodies, ignoring the fact that bodies, as they appear, are incompatible with metaphysical principles, like the identity of indiscernibles and sufficient reason. They satisfy the imagination, but contradict reason, insofar as they assume parts and composition to be fundamental to whatever exists.

III. From Metaphysics to Epistemology

The outcome of the previous section can be reframed as the expression of Leibniz’s disagreement that we should account for human thinkers’ ways of understanding bodies through a metaphysics that supports their basic intuitions about the world. He develops a highly counterintuitive metaphysics, although, as we show in the rest of the paper, it can account for human knowledge of the natural world. To this end, I shortly present its main points in order to argue that metaphysics grounds epistemology.[[22]](#footnote-22)

Arthur insightfully argued that the problem of the continuum, combined with Leibniz’s metaphysical and scientific interests, pushed Leibniz to develop a metaphysical understanding of matter that sees matter as *constituted* – not composed – by monads.[[23]](#footnote-23) The term ‘constitution’ is loaded with a peculiar meaning. Whatever exists cannot be understood as an aggregate of matter to which a monad is added as a part to form a being. Matter and monads cannot form an aggregate because they are not homogeneous: matter is actually divided, and monads are true unities. Monads cannot therefore be a part of a composite made out of matter. Monads rather *constitute* matter, meaning that monads instil a variety of actions in matter by giving it a form. As points are not parts of a line but their extremities, and lines are not parts of surfaces but rather their limit, so monads constitute matter as points constitute the continuum of a line: not by being a part of the line, rather by a being *possible* situation of points as coperceived, i.e., appearing continuous to an observer (more on this soon).

Leibniz hence applies to matter his theory of the actual infinite.[[24]](#footnote-24) Matter as infinitely actually divided implies that each part of matter is divided, such that for each division, there are always more. If we take a part of this matter, there will be divisions, and if we could take a smaller one, we could find other divisions. Since divisions in matter must be regarded as constituted by monads, for each part of matter there are monads that constitute it. Monadic constitution brings variety into matters: imagine your body, it divides into its organs; take an organ, it divides into tissues, tissues divide into cells, and cells into something else. At each level of this division, whatever there is has a form, a constitution: matter is neither pure *res extensa*, nor passive. So understood, matter does exist, but never without monads. The result is that for Leibniz bodies are organic and living, i.e., constituted by forms: “all bodies are either organic or collections of organic bodies,”[[25]](#footnote-25) a view he thought was confirmed by recent discoveries made by observations with microscopes.[[26]](#footnote-26)

What does this metaphysical view imply for cognition? Matter constituted by monads is metaphysically determined. This means that any single portion of matter we can individuate exploits the range of possible actual constitutions even if cognizers are not capable of grasping the totality of those constitutions. Think again of the organic constitution: if you see an organ in its functioning, you cannot see the cells and if you use a microscope to see the cells, you cannot see the organ in its functioning. Both divisions are ‘there’ but human cognizers cannot perceive them at once. Now, imagine a human observer who finds what appears to her the smallest piece of visible matter (maybe through a microscope). This matter will appear as a kind of unity (in one piece). Metaphysically, however, one must assume that what appears simple is a form of abstraction from further actual divisions that the observer cannot detect.

A similar argument can be drawn for time. Time depends on the coexistence of a certain distributions of parts perceived simultaneously as co-present. Succession is a change in how parts are co-present. If there are parts we cannot perceive, likewise there are durations and changes we cannot perceive. What we describe as a state of a body may be composed of states we do not know about.[[27]](#footnote-27) This causes a gulf between the discreteness of concrete existing things and the continuity of abstract, ideal bodies. There are divisions in matter – worlds into worlds – whose existence cannot be determined by human minds. The phenomena we detect are only a small part of what there is and are therefore appearances.

To summarize, Leibniz’s metaphysics builds on the denial that bodies are continuous and hence that matter is *res extensa*. What cognizers call bodies in their everyday experiences are, to a certain extent, *appearances* to said cognizers, i.e., phenomena. If bodies are those continuous things we observe in our everyday life, things that move together continuously through space, then what appears cannot be what there is metaphysically, the actual discrete constitution of matter with monads. The table in front of me appears to be smooth and rectangular, as having a weight or a breadth and length, but, *in rigore metaphysico*, I cannot attribute these properties to the body when considered metaphysically. *In rigore metaphysico*, I cannot even claim that there is *one* real thing corresponding to the one body I perceive, or a certain time in which the body is in the same state. Those things I perceive as one, and to which *I* bestow unity – both in the sense of composition (that thing is one, different from another thing) and permanence (that object has different states but endures as one same thing) – do not justify me in saying that its unity is metaphysically primitive. What appears to me as a continuum is *actually* discrete in a way I cannot grasp (more on this in the next section). We are drawn to idealism about bodies.[[28]](#footnote-28) Is it thus sufficient to argue that bodies are not metaphysically real?

Before turning to this question in the last paragraph of the paper, we can draw a first conclusion that seems to have drastic consequences for Leibniz’s epistemology. According to Leibniz, it is a mistake to infer properties of matter starting from its phenomenal appearance as bodies. This error led his predecessors to assume as fundamental some entities that cannot be substances.

This criticism also urges some intervention on the side of epistemology. If bodies are continuous while matter is not, *bodies* are mathematizable, but metaphysical entities are not. What justifies the application of mathematics to nature? What kind of certainty do predictions based on mathematical models for the description of nature have? And what do we refer to when we talk about bodies?

In short, Leibniz’s theory seems not just to collapse into body idealism, but also to lack any solid (metaphysical) ground justifying the application of mathematics to bodies. If bodies are constituted by matter, and matter is extended, then it *is* mathematizable, there is an identity relation between the mathematical description and the real thing. But if matter is discrete, how can Leibniz advocate for a mathematization of the natural world on this metaphysical basis?

The answer I give to this question is that he can because he identifies extension with bodies, although he does not analyse bodies in terms of matter, but in terms of space and time (IV.). Bodies are a scientific – not metaphysical – category that must be analysed in scientific terms, i.e., conformity to a subject matter that can be regimented through mathematical means, and, when regimented through mathematical means, this increases human capacity to acquire knowledge in a certain domain. This new vantage point generates different metaphysical and epistemological issues, first: the autonomy of scientific and metaphysical enquiries (V.).

IV. The Solution: Analysing Bodies as Spatiotemporal States

Each criticism of Leibniz, respectively addressing Hobbes, Atomists, and Descartes, can be subsumed under a more general one: postulating the metaphysical subsistence of entities human cognizers require for their mathematical understanding of nature. What satisfies their criteria of *knowledge* of observable phenomena also gives a criterion and a justification to postulate the existence of those entities as fundamental. Descartes’ assumption of matter as *res extensa* is one example; another is Newton’s ideas of absolute space and time, subject matter of the controversy between Leibniz and Samuel Clarke (1714-15).[[29]](#footnote-29)

The controversy with Clarke is about metaphysics insofar as it concerns Newton’s claim that, to justify our scientific understanding of physics, we must assume space and time as being absolute. If space and time are absolute, they must *exist* as containers of all bodies and their states. This postulation guarantees that our theory can determine which body moves absolutely and not only relatively to another body or a place. Leibniz criticizes the validity of an inference that moves from what minds require for conceiving of bodies to what there is metaphysically. Leibniz’s strategy rests on arguments designed to show that absolute space and time violates metaphysical principles, like the identity of indiscernibles and sufficient reason. There is another issue addressed in the correspondence: if space and time are not real entities, how do human minds form a notion of space and time as absolute? In the fifth letter to Clarke, §47, in a last attempt to convince his correspondent that space and time, as absolute, are ideal things that *minds* assume as being outside bodies for convenience in communication, Leibniz shows “how men come to form the notion of space to themselves.”[[30]](#footnote-30)

The text is long, but the overall idea is simple: forming the notion of absolute space starting from observable relational properties of space and time. Leibniz begins with assuming that there is an act, called coperception, according to which we perceive things as co-existent. To be co-existent means to be situated. One thing is situated when we perceive it in relation to other things, i.e., as having a distance from other things. Relation of distance and simultaneity are the most basic relations. Now, suppose that one of the coperceived bodies moves and takes the place of another coperceived body. Seeing something moving means seeing something as changing its relation of distance with other coperceived things. Changes in the arrangement of bodies are successive and there is a state in which the moving body moves another body away and occupies its place. Leibniz explains this “taking the place of” as a fiction of the imagination. *Minds* distinguish the body from its place and imagine that the body changes and takes the *same* place as if there were something identical existing without bodies. From this idea of place, minds form the idea of space: “Space is that which results from places taken together” (Ariew, 2000: 46):

“But the mind, not contented with an agreement, looks for an identity, for something that should be truly the same, and conceives it as being extrinsic to the subjects; and this is what we call place and space. But this can only be an ideal thing, containing a certain order, in which the mind conceives the application of relations.” (GP VII, 401, trans. In: R. Ariew (ed.): *Leibniz and Clarke*, p. 47)

When we refer to bodies as in space, hence, we tend to mistakenly think that they are ‘outside’ in some real place. What we mean by that is that bodies are those things that are subject to spatiotemporal relations and are therefore ideal. We will develop this view.

We observed that, as a philosophical term of the seventeenth century European philosophy, the term ‘body’ refers to an aggregate of parts each of which has the property of being extended, of having a degree of impenetrability, of resistance. Parts of a body are hence smaller bodies which can or cannot end in smallest parts, depending on the theory. We said that for Leibniz bodies so understood are ideal in the sense that they do not exist as cognizers perceive them.

The notion of a body stems from the mind’s capacity to apprehend an ordered arrangement of parts, each coperceived as occupying a different situation and hence, as external to the other parts. The parts are taken by a cognizer as a mereological unity, i.e., a whole of the parts, each of which submits to the rules of space and time as relational. This means that to form a whole, parts must be coherent. In its most fundamental meaning, ‘coherent’ refers to a property of imaginary points that merge together and appear *as if* there is no distance among them. So a certain kind of continuity among parts must characterize a body, although what makes a body *one* body is the fact that there is a cognizer taking it as *one*, i.e., coperceiving parts as members of a whole. Coherent in a more complex sense refers to a certain order according to which the body may change.[[31]](#footnote-31) A change in the arrangement of parts of a body corresponds to a change of state. States of bodies are simultaneous when they are coperceived; states of bodies are successive, when they are perceived as following each another. Conceiving bodies as aggregates of states means that the present state of a body (a coperception of parts as parts of a whole, each of which is external to the other, and still relevantly connected to the whole) is perceived as a possible outcome succeeding another combination, i.e., a previous state initiator of the actual one. Simultaneously, the present state is itself considered an initiator of another possible state. Bodies are a multitude of states that occurs in agreement with space and time and that can be explained mechanically.

V. Mathematizing Bodies – Its Epistemic Consequences

A body is an aggregate whose elements are states – spatial and temporal – apprehended by a cognizer because they appear in simultaneous and successive relations. Bodies are four-dimensional objects, i.e., an aggregate of coherent states that can or cannot be simultaneous and/or successive. The states must happen in space and time, meaning that they are subject to the law of continuity proper of continuous quantity.[[32]](#footnote-32) As an aggregate of states, the question about the existence of bodies changes in meaning. When we ask whether a body can exist, we ask if that aggregate of parts and states can be a *coherent object* of perception to an observer. ‘Can’ expresses the laws to which the object is subject to. They may be physical laws, statements we hold for true with respect to types of objects, but it cannot violate the continuity of spatiotemporal relations, since this is the minimum to have *physical* bodies: states perceived as coherent and possible phenomena of experience.

Since bodies are spatiotemporal objects, and space and time are continuous magnitudes subject to mathematics, we can apply mathematics to bodies and account for bodies and their parts however small mathematically. But if space and time, as absolute, do not exist, and bodies are constituted by space and time, then bodies do not exist as mind independent substances. If the subject matter of natural sciences is bodies and their states – broadly construed – then sciences are not about the ultimate constitution of whatever exists. But this does not prevent cognizers *from analysing bodies in terms of bodies*, i.e., as explicable in terms of processes that – to be *natural* and subject to sciences – must conform to mechanical laws.

For Leibniz, hence, bodies are mathematizable because they are aggregates of spatiotemporal states and, as such, *must* be explicable in mechanical terms. Leibniz maintains that bodies cannot exist without a monadic constitution or substantial forms. However, the introduction of substantial forms in metaphysics does not affect the principles and foundation of sciences.[[33]](#footnote-33) A form cannot interact mechanically with a body’s parts, therefore the assumption that there is a form in things, like the power to move, does not explain movement.

“But we perceive nothing distinctly in matter save magnitude, figure, and motion. If someone wishes in addition to ascribe to bodies a substantial form or a soul, and likewise sense and appetite, I do not contradict him, but I maintain that this contributes nothing toward explaining purely material phenomena and that it is not sufficient to say that a heavy body senses and desires the earth unless we explain at the same time how this sense and this desire arise […]

The passage continues in a puzzling way that I hope to make clear in the next section:

“In this way we should finally have to come to the construction of the organs of the sensing being, that is, to the mechanical principles. For what happens with perception happens nonetheless mechanically, and to the passions of the soul there correspond bodily motions in the organs which always follow mechanical laws.” (“An introduction on the value and method of natural science” trans. in: L.E. Loemker, *Philosophical Papers and Letters: A Selectio*n, Dordrecht- Boston-London 1989 (=L), p. 288)

As the first part of this passage suggests, metaphysics ends where natural science begins, but natural science ends where metaphysics begins. No matter how small we can go, the analysis of bodies leads to smaller bodies and hence cannot be a way of determining what subsists metaphysically.

Moreover, metaphysics plays a role in sorting out hypotheses, not in the sense that the object of metaphysics (substances) is the object of sciences (bodies). Metaphysical principles offer further criteria to support the ruling out of hypothesis that only seem valid,[[34]](#footnote-34) as we saw in discussing Hobbes’, atomists’, and Cartesians’ theories of matter. The problem with these theories was to pretend to analyse what there is metaphysically in terms of how we conceive of nature abstractedly, ignoring that metaphysics is about concretes and concretes are determined by indiscernibility. Any physical theory violating this metaphysical principle, or the principle of sufficient reason, must be rejected as metaphysically impossible. As Leibniz writes to Clark: “Those great principles of sufficient reason and identity of indiscernibles change the state of metaphysics. That science becomes real and demonstrative by means of these principles, whereas before it did generally consist in empty words.”[[35]](#footnote-35) In the end, what Leibniz advocates for is the fundamentality of metaphysics and its primacy among the sciences. Despite this priority, as I will explain soon, metaphysics must be scientifically informed to be a demonstrative science.

To conclude this section, although I agree with Garber[[36]](#footnote-36) that Leibniz wants to maintain questions about scientific analysis of bodies separate from questions about what matter metaphysically is, I do not think the result of this operation is that only immaterial substances are real. To highlight this point, the next section offers an answer to what makes metaphysics more fundamental than natural sciences. The answer is that metaphysics is more fundamental because it explains why phenomena and our knowledge of them are *real*.

VI. Mathematizing Bodies – Its Metaphysical Ground

Garber[[37]](#footnote-37) reports the following passage as an exemplar of the deep change that occurred in Leibniz’s metaphysical reflections at some point in the 1690s, when Leibniz allegedly realized that the theory of corporeal substances is untenable since only monads can be substances and bodies are mental phenomena:

“Indeed, considering the matter carefully, it should be said that there is nothing in things except simple substances and in them perception and appetite. Moreover, matter and motion are not so much substances or things as the phenomena of perceivers, the reality of which is located in the harmony of perceivers with themselves (at different times) and with other perceivers.” (Leibniz to de Volder, 30 June 1704 in: GP, II, 270, trans. in: P. Lodge (ed. and trans.): *The Leibniz-De Volder Correspondence*, New Haven-London 2013 (=LDV), pp. 306–7)

At first, such a statement may seem to contradict the interpretation here advanced. The passage might seem like an attempt to provide a metaphysical analysis of bodies, one that resolves bodies in immaterial entities and their modifications, perceptions and appetites. A closer look at what we have said so far shows that analysing bodies in terms of space and time *is* analysing bodies in terms of perceptions and their agreement, exhibiting a kind of metaphysics of bodies capable of establishing its foundational role towards other sciences.

If the former analysis is correct, bodies are constructed by cognizers as extended and, therefore, they can be mathematized because they are homogeneous with the subject matter of mathematics, quality and quantity, space and time. But how is analysing bodies in terms of space and time equivalent to analysing them in terms of perceptions and their coherence? To answer this question, we shall briefly look into Leibniz’s theory of perception.

Even if monads are simple and immaterial, they are subject to changes insofar as they perceive and appetize (*Monadology* §§1-17 GP VI 600-2/L 643-4). A perception is a multitude in the unity of the monad (*Monadology* §14 GP VI 600/L 644) and to exemplify this claim, Leibniz invites us to think of the constitution of what is considered simple sensation, like the sound of the sea (NE 53). When one hears the sound of the sea or sees the colour green, one thinks they are dealing with something simple, when indeed it is complex. If we reflect on the constitution of the sea, for instance, we must think that the parts are bodies (drops) that impact and produce ‘smaller’ sounds. What *we* perceive as a unity is in fact what a cognizer can capture of a more complex *process* whose actual divisions are precluded to her. Why do I say a process? Because time for Leibniz depends on the coexistence of a certain distribution of parts, i.e., the arrangement of space. But space for Leibniz is relational, which means that any arrangement of space is a certain configuration of parts of space, i.e., situated bodies. To be situated is to be perceived and, if two parts are co-perceived it means that they are perceived as distinct. Distinct here may assume two forms: it can be distinct because the parts are perceived at a certain distance, or because they are seen as contiguous, as parts of a bigger whole. Distinct in time means that two different configurations are taken as configurations of a same body, i.e., as two different states of a body.

This picture entails a major consequence. To be in space and time *is* for Leibniz to be perceived. But if ‘to be perceived’ is an expression of a multitude in the unity of the monads, perception has the same discrete structure we found in matter constituted by monads, even when what we perceive appears simple. It follows that any state we can distinguish in bodies never guarantees that what *we* perceive as one state *is* one state. Whatever we distinguish in bodies is a state of a body and hence expressible mathematically by a cognizer capable of doing so. Regardless of how small one goes, the question concerning the metaphysical ground for applying mathematics to nature remains unanswered. Leibniz’s solution in the passage to De Volder is to assert a correspondence – a harmony – between cognized states and discrete actual changes, and the correspondence is guaranteed in perceptions.

Experience – a repeated succession of phenomena and their states – manifests a coherence among phenomena and their states confirmed both by a single cognizer (who can judge and develop expectations about phenomena confirmed by new experiences) and by many similar cognizers (insofar as those cognizers can agree on facts and their consequences). This double-checked coherence, from a first and third person perspective, gestures toward a *harmony* among perceptions and appetites as changes of monads. What does this mean? The answer to this question is related to the second puzzling part of the passage quoted in V.: we must look into the construction of organs allowing for perceptions and observe that everything in matter happens mechanically, although it is expressed by monads without mechanical interactions.

Let us restart from the structure of matter as discrete. This ‘real matter’ is like a plenum, constantly acting and being acted upon because there is something active in it, i.e., monads. Monads’ activity *is* perception, i.e., an expression of how a certain corporeal substance is acted upon by and reacts to other corporeal substances. Each monad expresses with more clarity and distinction only those changes that aptly affect the organs of the organic body the monad is a constituent of. The result is that a perception that distinguishes parts and its successions is a *true* representation of the world from the point of view of the cognizer. As the cognizer constitutes bodies as being in space, she also constitutes her body as being in space,[[38]](#footnote-38) as having proportions and relations to other bodies. Other cognizers do the same and the result is a world of bodies represented from the perspective of the cognizer. In distinguishing bodies and their parts, each cognizer expresses one of the many actual divisions in matter, even though many other divisions are precluded to her. Any division we make in nature, nature makes too, although it may not be as precise as when we do it (LSS 328/II 4 352). Harmony is required to guarantee that although each cognizer expresses the world from her point of view, each view must agree with those of other cognizers relevantly similar among them.

Cognizers’ distinctions into bodies on a macrolevel are the results of ignorance of divisions that occur at a micro level, and so on to infinity. At each level, however, there are sentient beings, making distinctions in accord with experience and *expressing* some of the actual divisions in discrete matter constituted by monads. At the level of minds, insofar as phenomena are coherent, i.e., there are laws governing them, we have guarantees, but no geometric certainty, of expressing something real. We are like geometers sketching the figure on the sand; although not perfect, we can ignore the error because it is insignificant when accompanied by a calculation.

“However eternal truths based on limited mathematical ideas are still useful to us in practice, in as much as it is acceptable to set aside the inequalities too small to be able to cause significant errors in relation to the proposed purpose; just as an engineer who draws a regular polygon on the ground is not bothered if one side is longer than another by a few inches.” (Leibniz to Sophie, 31 October 1705, in: A II 4 353 trans. in: L. Strickland (ed. and trans.): *Leibniz and the Two Sophies: The Philosophical Correspondence*, Toronto 2011 (=LSS), p. 338)

“The ideal [unities] represent a whole which is not a perfect unity, but which our understanding takes as one thing, even though it is an accumulation of several, in order to have the convenience of reasoning about several things all at once, and that which is common to them and which has a connection not only to nature but also to existence.” (Leibniz to Sophie, 31 October 1705, in: A II 4 344 trans. in: LSS 328, my emphasis)

Harmony hence serves to guarantee the connection between bodies, their mathematical expressions, and their metaphysical ground – although this guarantee is not *a priori*. The necessity of monads and perceptions can be deduced *a priori*, since through introspection we can experience a variety of perceptual states.[[39]](#footnote-39) The harmony among perceptions is, however, just an inference to the best explanation, a hypothesis, as Leibniz writes to Lady Masham,[[40]](#footnote-40) because we can always imagine that there is no correspondence between phenomenal coherence and concrete beings, as Locke in his *Essay* seems to suggest – a position Leibniz opposes (NE 380-3). In this scenario, metaphysics is not possible, since we should assume that God creates a world where the principle of sufficient reason fails. This world can be ruled out *a priori* only if we assume that God cannot act against this principle, which is questionable; therefore harmony is just a metaphysical hypothesis, although it is the *best* hypothesis as it is the only one that guarantees the reality of phenomena without violating metaphysical principles: what exists is concrete, a thing that implies the maximum of varieties, although to explain variety one needs to assume the simplicity of substances.

VII. Conclusion: Back to Mathematics

The hypothetical metaphysical ground of bodies should not bother those who enquire scientifically into phenomena. When we pursue metaphysical enquiry, we must consider the distinction between bodies and substances, but, when we pursue scientific enquiries, we can do so *as if* only bodies and mechanic laws exist. Bodies as objects of sciences are phenomena whose existence is confirmed by coherence given by scientific laws. By showing that bodies, as we conceive of them, cannot be substances, metaphysics at the same time proves that this does not make any difference for scientific explanations since the error that may arise is too small to bother us. When we refer to bodies outside us, we are referring to something real.

Despite this autonomy, Leibniz believes in having a solid metaphysical ground to apply mathematics to nature, though avoiding mathematical realism. Nature is expressible in mathematical terms insofar as everything existing as discrete and concrete is expressible by cognizers as an extended body. What cognizers express as extended, having a size, moving and interacting mechanically with other bodies is ideal but it tracks something real – monads and their forces – because it is grounded in the perceptions of substances constantly expressing all other substances. Bodies appear with a certain unity deduced by observation: the parts move together, they are impenetrable to other bodies and are elastic, and cognizers may agree on these properties, although those are not the most fundamental ones. If there are changes that minds do not usually perceive, there must also be other smaller bodies. Experiments with the microscope confirm this view and incline Leibniz to think that there are not just smaller bodies but living bodies capable of perceptions. Since at each level there are living beings endowed with organs, at each level there are beings that track some changes, and their interactions harmonize. If at every level what is expressed tracks something real, every division actually exists, even if human cognizers cannot notice them and must rely on what appears. We *must* rely on experience and observation to know which divisions and connections of bodily states are real.

To conclude, natural sciences deal with phenomena, no matter how small, whose method and object are determined independently from questions about the metaphysical constitution of bodies. Since phenomena are spatiotemporal, they are quantifiable and potentially expressible in mathematical terms. Since appearances are grounded in the correspondence between perceptions and actual divisions, we can say that sciences are about what is real in phenomena. Nonetheless, the reality of phenomena is not guaranteed by harmony, but by the fact that cognizers succeed in the prediction of phenomena’s behaviours based on the assumption that everything has a cause explicable in mechanical terms. Even if human observers are compelled to use abstract and ideal entities to capture what concrete beings have in common, the bond to reality is preserved by observations, phenomenal agreement and agreement among members of the scientific community.

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2. G. Galilei, *Assayer* (1623) trans. in: S. Drake (ed.): *Discoveries and Opinions of Galileo*. Garden City, N.Y. 1957, pp. 237-38. [↑](#footnote-ref-2)
3. For a thorough discussion of Galileo’s epistemological and ontological basis for this claim, see C.R. Palmerino: “Reading the Book of Nature: The Ontological and Epistemological Underpinnings of Galileo’s Mathematical Realism”, in: G. Gorham et al (eds.)., *The Language of Nature*. *Reassessing the Mathematization of Natural Philosophy in the Seventeenth Century*, University of Minnesota Press 2016 (= *The Language of Nature*), pp. 30-47. [↑](#footnote-ref-3)
4. For a more comprehensive reconstruction of the position towards mathematization, see G. Gorham, B. Hill and E. Slowik: “Introduction”, in: G. Gorham et al (eds.)., *The Language of Nature*, pp. 1-28. [↑](#footnote-ref-4)
5. The word “disenchantment in this context is loaded with theoretical meaning, as Weber remarks: “W‌e are not ruled by mysterious, unpredictable forces, but that, on the contrary, we can in principle *control everything by means of calculation*. That in turn means the disenchantment of the world. Unlike the savage for whom such forces existed, we need no longer have recourse to magic in order to control the spirits or pray to them. Instead, technology and calculation achieve our ends” (M. Weber, “Science as a Vocation”. in: D. Owen – T. B. Strong (eds.), *The Vocation Lectures*, Indianapolis 2004, pp. 12-3). [↑](#footnote-ref-5)
6. “Scientia potentia est”, a phrase usually attributed to Bacon, but which can also be found in Thomas Hobbes. [↑](#footnote-ref-6)
7. For a teleofunctional account of Descartes, see R. De Rosa: *Descartes and the Puzzle of Sensory Representations*, Oxford: Oxford 2010, pp. 95-116. [↑](#footnote-ref-7)
8. D. Garber: *Leibniz: Body, Substance, Monad*, Oxford 2009 (= *Leibniz*), p. 115. [↑](#footnote-ref-8)
9. D. Rutherford: “Leibniz as idealist”, *Oxford Studies in Early Modern Philosophy* 4 (2008) (=*Leibniz as Idealist*), p. 152. [↑](#footnote-ref-9)
10. As I will extensively show, there are good theoretical reasons to argue that Leibniz rejects a mathematization of metaphysical entities. Therefore, I disagree with D. Jesseph (“Ratios, Quotiens and the Language of Nature”, in: G. Gorham et al., *The Language of Nature*, pp. 160-77) and K. Smith (“Leibniz on Order, Harmony and the Notion of Substance: Mathematizing the Sciences of Metaphysics and Physics”, in: G. Gorham et al., *The Language of Nature*, pp. 229-49). There is no mathematization of metaphysics, since the subject matter of metaphysics – substances – cannot be those of mathematics. And yet this claim does not contradict the fact that “my metaphysics”, as Leibniz writes “is all mathematics, so to speak, or it can become so” (Leibniz to De L’Hopital, 27 December 1694, GM II, 255–62). Metaphysics must be mathematically informed, although mathematics cannot settle metaphysical issues that rest on different principles, sufficient reason and indiscernible identity. [↑](#footnote-ref-10)
11. Cf. G. Gorham – E. Slowick: “Body and Extension in the Scientific Revolution”, in: D. Jalobeanu, C.T. Wolfe (eds), *Encyclopedia of Early Modern Philosophy and the Sciences*. Dordrecht 2020 <https://doi.org/10.1007/978-3-319-20791-9_24-1> [↑](#footnote-ref-11)
12. Cf. T. Hobbes: *Leviathan*, in: E. Curley (ed.), *Leviathan, with selected variants from the Latin edition of 1668*, Indianapolis 1994, and Id.: *Elements of Philosophy, The First Section, Concerning Bodies*, in: W. Molesworth (ed.), *The Collected Work of Thomas Hobbes*, Vol. I, New York. 1992. [↑](#footnote-ref-12)
13. For a reconstruction of Gassendi’s theory, see A. Lolordo: *Pierre Gassendi and the Birth of Early Modern Philosophy*, Cambridge 2006, Chap. 9, who shows that common-sense intuitions about properties of bodies are explained through atomism. [↑](#footnote-ref-13)
14. The works of Descartes are quoted according to the standard abbreviations: AT: C. Adam – P. Tannery (eds.), *Oeuvres de Descartes*, Paris 1897–1910 and 1964–1978, cited by volume and page; CSM: J. Cottingham – R. Stoothoff – D. Murdoch (eds.), *The Philosophical Writings of Descartes*. Vol. I-II, Cambridge 1984-5, cited by volume and pages. For Descartes, *Principles*, Cf. Part II, §23 AT VIII A 52-3/ CSM I 232. [↑](#footnote-ref-14)
15. R. Arthur: *Monads, Composition, and Force: Ariadnean Thread Through Leibniz’s Labyrinth*, Oxford 2018 (=*Monads, Composition, and Force*), Chap. 3. [↑](#footnote-ref-15)
16. Cf. D. Garber: *Leibniz*, pp. 63-5. [↑](#footnote-ref-16)
17. Letter from October 30, 1710 in GP III 507, my translation. For a discussion, see L. Oliveri: *Imaginative Animals. Leibniz’s Logic of Imagination* (=*Studia Leibnitiana, Sonderheft 57*), Stuttgart. 2021, pp. 242-3. [↑](#footnote-ref-17)
18. See “Système nouveau de la nature et de la communication des substances, aussi bien que de l’union qu’il y a entre l’âme et le corps” 1695 in: GP IV, 480. [↑](#footnote-ref-18)
19. The most influential reconstruction of Leibniz’s thought is offered by Garber (cf. D. Garber, *Leibniz*), who argues that the introduction of immaterial substances begins at the end of the 90s and culminates in his later metaphysics of monads. During the “middle period”, the time between is juvenile writings and *Monadology*, Leibniz tries to develop a metaphysics accepting corporeal substances, i.e., aggregates of matter and form, but this idea is dismissed in *Monadology*. [↑](#footnote-ref-19)
20. Cf. R. Arthur: *Monads, Composition, and Force*. [↑](#footnote-ref-20)
21. Although it is difficult to single out a cluster of statements we can univocally define as a nominalist position shared by early modern thinkers, there is one claim that we can consider the core of nominalism and to which people who referred to themselves as nominalists subscribe: abstract and general entities are not entities, since they cannot exist. For a discussion of Leibniz’s relationship to nominalist positions, see M. Mugnai: “Essences, Ideas and Truths in God’s Mind and in the Human Mind”, in: M.R. Antognazza (ed.): *Oxford Handbook of Leibniz*, Oxford 2018 (=*Oxford Handbook*), pp. 11-26; M.B. Bolton: “Theory of Knowledge: Mathematical and Natural Science”, in: M.R. Antognazza (ed.): *Oxford Handbook*, pp. 137-161. [↑](#footnote-ref-21)
22. I focus on Leibniz’s late period, in the late 1690s, when he already thought that spaces are relational. [↑](#footnote-ref-22)
23. Cf. R. Arthur: *Monads, Composition, and Force*. On the monadic constitution, see also D.Rutherford: *Leibniz as Idealist*. Although for Rutherford monadic constitution must be intended as the resolution of matter into mind-like substances, while for Arthur the monadic constitution does not imply that everything resolves to mind-like substances. Matter exists, although not like a substance, since it has the nature of being infinitely actually divided. [↑](#footnote-ref-23)
24. R. Arthur: “The Labyrinth of the Continuum”, in: M.R. Antognazza (ed.): *Oxford Handbook*, p. 282: “According to this view, it is perfectly legitimate to describe a plurality of things as actually infinite, without that committing you to there being an *infinite number* of them. To say that there are infinitely many prime numbers, for example, is to say that for any finite number you choose, there are actually (not merely potentially) more primes than this; it is *not* to say that there is a number of primes that is greater than any finite number, that is, an infinite number of them. The infinite, like the infinitely small, is thus treated as a *syncategorematic* term, one that derives its meaning from the sentence in which it occurs, but one for which there is no corresponding entity.” [↑](#footnote-ref-24)
25. See “Ex Cordemoii tractatu;” in: A VI 4, 1798/ R. Arthur: *Monads, Composition, and Force*, p. 285. [↑](#footnote-ref-25)
26. See A. Becchi: *Arlecchino e il microscopio. Saggio sulla filosofia naturale di Leibniz*, Milano 2018. [↑](#footnote-ref-26)
27. R. Arthur: *Leibniz on Space, Time and Relativity*, Oxford 2021. [↑](#footnote-ref-27)
28. Body idealism can be distinguished into three forms (cf. D. Rutherford, *Leibniz as Idealist*). Phenomenalism, divine phenomenalism, and qualified idealism. According to the first position, bodies are phenomena for minds. The second position adds that, even if bodies are appearances for minds, there is a true point of view: that of God. Qualified idealism (R. M. Adams, *Leibniz: Determinist, Theist, Idealist*, Oxford 1994) adds that even if bodies are ideal, there is a correspondence to something real. As Rutherford argues, this latter form of body idealism also implies matter idealism, or the view that matter is not real. Rutherford instead defends a form of matter realism according to which matter immediately derives its being real from its being constituted by monads, i.e., real beings. Arthur (*Monads, Composition and Force*) criticizes this view of Rutherford and argues that matter realism must be something stronger, i.e., matter actually divided never existing without the monadic constitution, as a line cannot exist without a point. Though veering towards Arthur’s position, I can at the moment remain neutral on what forms of idealism I subscribe to because, as Rutherford argues, all four accounts may be comprehended in a unified theory and are not mutually exclusive. My point is slightly different: I embrace body idealism, but I argue that it is not sufficient to claim that matter is not real. [↑](#footnote-ref-28)
29. As De Risi recently argued, it is questionable that Leibniz had a relational view about time early in his philosophical argumentation. This is not a problem for our account, because we focus on Leibniz’s view after the 1690s. [↑](#footnote-ref-29)
30. GP VII, 400, trans. In: R. Ariew (ed.): *Leibniz and Clarke: Correspondence*, Indianapolis 2000 (=*Leibniz and Clarke*), pp. 45-46. [↑](#footnote-ref-30)
31. The notion of coherence has recently been the object of study by Oliveri: *Imaginative Animals*, pp. 101-13. [↑](#footnote-ref-31)
32. For an analysis of the rules constituting space and time, see L. Oliveri: *Imaginative Animals*, pp. 129-138; 152-9. [↑](#footnote-ref-32)
33. A primary text on this topic is “Distinctiones Reflectionesque Metaphysicae*”*, where Leibniz distinguishes the different principles of certitude in science, metaphysics and cognition. See A VI 4 1394. [↑](#footnote-ref-33)
34. N. Hamid: “Teleology and Realism in Leibniz’s Philosophy of Science”, in: V. De Risi: *Leibniz and the Structure of Sciences. Modern Perspectives on the History of Logic, Mathematics, Epistemology,* Dordrecht-Boston-London 2019 (=*Leibniz and the Structure of Sciences*), pp. 271-98. [↑](#footnote-ref-34)
35. See GP VII, 372, trans. In: R. Ariew (ed.): *Leibniz and Clarke*, p. 22. [↑](#footnote-ref-35)
36. D. Garber: “Leibniz’s Trascendental Aesthetics”, in: V. De Risi (ed.): *Mathematizing Space. The Objects of Geometry from Antiquity to the Early Modern Age*, Dordrecht: Springer 2015 (= *Leibniz’s Transcendental Aesthetics*) pp. 231-44. [↑](#footnote-ref-36)
37. Ivi, pp. 247-8. [↑](#footnote-ref-37)
38. Cf. V. De Risi: “Leibniz's "analysis situs" and the localization of monads”, in: H. Breger (et al.): *Natur und Subjekt. Internationaler Leibniz-Kongress, Hannover, 26. September bis 1. Oktober 2011*, Hannover 2011, pp. 208-216. [↑](#footnote-ref-38)
39. Leibniz to Gottlieb Samuel Treuer, 1 June 1708, trans. in: Oliveri (ed.): “Leibniz-Treuer Correspondence”, in: *The Leibniz Review* 29 (2019), pp. 103-4. [↑](#footnote-ref-39)
40. Leibniz to Masham, 30 June 1704 (the same day he wrote the letter to De Volder quoted before) in: A II 4 354-8. [↑](#footnote-ref-40)