THE SUBJECT MATTER OF LOGIC

Explaining what logic is about

by

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To DJ, for a dream come true

ABSTRACT

Logics are formal systems with many different applications. The boundary between logics and other formal systems, like mathematics, is unclear. One way of clarifying this boundary is by appealing to the subject matter; defining their purpose, for instance, showing how the truth of the premises guarantees the conclusion, demonstrating reasoning, or modelling an argument.

There are two contemporary philosophical debates where the subject matter of logic is relevant. The first is that of logical pluralism, which needs a way to determine whether a logic is correct.[1,2] The second is the argument that the normativity of logic supplies a mechanism for determining whether a logic is correct.[3,4] In these debates, the subject matter of logic is relevant, but not discussed. And it does not need to be because the participants implicitly agree that the answer is validity. They also agree that this does not advance the debate.

Validity is a dead-end because it only transforms the question. Logicians clarify validity by giving a definition. But it is a well-established fact that there is more than one way of defining a relation of logical consequence. So, how does one determine whether a given definition is *correct*? Or to put it another way, how does one know whether that definition captures *validity*? On this point, opinions are deeply divided. Moreover, there is no clear strategy for resolving the difference of opinion. MacFarlane comments that appeals to intuitions about validity are prevalent in these debates. Furthermore, he remarks, these intuitions are a product of *education*.[5]

In many parts of the advanced literature, logicians agree that the subject matter of logic is validity. But this explanation won't do as an introduction for a beginner. As Newton-Smith says, this 'has the fault of explaining the obscure in terms of the equally obscure.'[6] Instead, teachers must give 'an informal and intuitive account of the matters with which logic is primarily concerned.'[7]

The constraints on introductory explanations provide an opportunity to investigate the subject matter of logic in a novel way and perhaps reveal the source of intuitions about validity, which might, in turn, shed light on questions of pluralism and normativity. My thesis examines the way that teachers introduce the subject matter of logic to beginners.

I begin by exploring a slice of history and a tradition of logic instruction. I argue that a reliance on this tradition leads to flawed teaching in the modern context. After that, I examine

1 J. C. Beall and Greg Restall, *Logical Pluralism*, Repr (Oxford: Clarendon Press, 2008).
2 Graham Priest, 'Logical Pluralism', chap. 12 in *Doubt Truth to be a Liar*, Reprint (Oxford: Clarendon Press, 2009), 194–211.
3 John MacFarlane, 'In What Sense (if Any) Is Logic Normative for Thought?' (2004).
4 Hartry Field, 'Pluralism in Logic', *The Review of Symbolic Logic* 2, no. 02 (9 June 2009): 342–359.
5 MacFarlane, 'In What Sense (if Any) Is Logic Normative for Thought?', p. 2.
6 W. H. Newton-Smith, *Logic: An Introductory Course* (Taylor & Francis, 2003), p. 1.
7 Benson Mates, *Elementary Logic* (New York: Oxford University Press, 1965), p. 1.

modern introductory texts and the strategies they use to present the subject matter of logic. I draw several lessons from this examination.

I conduct three interviews with Gillian Russell, Dave Ripley, and Johan van Benthem in which I ask philosophic and pedagogic questions directed by each of their interests. These dialogues are illuminating on their own. But brought together they show the interaction between the teacher's theory of the subject matter of logic and the practice of introductory logic teaching.

After this investigation into modern logic pedagogy, I present a framework for a solution to the pedagogic problem. It is a framework because there is no single best way to introduce logic to a beginner. But it is possible to develop a guiding structure. The solution which I present relies on a widely accepted 'trivial' form of pluralism and a kind of relativism which I introduce, and argue for, in this thesis. This discussion also includes practical suggestions for developing introductory logic courses.

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CONTENTS

INTRODUCTION: THE SUBJECT MATTER OF LOGIC

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> The decked boats on which the Bretons of the isle of Groix go fishing are marvellous machines. ...the curvature, the slope, the thickness are everywhere what they should be.
>
> Alain, Propos d'un Normand, September 1, 1908

There is a certain elegance to getting a beginning just right; a perfection to saying the most important thing *first*.

In philosophy, a traditional place to start is with a definition of terms, and of course, it is a crucial step. Though it is not the most important thing. The most important thing to give a reader at the very beginning is *a reason to care*.

You, dear reader, should care about this thesis because I will show you how to look at logic in a new way and explain why you might want to do that. My proposal is not radical, and not surprising, just something which has not been said before. Once I have said it, I am convinced that you'll feel that it was apparent all along. That is fine because novelty is not really what I am going for; *utility* is.

I want to say something useful about how to think about logic. I believe it will be helpful for both academic debates and logic pedagogy. It is a way of looking at logic that will benefit working logicians, logic students, and anyone with a casual interest in logic.

I will argue for a view on logic that combines a 'logic as modelling' view with subject matter pluralism to account for the diverse logics and logical investigations that take place within the discipline.

This thesis is devoted to providing evidence for subject matter pluralism. However, because I draw evidence from logic teaching, I also carefully consider both how pedagogic necessities may influence how the subject matter of logic is expressed in teaching and how the subject matter of logic may be better expressed within pedagogy.

1.1 WHAT IS LOGIC, AND WHY DOES IT MATTER?

The word 'logic' serves the dual purpose of both referring to the discipline and to systems within the discipline. To mark the difference I use 'a logic' or 'logics' to refer to systems within the discipline, logic. When I speak of the subject matter of logic I allow that to be ambiguous

between logic *quā* discipline and logic *quā* system, as the question of subject matter applies equally to both.

The subject matter of a discipline is, broadly speaking, what that discipline is about. A discipline's subject matter can be expressed in better and worse ways. For instance one could facetiously describe physics as the discipline that studies things bumping into other things. An inaccurate description of a discipline's subject matter is not a barrier to fruitful investigations in that dicipline.

In the philosophy of logic, however, this subject matter question is central. There is a long tradition of trying to distinguish logic from other disciplines in the philosophy of logic. What is a stake in these arguments is a claim to legitimacy. It is a way of laying claim to the place of some line of investigation within the discipline.

Despite a long history, there is no settled account of what unifies logic as a discipline, what belongs to the discipline and what does not. There is a collection of methods and some clusters of problems but a principled way to draw a sharp boundary remains elusive. An investigation into validity is a fair way to broadly characterise the discipline and is often tacitly accepted in debates in the philosophy of logic.

There is a sense in which the subject matter of logic is validity. Naturally, this explanation is no good for the beginner, since elucidation is supposed to come from a term that beginners cannot be expected to understand. However, the explanation is no better for experts – it is often precisely what is at issue in their debates: how do we define validity? What are its boundaries? These are questions that experts puzzle over in the philosophy of logic. So, explaining logic as being about validity is not any more elucidating for an expert than it is for a beginner.

One contemporary debate in which the subject matter of logic is relevant is logical pluralism; the normativity of logic is another. Both discussions are closely related to the subject matter of logic, but it is not explicitly discussed in either. Instead, there is an implicit agreement that the subject matter of logic is validity in both these debates. However, validity is a dead-end answer in these debates because it merely transforms the question.

Logicians clarify validity by giving a definition of a consequence relation. It is a well established fact that there is more than one way to provide such a definition. This introduces a problem: how does one determine whether a given definition is *correct*? Or to put it another way, how does one know whether that definition captures *validity*? G. Russell explains:

> Suppose we are also fortunate enough that one of the two logics is not mistaken – it is the right theory. Then in virtue of what is it right? The worry in the background here is that there might be no answer because there is nothing for the theories to be right about. There might be no such thing as 'valid simpliciter' but

> only 'valid$_I$' and 'valid$_C$' etc. But now what is to stop us making up any logic we like? ...The normativity of logic offers a possible solution: logic is about how one ought to reason.[1]

Like the debate over the normativity of logic, pluralism also has to do with the definition of validity. Pluralism is the thesis that within the normal meaning of validity more than one consequence relation may lay claim to being *correct*. Monism is the counter-thesis that there can be only one correct definition of validity.

Central to the notion of validity is the definition of a consequence relation. How a consequence relation is defined is an essential part of what it means for an argument to be valid in a logic. On the other hand, the meaning of validity that a logician is trying to articulate affects how they approach defining a consequence relation. The two concepts do not come apart easily, but they are not the same. Validity is the blanket concept; consequence relations are ways of spelling it out.

Making understanding validity more complicated is that there are two competing approaches to understanding logics: proof-theory, and model-theory. The proof-theoretic approach presents the definition of validity in terms of the logic's formation rules: a conclusion is validly derived when it is derived by following the logic's syntactic rules. The model-theoretic approach presents the definition by referencing the semantic meaning of the logic's symbols; and validity is the preservation of the inferential goal, e.g. truth. However, soundness and semantic completeness proofs in metalogic turn this into more of blue/black or white/gold dress phenomenon than a case of outright competition.

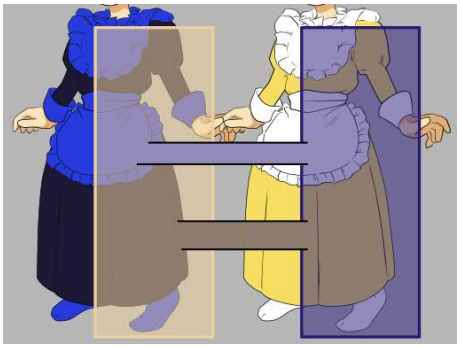


Figure 1.1: The dress can be perceived in two ways: black and blue under a yellow-tinted illumination (left); or gold and white under a blue-tinted illumination (right).

Just as figure 1.1 demonstrates how the dress can be perceived in two ways, soundness and completeness proofs can demonstrate equivalence between the two notions of validity. When there is both a soundness and a strong completeness proof for a logic, there are two ways to

1 Gillian Russell, 'Logic Isn't Normative', *Inquiry* 63, nos. 3-4 (April 2020): p. 378.

present a definition of validity that have the same extension.[2] Figure 1.2 shows the rough map I use to understand the terrain explored in logic.

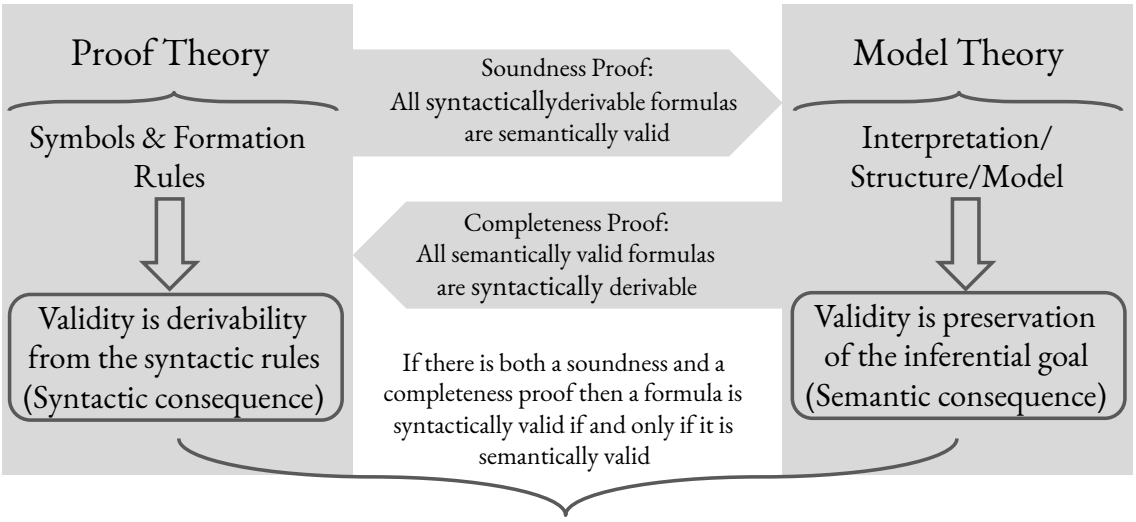


Figure 1.2: What a logic models; what it is *about*

Proof-theory focuses on logic's symbols and formation rules, but it is also a way of understanding validity. Model-theory offers a different way of understanding validity giving meaning to the symbols in a formal system beyond the formation rules. Metalogic explores the relationships between the proof-theoretic and model-theoretic components of formal systems. Like the blue & black/white & gold dress phenomenon, theorists are divided on the correct way to see validity.

1.2 LOGIC AS MODELLING

One way of looking at logical systems is to see them as models of logic's subject matter. Shapiro and Cook advance a logic-as-modelling view in which 'Different logics, viewed as models of various linguistic phenomenon, are correct relative to different theoretical goals, or relative to different ways of simplifying, idealizing, or precisifying the phenomena in question.'[3] Cook claims that the logic-as-modelling view is not controversial.[4] Indeed, it is not. This logic-as-modelling view is also like Priest's statement that: 'pure logics can be applied for many purposes, such as simplifying electronic circuits, or analysing certain grammatical structures. ...it is clear and uncontentious that different pure logics may be appropriate for each application.'[5]

2 There is, however, still a difference between the two notions because proof theoretic validity is finitary – it has a finite number of input values – whereas semantic validity is an infinitary notion

3 Roy T. Cook, 'Let a Thousand Flowers Bloom: A Tour of Logical Pluralism', *Philosophy Compass* 5, no. 6 (2010): pp. 500–501.

4 Cook, p. 501.

5 Graham Priest, 'Logical Pluralism', chap. 12 in *Doubt Truth to be a Liar*, Reprint (Oxford: Clarendon Press, 2009), p. 195.

However, Shapiro and Cook are pluralists, while Priest is a monist. One way to elucidate the disagreement is with an extended tool-use analogy. Just as with logics, many things are legitimate candidates for being considered a tool. Different tools are appropriate for different jobs, roughly corresponding to the distinction between pure and applied logic. Pure logic is just the investigation of logical systems themselves, whereas applied logic is when some system is used as a model of some phenomena. When we consider a particular job, we set a scope for judging between tools. Likewise, when we consider the application – what the logic models – we set a scope for judging between logics.

The tool-use analogy can help us explore an ambiguity. Consider the job of cutting. Cases of cutting include both chopping down trees and heart surgery. For both chopping down trees and heart surgery, a hammer is the wrong sort of tool. Weber says, 'An axe is good for chopping down trees, a chainsaw is better; a scalpel is good for heart surgery, but terrible for cutting down trees; while axes and chainsaws alike are bad for surgery.'[6] There is a family resemblance between the tools designed for cutting-type jobs and those for hammering-type jobs. When we consider cutting jobs, hammering tools are no good. Still, the scope set by cutting does not fully resolve which tools are better or worse for the tasks which fall within this domain. It is a similar ambiguity of scope that Beall and Restall exploit to generate their argument for pluralism.[7] They set a scope for logic – like selecting the job of cutting. Then they argue that within this scope, different logics are generated through different more precise, but equally correct, ways of spelling out validity.

Cook refers to the fact that there are multiple logics as trivial pluralism.[8] He is not trivialising the accomplishment of developing various logics. He is saying that there is nothing philosophically valuable to pointing it out. Field makes a similar point.[9] Another philosophically trivial point, relativism, is that judging a logic's value may depend on the task at hand – that a logic may be correct depending on what it is meant to be a model of.[10]

Shapiro and Cook use this relativist point on the way to arriving at their pluralist position. Priest uses it too on the way to his monist argument. Priest acknowledges the variety of logics – like acknowledging the variety of tools – but clarifies that his position is relative to a specific application.[11] Like explaining that we are only interested in the job of cutting down trees. Even within this scope, one could acknowledge variety, like admitting that there are a variety of axes. One could also acknowledge that there are other options, like chainsaws. But it seems entirely

6 Zach Weber, 'A Guide to Logical Pluralism for Non-logicians', *Think* 16, no. 47 (2017): p. 94.

7 Gillian Russell, 'One True Logic?', *Journal of Philosophical Logic* 37, no. 6 (2008): 593–611; J. C. Beall and Greg Restall, *Logical Pluralism*, Repr (Oxford: Clarendon Press, 2008).

8 Cook, 'Let a Thousand Flowers Bloom: A Tour of Logical Pluralism', p. 494.

9 Hartry Field, 'Pluralism in Logic', *The Review of Symbolic Logic* 2, no. 02 (9 June 2009): p. 348.

10 Cook, 'Let a Thousand Flowers Bloom: A Tour of Logical Pluralism', pp. 492-493.

11 Priest, 'Logical Pluralism', p. 196.

plausible that if we were specific enough about the job and how we want it done, there might be a single best tool; this is Priest's position.[12] However, it may be just as likely for there to be many equally good tools, which is Cook's claim.[13] These apparently different positions are compatible. Whether there is only one best tool, or many equally good tools depends on both the task at hand and the tools available. It is plausible for there to be only one, but equally possible for there to be many. The common ground between Cook and Priest is that whether something is *correct* is relative to what it is *for*.

The logic as-modelling-view modifies the approach to the question of the subject matter of logic. We are no longer asking what logic is *about*; instead we ask what it is *for*. Pluralism is an appealing position because it allows logic to be for many things.

Pluralism is opposed because it opens the door wide to unpalatable consequence relations. Some examples illustrate the problem. Two noteworthy examples of unpalatable consequence relations are: relations where any conclusion may follow from any premise – called the trivial logic; and relations where nothing follows from anything – called the empty logic. In this context, 'the' refers to a category, not a singular – many systems could be generated which are either trivial or empty – but there is little value in distinguishing among them. It is much easier to treat all instances in the category as being essentially the same. For many, an adequate philosophy of logic must explain why these consequence relations are illegitimate while also explaining the legitimacy of some other consequence relation.

I reject that position. The position I adopt is as Blake-Turner and G. Russell describe for the flat-footed logical pluralist: 'She concedes that we never deploy the trivial consequence relation, but insists that just because we don't put a consequence relation to use doesn't undermine its claim to being genuine.'[14] I insist that logics, whether trivial or empty, can still be meaningfully called logics. The role of a philosophy of logic is not to delegitimise logical systems.

The view I aim to develop is one flexible enough to cover all the investigations and systems which might reasonably and intuitively be called logic. I want the philosophy of logic that I advance to provide an adequate explanation of the practices and investigations of any sincere and competent speaker who claims to be a logician. What may make this seem far-fetched is that it is hard to see how the details could be worked out without the meanings of the words 'logic' and 'validity' collapsing into meaninglessness.

In this thesis, I will develop a position preserving the meaning of 'logic' in typical use and arguing that 'validity' is a concept with multiple meanings.

12 Priest, 'Logical Pluralism', p. 208.

13 Cook, 'Let a Thousand Flowers Bloom: A Tour of Logical Pluralism', p. 501.

14 Christopher Blake-Turner and Gillian Russell, 'Logical Pluralism without the Normativity', *Synthese*, September 2018, § 5.

The territory of what has meaningfully been included in the domain of logic is vast. It includes logics trivial and empty, and of all different stripes, be they deductive, inductive, or abductive. It ranges from the well-defined to the merely nominal. Meaning can be found in the notion of abductive validity, even in the absence of a system providing a defined consequence relation. The point of working in the field of abductive logic would be to generate a consequence relation that sorts the abductively valid from the abductively invalid.

Below I will lay out some features of systems that have been called logics which distinguish them from other systems which are not logics. However, these features give only a rough approximation for what could count as a logic.

The most important, even defining, feature of a logic is that it aims to give some definition of a consequence relation, roughly speaking, some relationship between input (premises) and output (conclusion). This is the way of spelling out what validity *means* in this system. Premises and conclusions related together in arguments that can be determined to be valid or invalid according to the definition of the consequence relation.

However, my next move is a daring one: to reject all historical constraints on how this relationship may be spelled out. In this view, all that is expected when someone claims that a system is a logic is that the system gives some definition of consequence which draws the line between valid and invalid arguments. To be in the business of logic is to be in the business of generating consequence relations, but no constraints on consequence relations need to apply. This is a radically pluralist thesis.

When explaining their pluralist thesis that within the standard meaning of 'validity' multiple consequence relations may be equally legitimate, Beall and Restall place three constraints on how consequence relations may be defined: necessity, normativity, and formality.[15] But by permitting that validity can have multiple meanings, all of these constraints can be dropped.

Blake-Turner and G. Russell retain the conditions of necessity and formality but consider dropping or replacing the normativity condition.[16] I begin by embracing what Blake-Turner and G. Russell call flat-footed logical pluralism, dropping the normativity constraint on what may count as a genuine consequence relation. As they discuss, the normativity constraint rules out unpalatable consequence relations. I consider this unnecessary.

Logic could still be considered normative since it gives rules, or a scheme, for deriving consequences such that if one wishes to apply a logic one must obey its rules. This normativity does not place any significant constraint on what may count as a logic and is relatively trivial. It is not what logicians mean when they claim that logic is normative. This type of normativity is one constraint I would consider for what may count as a logic: a logic should, at least loosely,

15 Beall and Restall, *Logical Pluralism*, p. 7.
16 Blake-Turner and Russell, 'Logical Pluralism without the Normativity', § 4-5.

be a system of rules. However, these rules might be spelled in different ways. For instance, in modern logic it is typical to spell out the rule system using mathematical language, but in older logics such as Aristotelian logic and Medieval logic, it is not.

I press beyond dropping the normativity constraint. I am similarly unconvinced of the need for the formality or necessity constraints. I think the best way to think of formality is as a process of abstracting away from particulars – applying some straightforward replacement scheme. For instance, take the argument:

The ball is red ; If the ball is red, then the ball is coloured ; So, the ball is coloured

Formalisation is the process of rendering that argument schematically either as:

P ; If P, then Q ; So, Q

or as:

x is y ; If x is y, then x is z ; So, x is z

Some accounts of validity explain validity by appealing to an argument's form. They might say, 'Logic does not speak at first of individual concrete arguments. Instead, it categorises *forms*.'[17] However, it can be noted that there may be some arguments like 'the ball is red, so the ball is coloured' which we might wish to count as valid, yet which do not so easily lend themselves to a demonstration of that validity through formalisation.

Many logics employ schematic abstraction. A *modus ponens* argument[18] in classical propositional logic appears as:

$$P, P \to Q \vdash Q$$

Aristotelian syllogistic logic cannot capture the validity of *modus ponens*, but it also employs schematic abstraction. A statement such as:

All men are mortal

has the form:

All S are P

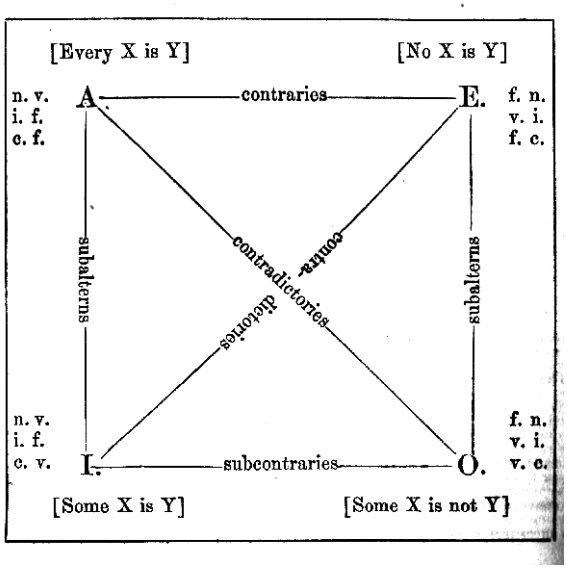
Statements such as this are called universal affirmative statements and given the code A. There are three other types of statements coded E, I, and O. Relations among statements of these types are described in the square of opposition. Figure 1.3 is an example of the square of opposition taken from Whately's *Elements of Logic*.[19]

17 Beall and Restall, *Logical Pluralism*, p. 20.

18 Loosely characterised as an argument with the form *if X then Y, and X, therefore Y*

19 Richard Whately, *Elements of Logic*, Ninth ed. (London: John W. Parker, 1853), p. 46.

Figure 1.3: Square of Opposition



An argument in Aristotelian logic is composed of two premises and a conclusion and has three terms – where a term is a replacement for S or P. For example:

All S are P
No P are Q
Therefore, No S are Q

Every argument may be described by the types of statements that appear in it combined with one of the four permitted arrangements of terms and given a label that describes this arrangement, such as $AAA-1$ or $EAO-3$. Validity is about classifying all 256 arrangements, and necessity helps explain the principle used for this classification. Valid arguments are the ones in which the truth of the premises *necessitate* the truth of the conclusion. Alternatively, valid arguments are the ones in which the premises cannot be true and the conclusion false.

Formality and necessity may form part of a logical system. However, as constraints on what may count as a genuine logic, they are artificial. Any system which could explicate – give an analysis or explanation of – the validity of the inference 'the ball is red, so the ball is coloured' should still be entitled to call itself logic.

Necessity is used to explain the purpose of the consequence relation in some system of rules – some logic – but it is not guaranteed to be the best way to explain the purpose of the consequence relation. Consequence relations link premises to conclusions, but there is no need, and it is not desirable, to place a constraint on what may count as a genuine consequence relation absent of an explanation of the overall purpose of the logic.

Sometimes consequence relations are described in terms of truth preservation. G. Russell does this when she says, 'Logical laws describe patterns of truth-preservation across the truth-bearers of languages.'[20] However, with *Nether Logic*, Sobel demonstrates the construction of a consequence relation that is falsity-preserving.[21] In *Nether Logic*, valid arguments are the ones in which the premises cannot be false and the conclusion true. Consequence relations can also be thought of in terms of grounding proof or preserving properties like information or beauty.

Besides a consequence relation, another typical feature of modern logics is that they have variables and constants. Variables are terms where what they stand for can change. Constants are terms where the meaning is the same in every interpretation. For instance, take the classical propositional statement:

$$P \rightarrow Q$$

in this statement 'P' and 'Q' are variables, what they stand for can change. In contrast, '$\rightarrow$' is a constant, which always stands for 'if ...then'. Concerning logical constants, I take Varzi's view that in principle any terms of a logic could take the role of logical constants.[22] I also have no issue with trivialising constants such as 'tonk'[23,24] since I have already accepted the trivial logic.

I have claimed that we should accept any system designed to sort valid from invalid augments as a legitimate logic. However, there is a challenge for this proposal. It is still sometimes meaningful to claim that one system is correct, and another is not.

Priest explains that when a pure logic (or formal system) is applied it becomes a *theory* of how the domain in which it is interpreted behaves.[25] Hjortland similarly explains:

> a logical theory is a theory of logical properties. The main logical property, most would agree, is validity. ...We should be careful, however, not to confuse a logical theory with a formal system. A logical theory is not a proof system or a formal semantics with a consequence relation. Granted, formal systems typically contribute to logical theories. Our understanding of the property of validity is improved by formal counterparts such as model-theoretic consequence. But a logical theory should not be equated with the model theory. The model theory is not an account of logical properties in its own right – it is merely a formalism. For it to be part of a theory it requires an application, and it is the logical theory that states what the application is. No one is disagreeing about, say, whether the law of double

20 Russell, 'Logic Isn't Normative', p. 387.
21 Jordan Howard Sobel, 'Nether Logic', *Teaching Philosophy* 17, no. 2 (1994): 161–171.
22 Achille C. Varzi, 'On Logical Relativity', *Philosophical Issues* 12 (2002): p. 200.
23 Arthur Prior, 'The Runabout Inference-ticket', *Analysis* 21, no. 2 (1960): 38–39.
24 The meaning of 'tonk' is given by the rules $P \vdash P \; tonk \; Q$ and $P \; tonk \; Q \vdash Q$. This combination of rules permits deriving Q from any P, thus rendering any logical system which contains it trivial.
25 Priest, 'Logical Pluralism', p. 195.

> negation is *classically valid*. It is, and that is uncontroversial. The disagreement is about whether or not it is *genuinely valid*.[26]

Priest's discussion of genuine validity vs. formal validity produces the following: 'A vernacular inference is valid iff its translation into the formal language is valid in the pure logic.'[27] This brings us dangerously close to doing what Hjortland warns against. The claim is that an inference is valid (genuinely valid) if and only if it is valid in the formal system. The biconditional nature of the claim introduces a risk to its overall value.

An obvious aim for a formal system is that its definition of validity should 'match up' with genuine validity. The idea here might be expressed by the conditional: if an inference is genuinely valid, then it should be valid in the formal system. However, the trivial logic fulfils this condition. The trivial logic validates every inference, so it will also validate all instances of genuinely valid inferences. A common approach to patching this is to use a biconditional instead, so that an inference is valid in the formal system, only if it is a case of genuine validity. However, the biconditional structure introduces the risk of circularity.

To be non-circular, claims of the form 'an inference is genuinely valid iff its translation in the formal system is valid' must have something more than just the formal system in question to appeal to for verification. Without this external basis, these claims become circular.

What is needed is some informal account of what validity is for – some inter-theoretic validity. A theory of inter-theoretic validity establishes a domain of investigation, clarifies the phenomena under investigation, and indicates what data would confirm or disconfirm rival theories. Normativity offers precisely this. It describes an area of inquiry and points to the data that would confirm or disconfirm competing theories. With that in place, it becomes possible to settle disputes between rival logical theories. Normativity is a perfectly good theory of inter-theoretic validity. However, it is only one of many possible inter-theoretic validities.

The logic-as-modelling view that I adopt allows for the possibility of multiple, equally legitimate, inter-theoretic validities – logical projects. Normativity is one, but there are others. The inter-theoretic validity which a logic models is its subject matter. This subject matter is what rival logical theories can be right about.

I suggest taking a closer look at what different logics could be models of and then asking questions about how one could determine those models' accuracy. Logicians can have quite different objectives when they develop logics. They could be exploring syntactic validity in natural language, or the provability of mathematical statements, or truth preservation (in natural language or a formal structure), or constraints on belief, or assertability and deniability conditions

26 Ole Thomassen Hjortland, 'What Counts as Evidence for a Logical Theory?', *The Australasian Journal of Logic* 16, no. 7 (November 2019): pp. 252–253.

27 Priest, 'Logical Pluralism', p. 196.

for speech acts. The objective can have quite an impact on how one considers the suitability of a logic. Priest calls the analysis of reasoning as logic's canonical application.[28] There is a long tradition of viewing logic this way. However, the analysis of reasoning is only one of many applications that appear in the history of logic. I agree with van Benthem when he says, 'a subject is, in some sense, the sum total of all topics that its practitioners have found interesting.'[29] The logic-as-modelling view supplies a way to construct explanations of logics by identifying what the logic in question could reasonably be a model of.

These explanations, the things which we could say logics are supposed to model, supply different inter-theoretic notions of validity. If two logics share the same notion of inter-theoretic validity – they are part of the same logical project – then they can meaningfully be compared to one another. If, however, the things they aim to model are different, then a comparison is meaningless. What matters when judging a logic is the degree of alignment between the system and the notion of inter-theoretic validity used to explain it.

Differing systems and notions of inter-theoretic validity are not right or wrong on their own. Judgement can only occur once a system has been combined with a logical theory, and that theory is compared against the notions it was meant to explain.

1.3 SUBJECT MATTER AND PEDAGOGY

This thesis is devoted to demonstrating the pedestrian point that logic has multiple subject matters – that it is about many things. This proof is drawn from introductory logic teaching. I turn to teaching as a source because the subject matter of logic is not directly debated in contemporary literature, but introductory texts talk about it in their introductions as a matter of course. However, examining the subject matter of logic this way requires attending to a particular problem: the practical problem of getting the introduction right.

Writing any introduction is hard work but writing an introduction to an elementary logic textbook is particularly challenging. The challenge of writing an introduction to an elementary logic textbook comes from the practical need to explain what the student will learn during the course. This forces the teacher to say *something* about the subject matter of logic, which is a hard question in the philosophy of logic. In saying something about the subject matter of logic, teachers must draw on advanced and controversial material that only those interested in the philosophy of logic will ever again re-visit. So, the difficulty confronting teachers is that the introduction must cover challenging material, be designed to introduce the subject matter to a beginner, and is only there to set the scene for what is otherwise a technical, almost mathematical course.

28 Priest, 'Logical Pluralism', p. 196.
29 See §6.21

However, there is more at stake than just how to explain the subject matter of logic to a beginner. Teachers must say things that are vital to an understanding of logic right at the point when those things are most likely to be accepted without critical examination. At the earliest stage to demanding to expect students to critically engage with hard questions in the philosophy of logic and they will not have the context they need. Furthermore, teachers cannot rely on a student's future studies to correct any misunderstanding since not all students will engage in further studies. So the introduction must walk a fine line: presenting the ideas accurately enough that students who will not study logic further do not form mistaken beliefs, and priming students who will progress so that they will be well equipped to join the conversation when they are ready.

So, what do teachers do to solve this problem? They fall back on tradition. They recall what they were taught and what their predecessors have written, and copy it. There is some change, but there's also a strong connection to what has come before.

I use Alain's poetic description of how the boats of the Isle of Groix are designed as a thematic backdrop for my discussion of the impact of tradition.

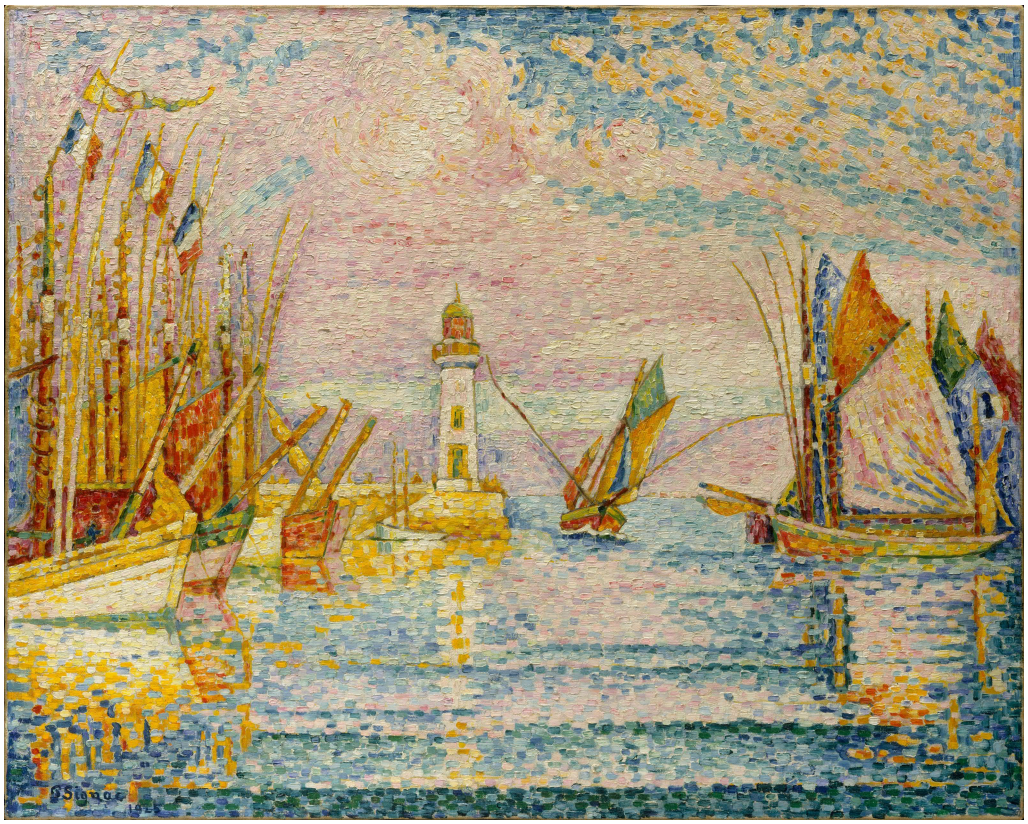


Figure 1.4: *Le phare, Groix* (Lighthouse at Groix) by Paul Signac, 1925, Oil on canvas

Alain says:

> Every boat is a copy of another boat... It is clear that a badly done boat will go down after one or two campaigns, and so will never be copied. ...We understand very well that, most often, such an old hull is precisely the most perfect of all, I mean the one that best meets the use that is made of it.[30]

Introductions to logic evolve like Alain's boats, copied time and again, evolving and changing slightly with each copy, but fundamentally built after the same model. This theory predicts that in an analysis of modern logic textbooks, there will be evidence of a traditional explanation of the subject matter of logic. I ask whether tradition has furnished us with an account that is precisely the most perfect of all. Does time fashion teaching as the sea fashions boats?

1.4 THE PLAN FOR THIS THESIS

The objective of the thesis is to conduct a critical analysis of the subject matter of logic and the way it is taught. My research questions are:

1. What concepts are embedded in introductory explanations of logic?
2. How should introductions to logic be structured?

I use methods that are not typical in philosophy: text analysis and interviews. I am using atypical methods, because there is not enough literature on the topic to take a more traditional approach.

With question 1, I want to understand the possible alternatives for what logic could be about. Without a body of established literature, I have turned to textbooks. The focus of question 1 is analysing explanations of the subject matter of logic found in introductory logic textbooks. However, examining textbooks rather than philosophical literature introduces a new challenge. A challenge alluded to later in the interview with van Benthem where he says that his textbook is not a considered view. [31] It would be unjust to read and criticise textbooks as if they do represent fully considered philosophical views. Textbooks can only justly be criticised against their original purpose – as a teaching resource.

In my investigation of textbooks a principal assumption is that there is a particular challenge involved in producing an introduction for a logic textbook. This challenge leads me to examine a slice of the history of modern logic instruction, and ten textbooks from that history. In Chapter 2 I examine some of the history of modern logic and textbooks produced in that history. I argue that none of the approaches to the subject matter of logic appearing in that

30 Alain, 'XXXVIII- 1er Septembre 1908', in *Propos d'un Normand 1906-1914* (Paris: Gallimard, 1956).
31 See §6.47

history are suited to introducing the subject in the modern context. The explanations of logic supplied by tradition are flawed.

Chapter 3 explores the strategies used to introduce logic in contemporary textbooks via text analysis. I consider the approaches to introducing logic in a corpus of 38 introductory logic textbooks used in contemporary logic courses. This allows me to capture a broad sample but necessitates a shallower analysis of each text. However, a shallow analysis is in keeping with the need to treat textbooks as textbooks, not expressions of fully considered philosophical views. I use NVivo – a qualitative data analysis software package that contains tools for textual data analysis – to track appearances of words and word forms (e.g. reason, reasoning, reasons) as instances of the concepts used to explain logic in these texts. I then explore what these concepts mean for the subject matter of logic, arguing for a pluralistic understanding of logic, the discipline, which accepts all the alternatives as legitimate subject matters for logic. At the same time, I argue that in the context introducing classical logic, a common choice for a first introduction to logic, not all of these alternate subject matters make sense.

The second question comes about as a natural response to the examination of textbooks. There is a need to understand why textbooks, and particularly their introductions, are constructed as they are. Further, and arising from my criticism of the approaches to introducing logic that I examine, there is a need to provide constructive recommendations for change. The focus of question 2 combines the philosophy of logic and pedagogy to construct a positive solution to the pedagogic problem. I interview three logicians and teachers: Gillian Russell, Dave Ripley, and Johan van Benthem. This approach sidesteps the void in the literature and, at the same time, generates discussions that deal precisely with my topics of interest. In the the interviews I want to understand more about what teachers are trying to do when explaining what logic is about. I also want to investigate the relationship between how teachers explain the subject matter and the instructor's views on the subject matter of logic.

These interviewees were selected for the different views on logic suggested by their published research. G. Russell's writings indicate a model-theoretic perspective coupled with an interest in language. This can be contrasted with Ripley's proof-theoretic perspective. G. Russell and Ripley represent different but common perspectives in logic, while van Benthem has an entirely different way of conceiving logic. The interviews are an opportunity to dig deeper into their individual views on the subject matter of logic. At the same time, it is an opportunity to learn about their pedagogic practice and the reasons behind their approach. In this way, the interviews are a vital supplement to the examination of textbooks, supplying insight into potential motivations for the choices made in textbooks.Chapters 4, 5 & 6 contain lightly edited verbatim transcripts of the interviews.

In Chapter 7 I discuss the outcomes they each aim for in teaching and their thoughts talking

to students about logic in contrast to their own views on the subject matter of logic. I then build on the evidence produced in the text analysis and interviews which demonstrate the plurality of views on the subject matter of logic, developing the point that there are many things that logic could be about by examining different options in greater depth.

Chapter 8 synthesises the interviews and the lessons drawn from examining introductory textbooks. In this chapter I return to pedagogy, gathering threads from the interviews and my analysis of introductory logic texts to construct a solution to introducing the subject matter of logic in introductory teaching. I discuss the context of logic teaching and other practical problems that logic teachers face, like the need to deliver inclusive teaching and the two-hump grade curve, to situate my discussion of logic's subject matter. The solution I propose draws from course design theory and challenges the textbook's role in contemporary logic teaching in tertiary programmes. I present a framework for solving the pedagogic problem of introducing to the subject matter of logic.

In the conclusion I return to philosophy and the discussion of validity and the subject matter of logic.

INTRODUCTORY LOGIC TEXTBOOKS

GENESIS OF THE MODERN LOGIC TEXTBOOK

> Every boat is a copy of another boat; all their science stops there: copying what is, doing what one has always done.
>
> Alain, Propos d'un Normand, September 1, 1908

2.1 RICHARD WHATELY, A BEGINNING

Disciplines develop continually, and eras in their development come and pass, leaving practices that make the tradition. Tradition becomes imprinted in textbooks, yet is continually shifting with each new generation of teachers and learners. Richard Whately is not the beginning, there is no original beginning to the development of any discipline, but he is a beginning. Whately exists at the ending of one era in the development of logic and the beginning of another.

Whately was born in 1787 in London. He was educated at a private school near Bristol and then at Oriel College, Oxford from 1805. He is most well known for his role as Archbishop of Dublin and work as an education reformer.

In the article 'History of logic' for *Encyclopaedia Britannica* Spade and Hintikka refer to Whately's logic textbook, *Elements of Logic*, as arising from a tradition which produced considerable numbers of English logic textbooks in the 17th and 18th centuries including the textbooks of Aldrich and Mill. They say that Whately's textbook, 'reformulated many concepts in such a thoughtful and clear way that it is generally (and first by De Morgan) credited with single-handedly bringing about the "rebirth" of English-language logic.'[1] The role in the revival of the study of formal logic in England is what makes Whately's textbook special – far more than its content. Whately's textbook presents Aristotelian syllogistic logic, the then-dominant tradition in western logic, and contains the familiar square of opposition (see figure 1.3).[2]

In 1899 in the Dictionary of National Biography,[3] the effect of Whately's *Elements of Logic* was eloquently described:

1 Paul Vincent Spade and Jaakko J. Hintikka, 'History of Logic', Encyclopædia Britannica Online, September 2019, §4.4.3 'Other 18th-century logicians', accessed 12 November 2019, https://www.britannica.com/topic/history-of-logic.

2 Whately, *Elements of Logic*, p. 46.

3 The DNB contains articles on more than 29,000 notable figures from British history. It was originally published by Smith, Elder & Co. in 63 volumes between 1885 and 1900. It is now available in the wikisource project: https://en.wikisource.org/wiki/Dictionary_of_National_Biography,_1885-1900

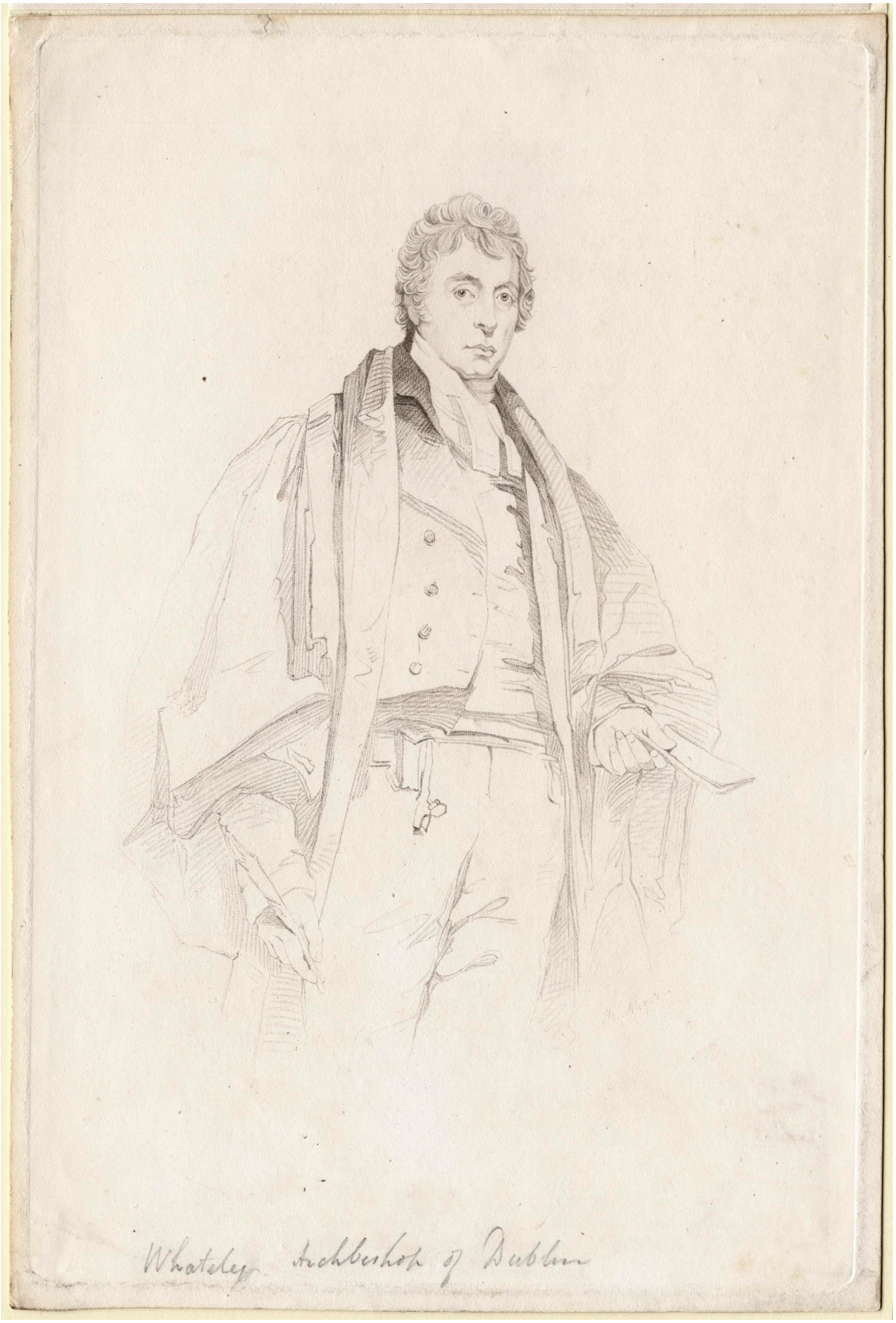


Figure 2.1: Richard Whately by Henry Meyer after Charles Grey, 1830s-1840s, Stipple engraving.

> In 1825 Whately returned to Oxford as principal of St. Alban Hall. ...Learning was then at a low ebb in Oxford, where outside the precincts of Oriel there was little stir of intellectual life. Aristotle was more venerated than read, and Aldrich was still the text-book on logic. This reproach Whately did much to remove. To the 'Encyclopaedia Metropolitana' he contributed articles on 'Logic' and 'Rhetoric' which appeared in separate form, the one in 1826, the other in 1828... Neither work was of the kind which lays posterity under permanent obligation; but the logic unquestionably marks, if it did not make, a new epoch in the history of the science. ...The effect of the work was twofold: with certain thinkers it served to rehabilitate the discredited formal logic; to others it suggested the deeper questions as to the nature of the scientific method which it so airily dismissed from its purview, and of the illative process in general, to the solution of which John Stuart Mill addressed himself.[4]

Whately contributed significantly to the cultural milieu that gave rise to Boole and De Morgan, who Spade and Hintikka call 'the two most important contributors to British logic in the first half of the 19th century.'[5] Yet Whately himself is not a logician. His biographies mention his *Elements of Logic* and its popularity and importance, but they do not focus on it or discuss any other contribution to logic. Instead, they are occupied with his contributions to education reform, his involvement with the Noetic school, and his role as the Archbishop of Dublin.

Though he had a profound effect on the course of modern logic, Spade and Hintikka comment that Whately's textbook is 'now largely relegated to a footnote.'[6] In charming harmony, it is footnotes that reveal both Whately's location in the history of logic instruction, and how introductions to logic may develop in a tradition. The first footnote comes from the first page of Whately's *Elements of Logic* first published in 1826. In the section titled 'Definition of Logic', Whately says:

> Logic, in the most extensive sense in which it is advisable to employ the name, may be considered as the Science, and also as the Art, of Reasoning. It investigates the principles on which argumentation is conducted, and furnishes such rules as may be derived from those principles, for guarding against erroneous deductions. ...Logic (as well as any other system of knowledge) becomes, when applied to practice, an *art*; while confined to *theory* of reasoning, it is strictly a *science*.[7]

4 James McMullen Rigg, 'Whately, Richard', in *Dictionary of National Biography: Watson - Whewell*, ed. Sidney Lee, vol. 60 (London: Elder Smith & Co., 1899), p. 424.
5 Spade and Hintikka, 'History of Logic', §4.4.4 'Boole and De Morgan'.
6 Spade and Hintikka, §4.4.3 'Other 18th-century logicians'.
7 Whately, *Elements of Logic*, p. 1.

The footnote then says, 'It is surely strange, therefore, to find in a treatise on Logic, (Aldrich's,) a distinct dissertation to prove that it is an Art, and *not* a Science!' Thus Whately connects his discussion of the subject matter of logic to Aldrich's.

As well as containing a footnote revealing Whately's place in the history of logic instruction, the second quote also shows a similar preoccupation with whether logic is an art or a science. It comes from John Stuart Mill's enormously popular *A System of Logic, Ratiocinative and Inductive* published in 1843. As with the footnote in Whately's textbook, this is also the first footnote to occur in Mill's textbook. It is from the second section titled 'Is logic the art and science of reasoning?' In this section, Mill begins:

> Logic has often been called the Art of Reasoning. A writer who has done more than any other person to restore this study to the rank from which it had fallen in the estimation of the cultivated class in our own country, has adopted the above definition with an amendment; he has defined logic to be the Science, as well as the Art of reasoning; meaning by the former term, the analysis of the mental process which takes place whenever we reason, and by the latter, the rules, grounded on that analysis, for conducting the process correctly. There can be no doubt as to the propriety of the emendation. A right understanding of the mental process itself, of the conditions it depends on, and the steps of which it consists, is the only basis on which a system of rules fitted for the direction of the process, can possibly be founded.[8]

The footnote removes the mystery of who it is that Mill refers to with such esteem, saying only, 'Archbishop Whately.' Whately's textbook is a demonstration of the powerful effect a textbook can have, particularly a popular one. It may be only a footnote in history, but it is nevertheless an important note.

2.2 MILL, FREGE, RUSSELL, AND THE REVOLUTION

Like Whately, Mill is the author of an influential logic textbook. Also like Whately, his biographers do not refer to him as a logician. Instead, they call him a British philosopher, political economist, and civil servant.

In 1843, when Mill's logic textbook was published, Whately's textbook had dominated reading lists for almost 20 years.[9] Mill's *A System of Logic, Ratiocinative and Inductive* challenged

8 John Stuart Mill, *A System of Logic, Ratiocinative and Inductive: Being a connected view of the principles of evidence, and methods of scientific investigation.*, vol. 1 (London: John W. Parker, West Strand, 1843), p. 2.

9 Richard Brent, 'Whately, Richard (1787–1863), Church of Ireland Archbishop of Dublin and Philosopher." Oxford Dictionary of National Biography', in *The Oxford Dictionary of National Biography* (Oxford: Oxford University Press, 2004).

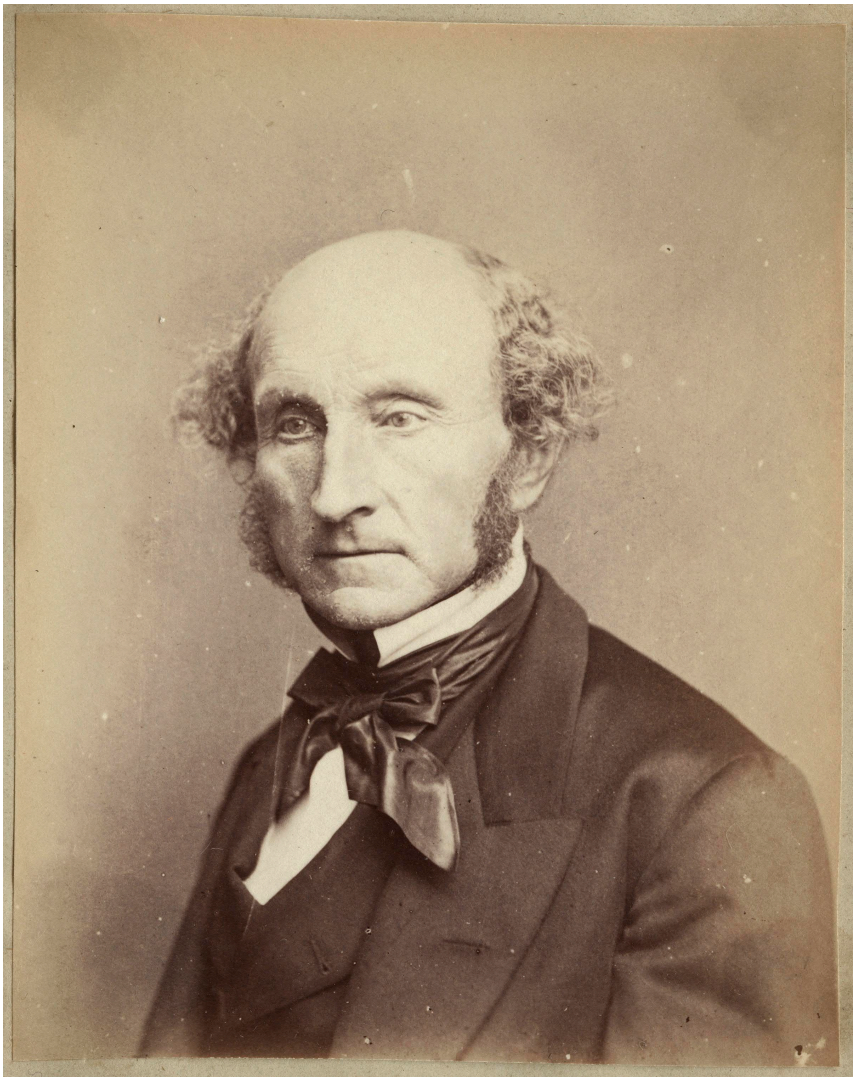


Figure 2.2: John Stuart Mill by John Watkins of John & Charles Watkins Photographers, 1865

this dominance. It is no surprise then, that Mill refers to Whately with equal parts criticism and respect. In places, such as the definition of logic quoted above, Mill takes Whately to be correct and refers to him as an authority. In other areas he is in direct disagreement. The most salient point of dispute is:

> Archbishop Whately has contended that syllogising, or reasoning from generals to particulars, is not, agreeable to the vulgar idea, a particular *mode* of reasoning, but the philosophical analysis of *the* mode in which all men reason, and must do so if they reason at all. With all deference due so high an authority, I cannot help thinking that the vulgar notion is, in this case, the more correct.[10]

This is apropos as Mill's text develops a system of inductive reasoning composed of five prin-

10 Mill, *A System of Logic, Ratiocinative and Inductive*, Book II; Chapter III §3.

ciples known as Mill's Methods. It is an entirely different style of logic from that delivered in Whately's *Elements of Logic*. However, as the earlier quote makes plain, it borrows Whately's philosophy of logic wholesale.

The impact of Mill's *System of Logic* most salient to the development of introductory textbooks is the impact on Gottlob Frege, whose 1884 *Grundlagen der Arithmetik* contains 'a detailed criticism of Mill's philosophy of mathematics.'[11] In the *Grundlagen der Arithmetik* Frege argues that mathematics and logic are not part of psychology: that the terms or rules in mathematics and logic cannot be defined, demonstrated, or explained by psychological facts.[12] This point becomes particularly relevant to the analysis of introductory logic textbooks when we consider that early on in the *System of Logic* Mill defines logic as the Science and Art of reasoning, clarifying that 'Science' means 'the analysis of the mental process which takes place whenever we reason,'[13] and that 'Art' means 'the rules, grounded on that analysis, for conducting the process correctly.'[14] Mill's definition contains both descriptive and prescriptive elements. Frege makes a distinction between two types of psychologism: identifying the laws of logic with descriptive psychological laws; or understanding the laws of logic as prescriptive laws based on descriptive psychological laws.[15] Frege rejects both types of psychologism.[16] Instead, he has famously argued that 'laws of truth' better describes what logic is about.

In contrast to Mill and Whately, Frege is famous for his work as a logician and he did not author a logic textbook. One of Frege's works, his 1879 *Begriffsschrift: Eine der arithmetischen nachgebildete formelsprache des reinen denkens*,[17] is a key text in the history of modern logic. Between the mid-19th century and the mid-20th century there was a significant shift in the development of logic. This shift is so significant that it is commonly referred to as a revolution.[18] Frege's *Begriffsschrift* is a key contribution to this revolution.

But the revolution did not begin in logic. The revolution began in Geometry with non-Euclidean geometries which challenged Euclidean axioms and disturbed the typical appeal to 'obviousness' which had been used to justify them. Frege's doctoral work in geometry and his revolutionary contribution to logic are part of this same tapestry, the salient parts of which are best illustrated in the Frege-Hilbert Controversy:

11 Martin Kusch, 'Psychologism', in *The Stanford Encyclopedia of Philosophy*, Winter 2015, ed. Edward N. Zalta (1 December 2015), §4, para. 2.

12 Martin Kusch, *Psychologism: A Case Study in the Sociology of Philosophical Knowledge*, Philosophical Issues in Science (London; New York: Routledge, 1995), p. 30.

13 Mill, *A System of Logic, Ratiocinative and Inductive*, p. 2.

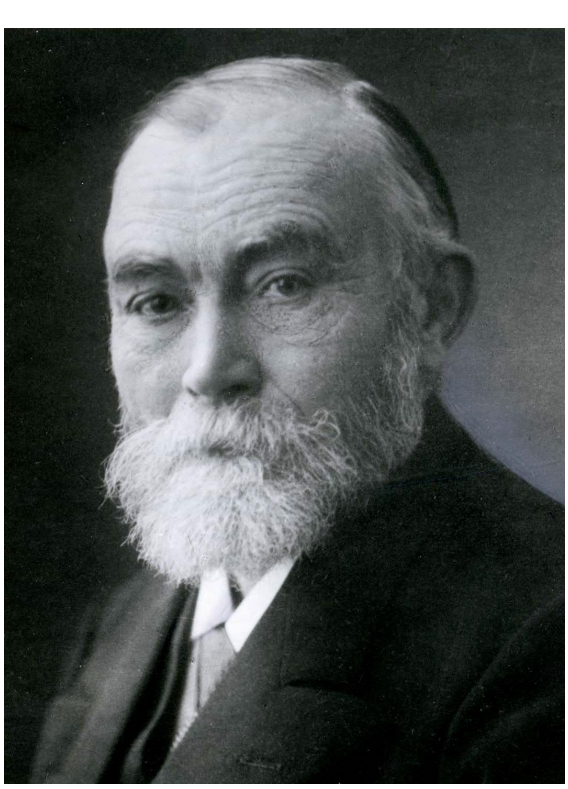
14 Mill, pp. 2-3.

15 Kusch, 'Psychologism', §4, para. 4.

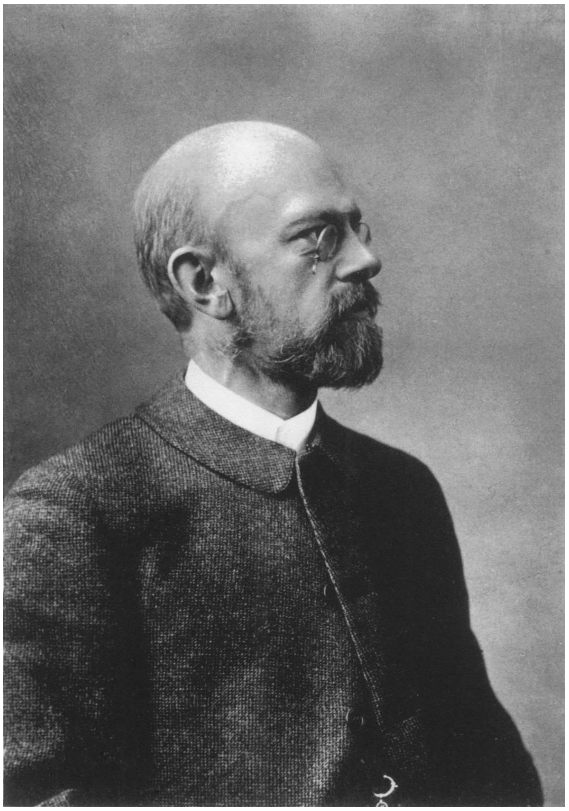
16 Kusch, *Psychologism: A Case Study in the Sociology of Philosophical Knowledge*, p. 34.

17 Concept Notation: A formula language of pure thought, modelled upon that of arithmetic; commonly known as the *Begriffsschrift*

18 Leila Haaparanta, ed., *The Development of Modern Logic* (Oxford University Press, June 2009).



(a) Gottlob Frege



(b) David Hilbert c.1907

Figure 2.3: Frege and Hilbert: titans of mathematical logic

> In the early years of the twentieth century, Gottlob Frege and David Hilbert, two titans of mathematical logic, engaged in a controversy regarding the correct understanding of the role of axioms in mathematical theories, and the correct way to demonstrate consistency and independence results for such axioms. The controversy touches on a number of difficult questions in logic and the philosophy of logic, and marks an important turning-point in the development of modern logic.[19]

This dialogue was brought to light by Resnik in 1974 in an article which called attention to the letters that passed between Frege and Hilbert on the topic of Hilbert's *Grundlagen der Geometrie*.[20]

Hilbert's aim in *Grundlagen der Geometrie* is to develop a consistent and independent set of axioms for geometry. Naturally, it follows that the work consists mostly of a set of axioms for Euclidean geometry and demonstrations of consistency and independence for those axioms.

19 Patricia Blanchette, 'The Frege-hilbert Controversy', in *The Stanford Encyclopedia of Philosophy*, Fall 2018, ed. Edward N. Zalta (9 August 2018).

20 Michael David Resnik, 'The Frege-hilbert Controversy', *Philosophy and Phenomenological Research* 34, no. 3 (March 1974): 386–403.

Frege generally agrees with Hilbert's axiomatic approach but he disagrees with Hilbert's approach to definitions and techniques for proving consistency and independence.

In June 1899 Hilbert delivered the lecture which formed the basis of his 1900 *Grundlagen der Geometrie*. In December 1899 Frege initiated an exchange of correspondence which continued until September 1900. The discussion was not very fruitful. Frege's criticisms did not convince Hilbert, and Frege's critical attitude and general refusal to accept the significance of what Hilbert had accomplished meant that the criticisms he published in 1903 and 1906, both titled 'Uber die Grundlagen der Geometrie', received little attention from Hilbert or anyone else. The only exception to the general fruitlessness of Frege and Hilbert's interaction was its role in convincing Hilbert that 'a fundamental investigation of logic and its relationship with mathematics was needed.'[21]

Resnik's article and Blanchette's contribution to the Stanford Encyclopedia of Philosophy revive Frege's criticisms, providing commentary which extracts value from Frege's insights. Resnik argues that Frege's response to Hilbert's work should receive a fairer hearing and that 'much of the confusion concerning "implicit definitions" can be traced to Hilbert, while Frege's essays contain excellent accounts of the roles of axioms and definitions in mathematics.'[22]

Although in the early years of the 20th century Hilbert's primary interest was in geometry, he was also interested in the principles which underpin the axiomatic method and aware of the importance of logic.[23] In 1905 he taught a course called 'Logical Principles of Mathematical Thought'. The course focused on set theory – axiomatised in natural language like his treatment of geometry – but included a basic calculus of propositional logic.[24] Hilbert's interest in logic continued to develop, and his 1917/18 lectures on the principles of mathematics contained propositional and first-order logic, as well as Russell's type theory.[25] Hilbert lectured on logic for the next decade. Finally, in 1928, with the aid of his student Wilhelm Ackermann, Hilbert published *Grundzüge der theoretischen Logik* the first introductory textbook for the new logic emerging in the late 19th century and early 20th century.

21 Paolo Mancosu, Richard Zach and Calixto Badesa, 'The Development of Mathematical Logic from Russell to Tarski, 1900–1935', in *The Development of Modern Logic*, ed. Leila Haaparanta (Oxford: Oxford University Press, 2009), p. 368.

22 Resnik, 'The Frege-hilbert Controversy', pp. 386-387.

23 Mancosu, Zach and Badesa, 'The Development of Mathematical Logic from Russell to Tarski, 1900–1935', pp. 368-369.

24 Mancosu, Zach and Badesa, p. 369.

25 Mancosu, Zach and Badesa, p. 370.

Figure 2.4: Timeline of people and texts discussed in §2.1 and §2.2

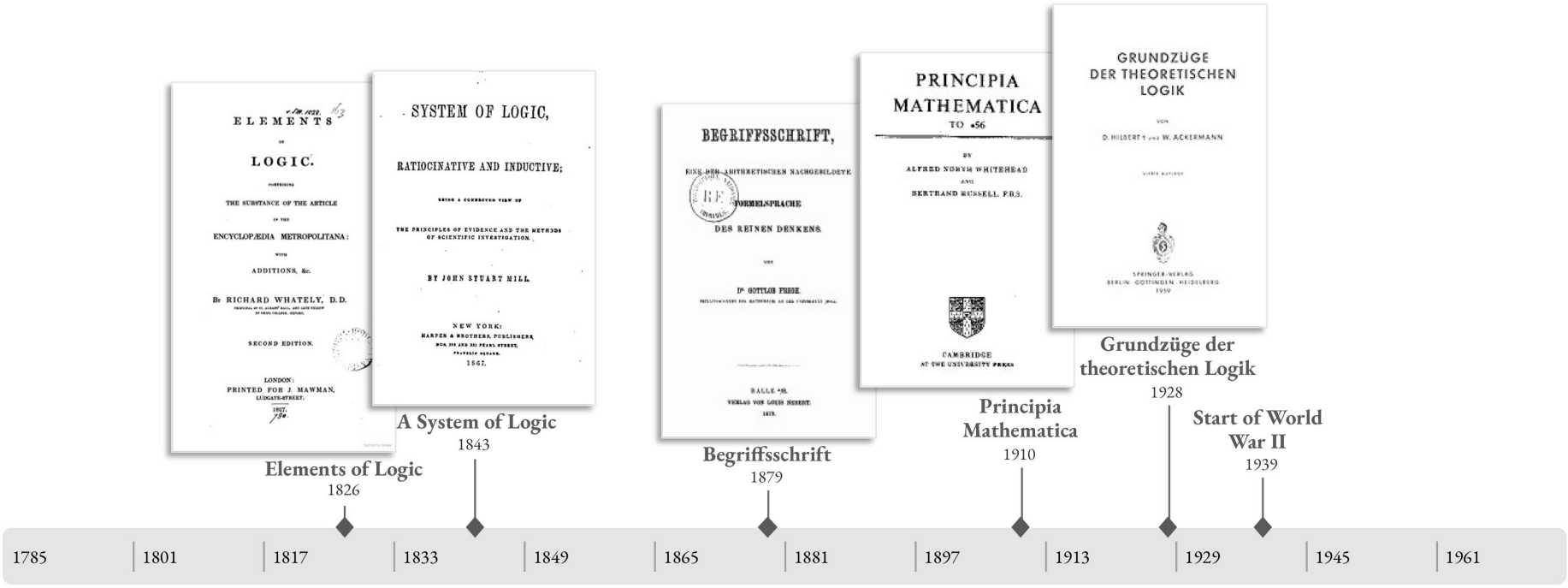


Figure 2.5: Paris, Exposition Universelle 1900 by Eugene Trutat, October 1900, photograph preserved by the muséum de Toulouse.

The last major contributor to the logical revolution left to discuss is Bertrand Russell. Russell's contribution to logic and the revolution is significant. An overwhelming amount could be said of him and his contribution. To avoid being washed away, let us focus on a single moment – the Paris conference and its impact on Russell's work. Spade and Hintikka tell us: 'Logic in the 19th century culminated grandly with the First International Congress of Philosophy and the Second International Congress of Mathematics held consecutively in Paris in August 1900. The overlap between the two congresses was extensive and fortunate for the future of logic and philosophy .'[26]

The Paris conference marks a turning point in Russell's intellectual development. Early in his career Russell was interested in non-Euclidian geometry. In 1894 he chose a fellowship thesis topic on the philosophy of non-Euclidian geometry. In 1896 he published 'The Logic of Geometry' and *An Essay on the Foundations of Geometry* in 1897. In 1900, at the time of the Paris conference, Russell was wrestling with the foundations of mathematics. However, he had not read the works of Peano and his Italian school.[27] At the Paris conference, Russell met Peano and

26 Spade and Hintikka, 'History of Logic', §4.4.9 'Other 19th-century logicians'.

27 Mancosu, Zach and Badesa, 'The Development of Mathematical Logic from Russell to Tarski, 1900–1935', p. 330.

was impressed by his precision and ability to argue.[28] Russell's excitement after this meeting is palpable in his autobiography. He says:

> as soon as the Congress was over I retired to Fernhurst to study quietly every word written by him and his disciples. It became clear to me that his notation afforded an instrument of logical analysis such as I had been seeking for years, and that by studying him I was acquiring a new and powerful technique for the work that I had long wanted to do.[29]

Like Frege, Russell was interested in the project of attempting to define mathematical concepts in terms of logical concepts and Peano gave him the tools to advance that project. In 1901 Russell published two articles, one that was Russell's first substantial contribution to logic, the other contains his first logicist claim that 'All pure mathematics - Arithmetic, Analysis, and Geometry - is built up of the primitive ideas of logic, and its propositions are deduced from the general axioms of logic.'[30]

Russell's next advance occurred mid-1902 when he began to study Frege's work, and 'notice the limitations in Peano's treatment of symbolic logic.'[31] Russell had started work on the ideas that form the basis of *Principia Mathematica* in late 1901 and, a year or so later recruited his mentor, Alfred Whitehead, to his aid. However it was not until after he developed the 'theory of types' – which offered a solution to the set-theoretic paradoxes that he wrote to Frege about in 1902 – that he began to write in earnest. The most intense period of writing occurred between 1907 and 1910. Russell and Whitehead's *Principia Mathematica* was finally published in three volumes in 1910, 1912, and 1913.

2.3 THE VIENNA CIRCLE: LEARNING THE NEW LOGIC

The origin and development of the new logic is a complicated story. Peckhaus tells us:

> Today historians have recognized that the emergence of the new logic was no isolated process. Its creation and development ran parallel to and was closely intertwined with the creation and development of modern abstract mathematics which emancipated itself from the traditional definition as a science which deals with quantities and geometrical forms ...the new logic is not conceivable without the new mathematics. In recent research on the history of logic, this intimate re-

28 Bertrand Russell, *The Autobiography of Bertrand Russell 1914-1944* (Boston: Little, Brown, 1969), p. 218.
29 Russell, p. 218.
30 Mancosu, Zach and Badesa, 'The Development of Mathematical Logic from Russell to Tarski, 1900–1935', p. 331.
31 Mancosu, Zach and Badesa, p. 331.

> lation between logic and mathematics, especially its connection to foundational studies in mathematics, has been taken into consideration.[32]

The Vienna Circle provides a way to narrow the focus of this otherwise complex story; it serves as a backdrop for the characters of this next chapter.

The Vienna Circle was a group of philosophers and scientists drawn from the natural and social sciences, logic and mathematics that met regularly through the 1920s in Vienna. Their purpose was to investigate scientific language and scientific methodology, and 'they discussed philosophical questions such as: What characterizes scientific knowledge? Do metaphysical statements have any meaning? What makes logical propositions so certain? Why is mathematics applicable to the real world?'[33] Among the thinkers that this group held in high regards are the physicist Albert Einstein, the mathematician David Hilbert, and the philosopher Bertrand Russell.[34]

There is a wealth of historical discussion of the Vienna Circle and their influence. In a review of Karl Sigmund's history Franklin comments:

> Mostly outside the scope of Sigmund's story is the long-term effect of the Circle's logical positivism on Anglophone philosophy. From the 1950s, Wittgenstein's influence increasingly permeated British linguistic philosophy. Willard Van Orman Quine, who had visited Europe and sat at the feet of Carnap in 1933, became the leading American philosopher. Karl Popper, sometimes regarded by the Circle as the in-house 'official opposition,' and his followers such as Thomas Kuhn occupied nearly the whole scene in the philosophy of science for decades. These figures naturally disagreed vigorously among themselves and attacked various theses of the original Viennese logical positivism. But, on essentials, they agreed with the Vienna Circle's approach and assumptions. Metaphysics was out and stayed out, symbolic logic stayed in. English-language philosophy in the later twentieth century was long on careful logical analyses of what one might mean by this or that claim, often replete with Greek symbols. No one wrote 'later' if they could write 'at time $t_2 > t_1$.' It was short on the entities traditionally discussed by philosophers, such as causes, consciousness, virtues, and God. In recent decades, a certain diversity on these questions has become permissible. But philosophy has not yet entirely recovered from its walk in the Vienna Woods.[35]

32 Volker Peckhaus, 'The Mathematical Origins of Nineteenth-century Algebra of Logic', in *The Development of Modern Logic*, ed. Leila Haaparanta (Oxford: Oxford University Press, 2009), 159–195.

33 Karl Sigmund, *Exact Thinking in Demented Times: The Vienna Circle and the Epic Quest for the Foundations of Science* (Basic Books, 2017), §1.1.

34 Sigmund, §1.1.

35 James Franklin, 'Ludicrous, but Interesting', *New Criterion* 36, no. 4 (2017): p. 82.

Figure 2.6: Meeting place of the Vienna Circle

For my purposes though the Vienna Circle's importance is in popularising a style of philosophy which was heavily reliant on the new logic. So much so that for decades it would have been difficult to engage with those dialogues effectively without some basic familiarity. This leads naturally to offering philosophy students instruction in the new logic.

So the stage is set. We meet our cast of characters in their youth, living through the anxious period between the end of World War I and the start of World War II. The story is composed of four biographical snippets. First, of L. Susan Stebbing, whose textbook *A modern introduction to logic* comes at a pivotal point in logic teaching. Next, the friends Rudolf Carnap and Hans Reichenbach, central figures in the dissemination of logical positivism. Then, the renowned Alfred Tarski, one of the greatest logicians of all time. Finally, Willard Van Orman Quine, one of the most famous philosophers of the 20th century.

Our first introduction is to L. Susan Stebbing who is often credited for bringing the thought of the Vienna Circle to Britain.[36,37] For us she will serve as a bridge the other way: to go from a discussion of British logic teaching to the Vienna Circle. In her day, she was recognised most

36 Siobhan Chapman, *Susan Stebbing and the Language of Common Sense*, Kindle Edition (London: Palgrave Macmillan UK, 2013), p. 87.

37 Michael Beaney, 'Stebbing, Lizzie Susan (1885–1943)', in *The Dictionary of Twentieth-Century British Philosophers*, ed. Stuart Brown (Bristol: Thoemmes, 2005), p. 348.

for her work on logic, and particularly her textbook *A Modern Introduction to Logic*.[38] In 1933 Stebbing's appointment at Bedford College was reported in national newspapers as she was the first female professor of philosophy in the U.K.

Stebbing's textbook is the first English language textbook to introduce the new logic. In the preface Stebbing explains, 'During the last half-century greater advances have been made than in the whole of the preceding period from the time of Aristotle. But the introductory text-books now being used in British Universities show no trace of these developments.'[39]

Like Whately, Stebbing's impact has mostly been forgotten; a sad oversight which her biographer Siobhan Chapman aims to rectify. While Chapman's focus is on bringing to light the impact Stebbing had beyond her textbook, I aim to highlight the important role of textbooks in the development of the discipline.

Stebbing's textbook is noteworthy. It was written at a pivotal point in the development of logic and as a consequence, Stebbing combines a presentation of traditional Aristotelian logic with the new symbolic logic. She also discusses issues of scientific method, including the problems surrounding deduction and induction[40] with frequent references to J.S. Mill. Stebbing's textbook meets the challenge of preparing students for university examinations which did not require any knowledge of the new logic while also introducing that logic by showing the continuity between it and the traditional logic. She says: '...my purpose has been to emphasize the connexion between Aristotelian logic and symbolic logic, and thus to write a text-book which will include little possible that the student has subsequently to unlearn, or for the teaching of which the modern logician feels it necessary to apologize.'[41]

Chapman remarks, 'Stebbing confronts her readership of beginning students with some very challenging material indeed. She is not interested in presenting a tidied up version of her subject matter, or in pretending that the issues under discussion are more fully resolved or less controversial than is the case.'[42] Stebbing's text does not open in a way that connects immediately with Whately and Mill. Instead it is towards the back of the book, in Chapter XXIV §3 'The Normative Aspect of Logic', that Stebbing begins a discussion of whether logic is an art or a science, concluding that it is a science. She says, 'There may be an art of thinking ...But the art of thinking must not be confused with logic.'[43] Shortly after this Stebbing begins to discuss whether logic is normative. She argues:

> In so far as logic is concerned with the criticism of modes of thinking it has a norm-

38 Chapman, *Susan Stebbing and the Language of Common Sense*, p. 172.
39 L. Susan Stebbing, *A Modern Introduction to Logic* (London: Methuen & co. ltd., 1930), p. vii.
40 Chapman, *Susan Stebbing and the Language of Common Sense*, p. 50.
41 Stebbing, *A Modern Introduction to Logic*, pp. x-xi.
42 Chapman, *Susan Stebbing and the Language of Common Sense*, p. 51.
43 Stebbing, *A Modern Introduction to Logic*, p. 473.

> ative aspect. ...But this normative aspect is, as it were, a by-product. We do not study logic in order to establish norms by reference to which the validity of reasoning may be tested. The discovery of norms of thinking – when, indeed, they are discovered – results from the fact that valid thinking is formal and that logic is the science of possible forms. It is a mistake to regard the normative aspect of logic as constituting its distinguishing characteristic.[44]

Stebbing's text connects the traditional Aristotelian logic and the new symbolic logic, but it also connects the old discussion of whether logic is an art or a science and the new discussion of whether logic is normative or descriptive.

Figure 2.7: L. Susan Stebbing

Stebbing comes from the British logic tradition, but she devoted research to dialogues outside of Britain. In the 1930s Stebbing 'was a central figure in the early dialogue between British philosophy and logical positivism, and did more than anyone else at the time to introduce the

44 Stebbing, *A Modern Introduction to Logic*, p. 474.

Vienna Circle to the English-speaking world.'[45]

Stebbing's interactions with the Vienna Circle began in 1930 when she met Moritz Schlick, the founder and leader of the Vienna Circle.[46] In 1934 Stebbing invited Rudolf Carnap to lecture at Bedford College, and she was the 'only British philosopher on the first "Organisation Committee of the International Congress for the Unity of Science".'[47] She attended the Paris congress in 1935 and in 1938, when the International Congress of the Unity of Science came to Cambridge, she gave the inaugural lecture.[48] Stebbing is remembered for her 'role as an early British interpreter and critic of logical positivism and the dialogue she promoted between the two main branches of analytic philosophy.'[49]

From Stebbing, we cross to meet the good friends Rudolf Carnap and Hans Reichenbach. Carnap and his friend were both born in 1891 in Germany. They are six years younger than Stebbing who was born in 1885. From 1910 to 1914 Carnap attended the University of Jena where he took Frege's courses in mathematical logic. World War I interrupted his studies, but after three years of military service, he returned to study, now at the University of Berlin.

Carnap's early studies centred on physics, albeit spiced with philosophy and mathematical logic. In contrast, his friend Reichenbach travelled a meandering path through civil engineering, physics, mathematics and philosophy attending various universities including Berlin, Erlangen, Göttingen and Munich. Among Reichenbach's teachers were David Hilbert, Max Planck, and Albert Einstein.

Carnap and Reichenbach began corresponding some months before they met in person in March 1923 at a conference in Erlangen. After this, they remained in contact until Reichenbach's death in 1953.

In 1926 the friends, now both 35 years old, each received university appointments. Carnap was offered a position as a private lecturer on theoretical philosophy at the University of Vienna. At the same time Reichenbach became an assistant professor in the physics department of Humboldt University in Berlin. In February 1930 Alfred Tarski lectured in Vienna, and during November 1930 Carnap visited Warsaw, Tarski's home town. Also during 1930, Carnap and Reichenbach began editing the journal *Erkenntnis* together.

In 1931 Carnap moved from Austria to Germany, accepting a position as an associate professor for natural philosophy at the School of Natural Science at the German University in

45 Michael Beaney, 'Susan Stebbing on Cambridge and Vienna Analysis', in *The Vienna Circle and Logical Empiricism: Re-evaluation and Future Perspectives*, ed. Friedrich Stadler, vol. 10 (Kluwer Academic Publishers, 2003), p. 348.

46 Chapman, *Susan Stebbing and the Language of Common Sense*, p. 81.

47 Chapman, p. 89.

48 Chapman, p. 97.

49 Michael Beaney and Siobhan Chapman, 'Susan Stebbing', in *The Stanford Encyclopedia of Philosophy*, Summer 2017, ed. Edward N. Zalta (8 May 2017).

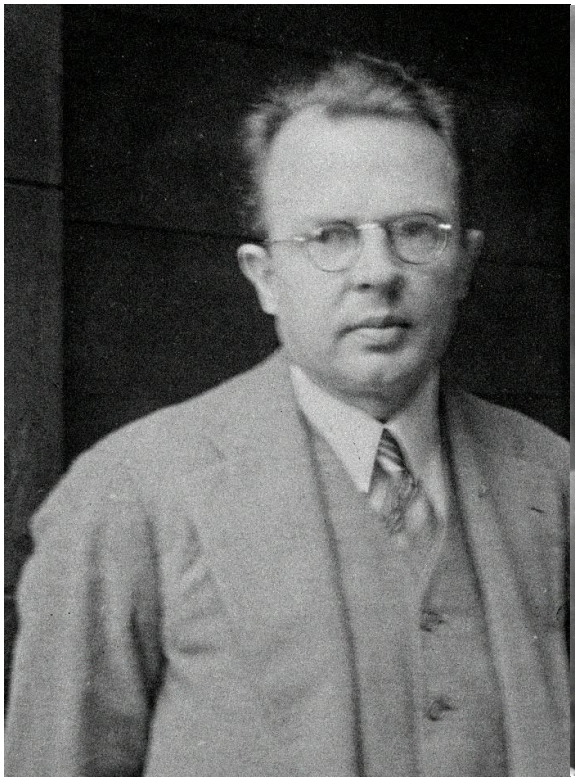


Figure 2.8: Rudolf Carnap

Figure 2.9: Hans Reichenbach

Prague (which ceased to exist in 1945).

1933 was the beginning of the end for these intellectuals in Germany. Adolf Hitler became Chancellor of Germany and Reichenbach was dismissed from his appointment at the University of Berlin due to his Jewish ancestry. He moved to Turkey to become the head of the Department of Philosophy at the University of Istanbul. Willard van Orman Quine was then on a travelling fellowship enabling Carnap and Quine's 1933 meeting in Prague.

Susan Stebbing invited Carnap to London to give a series of lectures in 1934. Through this invitation, Carnap met Bertrand Russell and A.J. Ayer for the first time.[50]

With the political situation in Europe becoming increasingly unstable, both Carnap and Reichenbach sought refuge in the United States. Carnap emigrated to the U.S. in 1935. In 1936 he took a position as a professor of philosophy at the University of Chicago. It took a few more years for Reichenbach to emigrate. However, in 1938 he finally secured a position as a professor in philosophy at the University of California, Los Angeles.

The next of our cast of characters is Alfred Tarski, one of the greatest logicians of all time. Younger than Carnap and Reichenbach by ten years, Tarski was born in 1901 in Warsaw – then a part of the Russian Empire. In 1918, when Poland gained its independence at the end of World

50 Michael Beaney, 'Series Editors Forward', in *Susan Stebbing and the Language of Common Sense*, Kindle Edition, by Siobhan Chapman (London: Palgrave Macmillan UK, 2013).

War I, Tarski entered the University of Warsaw. Tarski's study at the University of Warsaw from 1918 to 1924 was devoted almost entirely to mathematics and logic. He studied the foundations of arithmetic, geometry, and logic. His teachers included Jan Łukasiewicz and Stanisław Leśniewski.[51]

After becoming the youngest person ever to complete a doctorate at Warsaw University, Tarski supported himself, and later his wife and children, teaching mathematics at a high school and with teaching positions at the University of Warsaw.

Figure 2.10: Alfred Tarski and Kurt Gödel in Vienna 1935

From 1924 to 1939 Tarski wrote about logic and set theory, and built a strong international reputation. In 1930 he was invited to lecture in Vienna, where he met Kurt Gödel and Rudolf Carnap.[52] Tarski invited Carnap to visit Warsaw, which he did in November 1930.[53] From May to June of 1933, after visiting Carnap in Prague, Willard Van Orman Quine visited Warsaw and

51 Anita Burdman Feferman and Solomon Feferman, *Alfred Tarski: Life and Logic* (Cambridge, UK: Cambridge University Press, 2004), p. 30.
52 Feferman and Feferman, p. 81.
53 Feferman and Feferman, p. 83.

met Tarski. In a letter to Tarski, Quine recalls attending Tarski's seminar saying, 'It was an impressive seminar, a research center. You were already a great teacher and trainer of research logicians, as you have now been for fifty years.'[54] In 1935 the Unity of Science Movement[55] breathed new life into the Vienna Circle creating new opportunities for social interaction through a series of conferences. In 1934 the organisers had met in Prague to plan the 1935 Paris conference with Carnap, Reichenbach, and Tarski among them. In addition to attending the Paris conference in 1935 (for which Bertrand Russell gave the opening address), Tarski spent four months in Vienna meeting again with Kurt Gödel.

In 1939 Quine invited Tarski to speak at the Fifth International Unity of Science Congress, which would be held at Harvard University in September.[56] The same month as the Congress World War II began with the invasion of Poland forcing Tarski to stay in the United States, unable to reunite with his family. His wife and children survived WWII and were finally able to join him in 1946.[57]

The last of our cast is also the youngest. He has been called 'one of the best-known, most influential philosophers of the 20th century.'[58] Willard Van Orman Quine was born in 1908 in Akron, Ohio, in the United States. At 18 he entered Oberlin College where he studied mathematics and mathematical philosophy. Then from 1930 to 1932, he attended Harvard University where he wrote his doctoral dissertation on Whitehead and Russell's *Principia Mathematica* with Whitehead as his supervisor. Quine was affiliated with Harvard University in various capacities from his time as a student until his death.

After completing his Ph.D, he was a recipient of the Sheldon Traveling Fellowship offered by Harvard University. In 1932-33 this fellowship allowed him to visit Europe, including Vienna, Prague, and Warsaw. In 1975 Quine recalled his meeting with Carnap saying:

> Carnap was my greatest teacher. I got to him in Prague 38 years ago, just a few months after I had finished my formal studies and received my Ph.D. I was very much his disciple for six years. In later years his views went on evolving and so did mine, in divergent ways. But even where we disagreed he was still setting the theme; the line of my thought was largely determined by problems that I felt his position presented ...I was then an unknown young foreigner of 23, with thirteen

54 Feferman and Feferman, *Alfred Tarski: Life and Logic*, p. 87.

55 A philosophic program whose advocates strove to develop a single, comprehensive scientific language. They aimed to produce a unified philosophical and methodological foundation for science and remove nonempirical concepts from science.

56 Feferman and Feferman, *Alfred Tarski: Life and Logic*, p. 106.

57 Mario Gómez-Torrente, 'Alfred Tarski', in *The Stanford Encyclopedia of Philosophy*, Spring 2019, ed. Edward N. Zalta (29 January 2019).

58 Friedrich Stadler, 'The Vienna Circle and Its Periphery: Biographies and Biobibliographies', chap. 12 in *The Vienna Circle: Studies in the Origins, Development, and Influence of Logical Empiricism*, vol. 4, Vienna Circle Institute Library (Cham: Springer, 2015), p. 555.

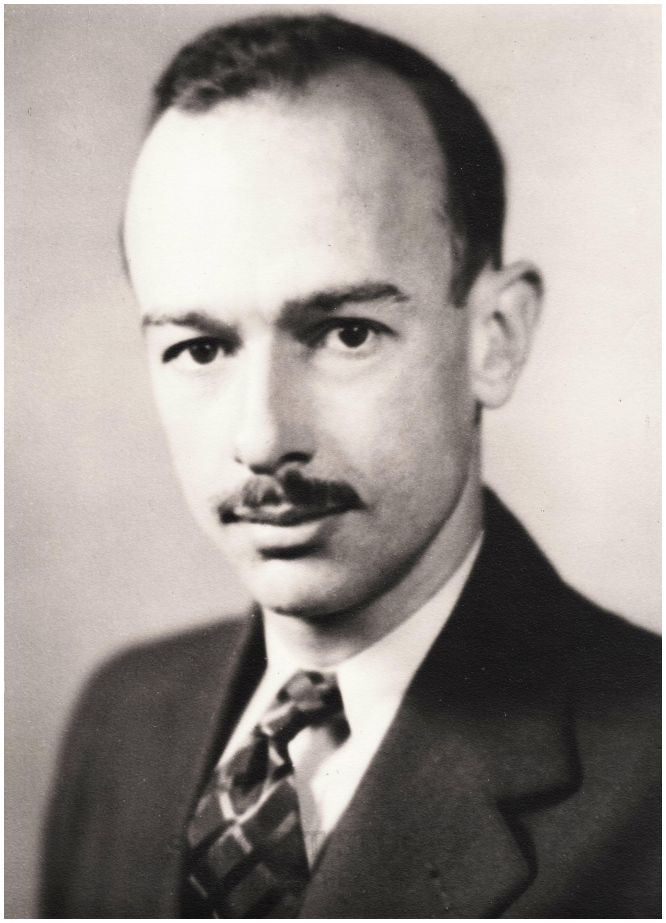


Figure 2.11: W.V.O Quine in 1934, Junior Fellow at Harvard University

> inconsequential pages in print and sixteen at press. It was extraordinary of anyone, and characteristic of Carnap, to have been so generous of his time and energy. It was a handsome gift. It was my first experience of sustained intellectual engagement with anyone of an older generation, let alone a great man. It was my first really considerable experience of being intellectually fired by a living teacher rather than by a dead book. ...At Harvard the following year, I lectured on Carnap's philosophy. Our correspondence was voluminous. ...By Christmas 1935 he was with us in our Cambridge [Massachusetts] flat.[59]

In 1939, held for the first time outside of Europe, the 5th International Congress for the Unity of Science was hosted at Harvard. Quine says, 'Basically this was the Vienna Circle, with some accretions, in international exile.'[60] The theme of the Congress was 'the Logic of Science' and the organising committee features L.S. Stebbing, R. Carnap, H. Reichenbach and, as sec-

59 W. V. Quine, 'Homage to Rudolf Carnap', in *Rudolf Carnap, Logical Empiricist: Materials and Perspectives*, ed. Jaakko Hintikka, vol. 73, Synthese Library (Springer Netherlands, 1975), pp. XXV-XXVI.

60 Willard Van Orman Quine, *The Time of My Life* (Cambridge, Mass.: MIT Press, 1985), p. 140.

retary of the committee on arrangements at Harvard, W.V. Quine.[61]

2.4 THE DIASPORA: TEACHING THE NEW LOGIC

The tumult of WWII brought with it a European diaspora carrying experts on the new logic to the United States.

In late 1938, a year before the start of world war II, Russell left England for the United States where he remained until May 1944. While in the U.S. Russell spent time teaching at three universities. At these universities, he connected with Carnap, Reichenbach, Tarski, Quine and Copi, a student who would go on to write an introductory logic textbook, the popularity of which would outshine that of both Whately's and Mill's.

Figure 2.12: Chicago Lakefront, 1938 Photographic vista by Kaufmann & Fabry

Russell's first stop in the U.S. was Chicago. When Russell arrived in Chicago in 1938, Carnap was a professor of philosophy at the University of Chicago. Russell describes this time and his meeting with some outstanding students saying:

> In Chicago I had a large seminar, where I continued to lecture on the same subject as at Oxford... It was an extraordinarily delightful seminar. Carnap and Charles Morris used to come to it, and I had three pupils of quite outstanding ability—Dalkey, Kaplan, and Copilowish. We used to have close arguments back and forth, and succeeded in genuinely clarifying points to our mutual satisfaction, which is rare in philosophical argument.[62]

Irving Marmer Copilowish was born in Duluth, Minnesota in 1917.[63] He studied at the University of Michigan at Ann Arbor between 1934 and 1948. For the academic year of 1938-39, he

61 'The Fifth International Congress for the Unity of Science at Harvard University', *Science* 88, no. 2292 (December 1938): 519.

62 Russell, *The Autobiography of Bertrand Russell 1914-1944*, pp. 331-332.

63 Duluth Public Library, 'Irving Copi', Vintage Duluth, 21 June 2007, accessed 25 May 2020, https://dplreference.wordpress.com/2007/06/21/irving-copi/.

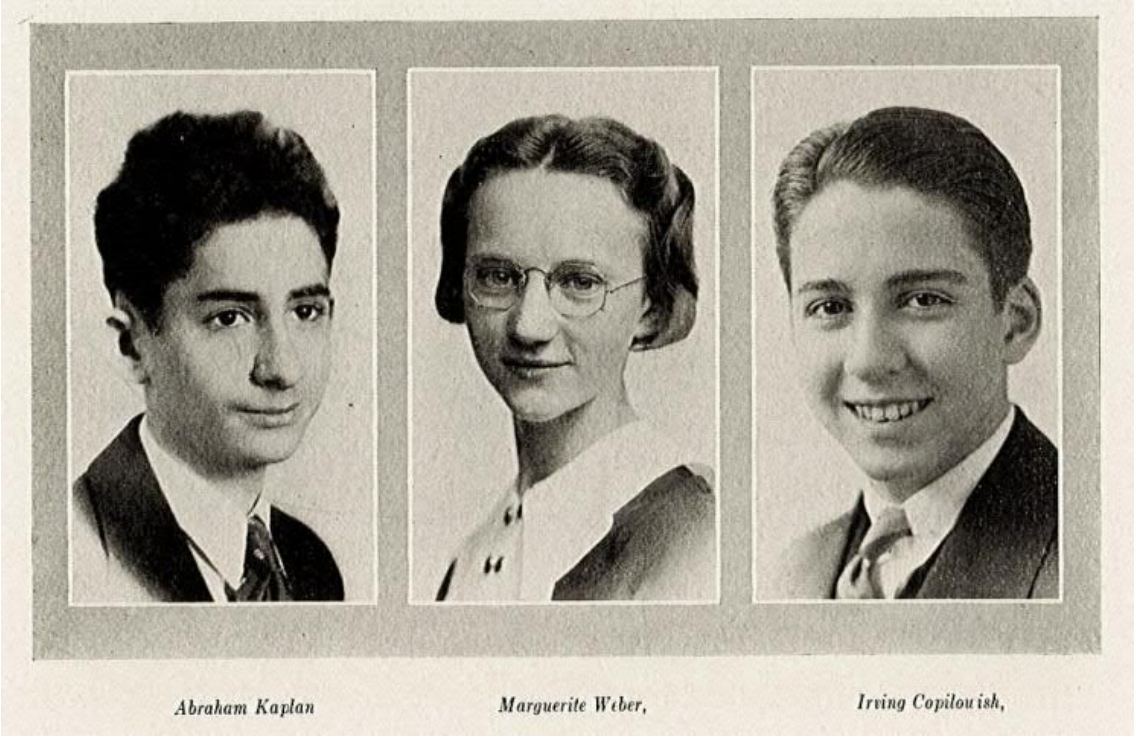


Figure 2.13: Duluth debate team 1933 including Abraham Kaplan and Irving Copi

travelled to the University of Chicago meeting with his high school friend Abraham Kaplan.[64] He later changed his last name to Copi.

In many ways, Copi's contribution to logic is like that of Whately's and Mill's: as the author of a staggeringly popular introductory logic textbook, more than through any other publication. Whately's textbook was in its 9th edition and had been in circulation for 20 years when Mill's textbook was published. This monumental achievement is dwarfed when compared to the success of Copi's textbook since its first publication in 1953. The textbook is in its 15th edition, has remained in print for over 60 years, and has found several additional co-authors over the years. The textbook is still used in reputable universities today despite an abundance of alternatives.

Determining who, of Irving Copi or Bertrand Russell, has had a greater impact on the development of logic would be a challenge. If you're surprised by this claim, consider that as the author of the widely used textbook *Introduction to Logic* and the less famous but similar textbook *Symbolic Logic*, Copi's work will have formed the basis of many students' early learning.

In 2002 when Copi died, Kenneth Kipnis, a University of Hawaii professor of philosophy, contributed to Copi's obituary in the Honolulu Star-Bulletin. Kipnis spoke of Copi's contribution to logic through his introductory texts, saying that in the 1950s his textbook was:

64 Duluth Public Library, 'Abraham Kaplan and Irving Copi', Vintage Duluth, 29 June 2007, accessed 25 May 2020, https://dplreference.wordpress.com/2007/06/29/abraham-kaplan-and-irving-copi/.

> head and shoulders above anything else out there, which is why people talk about Copi as being, in a sense, the father of logic courses now taught at every university. As logic courses boomed, philosophy departments hired more logicians, who were doing more publishing, and all of a sudden by the 1960s and 70s there was a huge number of people with background in logic.[65]

One might argue that it is Bertrand Russell's influence that extends through Copi, as Copi was Russell's student. But given Russell's high opinion of Copi as a student one should not assume that Copi is a passive recipient of Russell's teaching and should give Copi credit for understanding and contributing to the development of logic.

In a way that harks back to Stebbing, and before her Mill and Whately, Copi begins his discussion of the subject matter of logic saying, 'logic has frequently been defined as the science of the laws of thought,'[66] he goes on to caution that, 'the logician is not in the least concerned with the dark ways by which the mind arrives at its conclusions during the actual process of reasoning. He is concerned only with the *correctness* of the completed process.'[67] The enduring description of logic, which can be found opening the 14th edition, is: 'The study of logic is the study of the methods and principles used in distinguishing correct from incorrect reasoning.'[68]

The University of California, Los Angeles (UCLA) was Russell's next stop. Wienpahl recollects that 'In 1940, Russell joined the department, bringing with him from Chicago Abraham Kaplan and Norman Dalkey.'[69] By the time Russell arrived at UCLA Reichenbach had been teaching in the philosophy department for a year, and during Russell's stay they shared an office.[70,71]

It's possible that Russell at first intended to stay for more than just one year, but his biography indicates that while the weather in Los Angeles pleased him, the academic climate did not. Eager to leave, Russell resigned from his position at UCLA before his next post was secure, and from then until his return to England in 1944 Russell had difficulties with employment and income.[72]

65 Helen Altonn, 'Educator Earned Worldwide Fame for Work in Logic', *Honolulu Star-Bulletin*, 1 September 2002,

66 Irving M. Copi, *Introduction to Logic*, 2nd ed. (New York: The Macmillan Company, 1961), p. 4.

67 Copi, p. 6.

68 Irving M. Copi, Carl Cohen and Kenneth McMahon, *Introduction to Logic*, 14th ed. (ProQuest Ebook Central, New York: Routledge, Taylor & Francis Group, 2016), p. 3, ISBN: 9781315510873.

69 Hans Reichenbach, *Selected Writings 1909-1953: With a Selection of Biographical and Autobiographical Sketches*, ed. Maria Reichenbach and Robert S. Cohen, trans. Elizabeth H. Schneewind, vol. 1, Vienna circle collection (Dordrecht: D. Reidel, 1978), p. 48.

70 Güven Güzeldere, 'An Interview with Maria Reichenbach and David Kaplan', in *Turkish Studies in the History and Philosophy if Science*, ed. Gürol Irzik and Güven Güzeldere (Dordrecht: Springer, 2005), 7–24.

71 Reichenbach, *Selected Writings 1909-1953*, p. 79.

72 Russell, *The Autobiography of Bertrand Russell 1914-1944*, pp. 333-342.

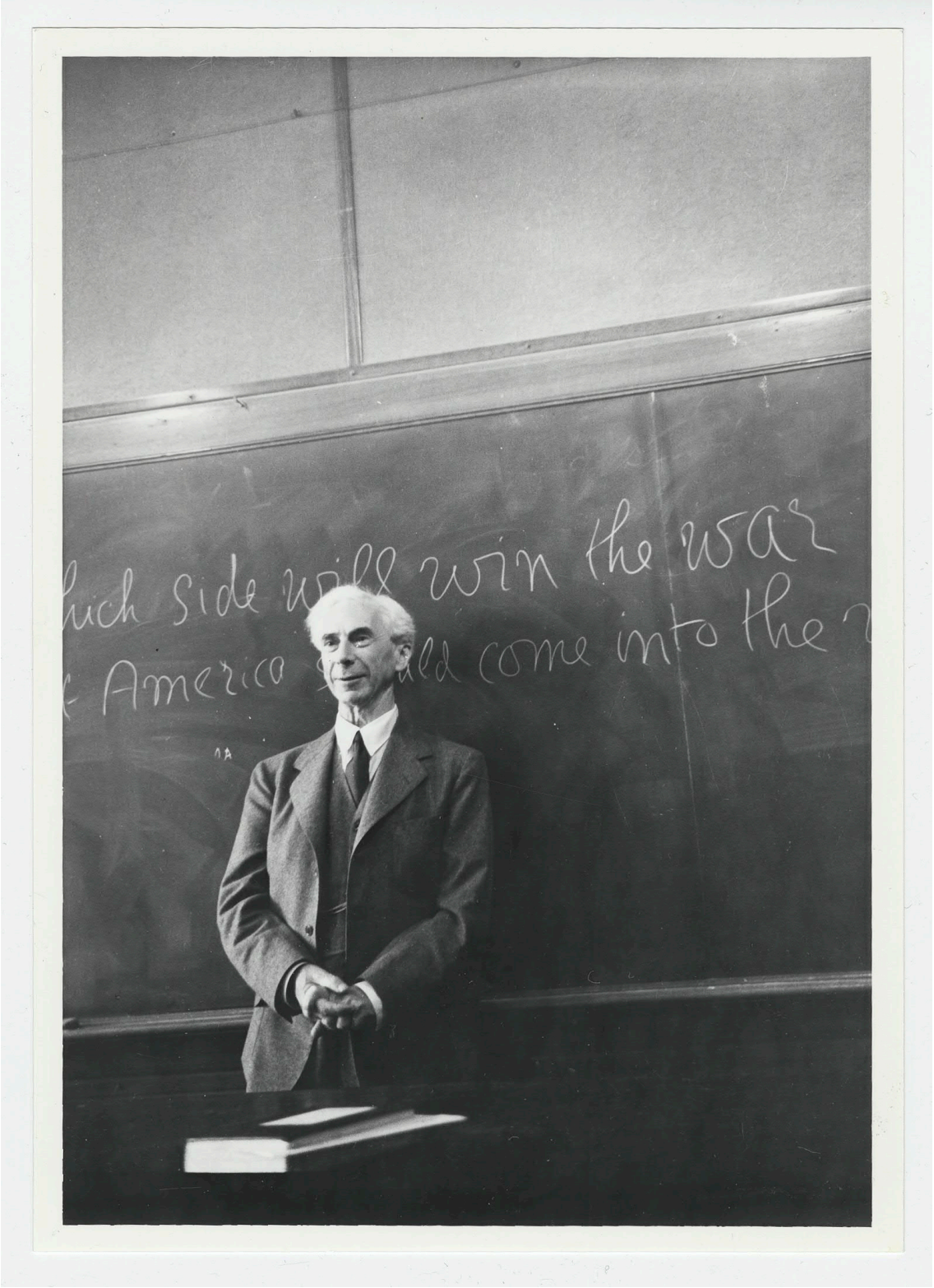


Figure 2.14: Bertrand Russell Teaching at UCLA in 1940

After transfering to UCLA in 1940, Kaplan finished his Ph.D in 1942.[73] From 1946 to 1963 he taught alongside Reichenbach at UCLA. In 1946 Reichenbach mentions the assistance & criticism for his *Elements of Symbolic Logic* provided by Norman Dalkey and Abraham Kaplan. Reichenbach also gives special mention to two of his teachers: David Hilbert from his days as a student at Göttingen and Bertrand Russell during his time at UCLA.

Given the interconnection of teachers and colleagues between Reichenbach and Copi, the similarity in approach to introducing the subject matter of logic in each textbook is not a surprise. They both express the idea that logic has something to do with reasoning, note that logic is not meant to describe reasoning, and argue that logic shows us how to reason *correctly*. The presence of these ideas in Reichenbach's textbook is also not surprising when we consider that anti-psychologism was a hot topic in the philosophy of logic during his time as a student.

Reichenbach begins, 'Logic has often been defined as the science that deals with the laws of thought.'[74] After explaining that logic is not an exploration of the psychological laws of thought which would have to account for both correct and incorrect thinking, he says, 'If we want to say that logic deals with thinking, we had better say that logic teaches us how thinking *should* proceed and not how it *does* proceed.'[75]

By 1946, when Reichenbach was writing, anti-psychologism had cooled as a topic with everyone quite convinced that logic does not *describe* the reasoning process. But, according to Frege, a discussion which focuses on *correct* reasoning rather than *actual* reasoning merely replaces one type of psychologism with another. Reichenbach avoids this by shifting the target of analysis from thought to language concluding, 'logic is *analysis of language*.'[76]

Returning to the story of Russell's tour of the United States: in the 1940-41 academic year Russell, Tarski, Quine, and Carnap were all in Cambridge, Massachusetts. In 1940 a moral panic was making it difficult for Russell to find employment. However, the arrangement to give the William James Lectures at Harvard had been made before the trouble began and proceeded despite it. Carnap was on loan from Chicago that year, and Tarski had not yet found permanent employment after finding himself unable to return to Poland. In 1936 Quine had taken a position as an Instructor at Harvard, in 1941 he would be promoted to Assistant Professor. It was just a year after the Fifth International Unity of Science Congress had been held at Harvard. Both Quine and Tarski were working on introductory logic textbooks. Quine was writing *Elementary Logic* and Tarski was working on converting the 1937 German translation of his 1936 Polish text *On Mathematical Logic and Deductive Method* from a popular scientific

73 Duluth Public Library, 'Abraham Kaplan', Vintage Duluth, 4 June 2007, accessed 25 May 2020, https://dplreference.wordpress.com/2007/06/04/abraham-kaplan/.

74 Hans Reichenbach, *Elements of Symbolic Logic* (New York: The Free Press, 1966), p. 1.

75 Reichenbach, p. 1.

76 Reichenbach, p. 2.

book into a textbook which could be used for an elementary university course.[77] Quine writes: 'The fall term of 1940 is graven in my memory for more than just the writing of *Elementary Logic*. Russell, Carnap, and Tarski were all at hand. Tarski had a makeshift research appointment at Harvard and was in need of a job. Carnap had a visiting professorship with us. Russell was giving the William James Lectures, backed by a seminar.'[78]

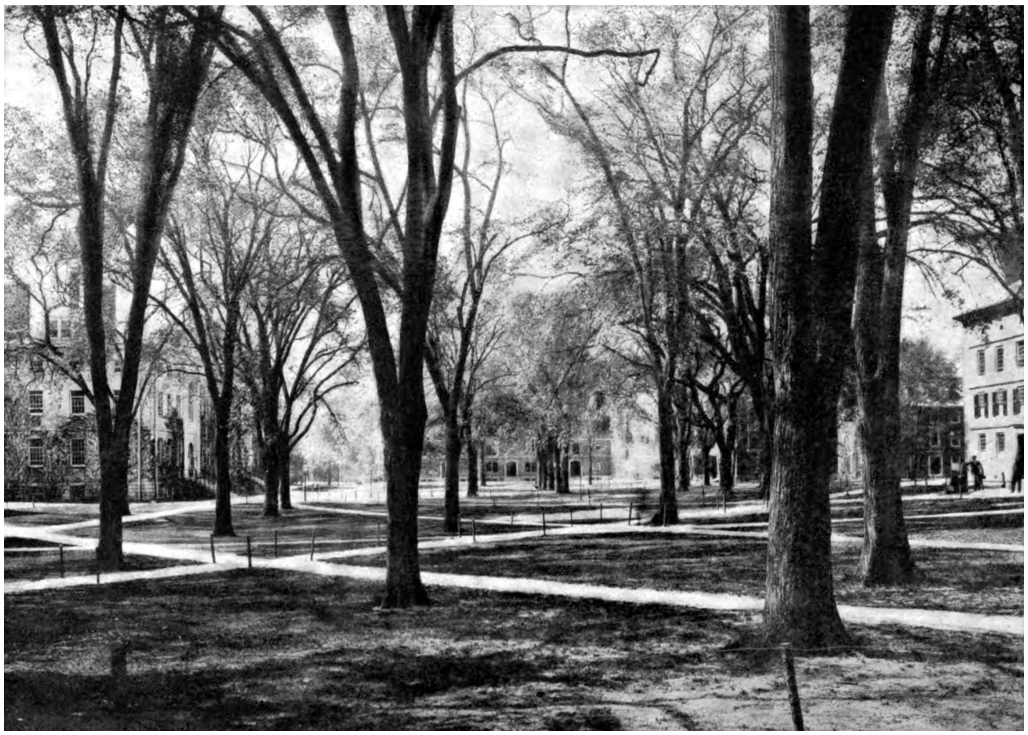


Figure 2.15: Harvard Yard

Gregory Frost-Arnold discovered Carnap's notes from the time and, in his book inspired by this discovery, discusses the conversations held and their relevance to current topics in the philosophy of science. He says:

> In 1940-41, when many of the greatest scientific philosophers of the twentieth century spent a year together, the plurality of their academic collaboration focused on the question: 'What form should an intelligible language adequate for analyzing science take, if the number of physical things in the universe is possibly finite?' And, as a corollary, 'How will this force us to change arithmetic?'[79]

This question stands in the background of the textbooks produced by Carnap, Tarski, and Quine. It sheds light on the way they each define logic, and it is what makes their textbooks

77 Alfred Tarski, *Introduction to Logic: And to the methodology of deductive sciences*, Second Edition, Revised, trans. Olaf Helmer-Hirschberg (New York: Oxford University Press, 1946), p. xi.

78 Quine, *The Time of My Life*, p. 149.

79 G. Frost-Arnold, *Carnap, Tarski, and Quine at Harvard: Conversations on Logic, Mathematics, and Science*, vol. 5, Full Circle: Publications of the Archive of Scientific Philosophy (Chicago: Open Court, 2013).

distinct. While Tarski and Quine were both working on their textbooks in 1940, Carnap did not write one until 1954. Carnap's 1954 *Introduction to Symbolic Logic and its Applications* was initially published in German; it was translated into English in 1958 while Carnap was at UCLA.

At the end of the 1940-41 academic year, the group again dispersed. Russell moved on from Harvard to Philadelphia to lecture at the Barnes Foundation. After his year as a visiting professor at Harvard, Carnap returned to the University of Chicago. And finally after a period of tenuous employment after being stranded after the Fifth International Unity of Science Congress in 1939, Tarski was offered a position at the University of California, Berkeley in 1942. There he was finally reunited with his wife and children and remained until he died in 1983.

The final act takes place back in California at the University of California, Los Angeles (UCLA) and the University of California, Berkley (UCB). It is the story of the students who learned logic from Reichenbach, Tarski, and Carnap, and who then taught alongside them. We open into 1940. At UCLA Reichenbach was teaching, Russell had departed, and Kaplan had transferred in. Meanwhile over at UCB, Donald Kalish had begun his studies in psychology.

In 1942, Tarski arrived at UCB, and Kaplan finished his Ph.D at UCLA. Then, in 1945 the Second World War ends and the Servicemen's Readjustment Act encourages returning soldiers to get an education. Among the beneficiaries is Benson Mates, who joined the graduate program in philosophy at UCB in 1945. By this time Donald Kalish had completed a BA and a Masters in psychology and joined the graduate program in philosophy. Both Mates and Kalish go on to completed their doctorates in 1948 at UCB.

During these years, a dramatic shift in the tertiary education landscape was taking place. In the late 19th and early 20th centuries, enrolment in higher education was limited mainly to the wealthy, with only a relatively small proportion of the population participating in higher education. In the United States, 'when the federal Office of Education began collecting education data in 1869–70, only 63,000 students were attending higher education institutions throughout the country ...This small number of students was divided among 563 campuses.'[80] By 1991 in the U.S. over 14 million students were attending some 3,600 institutions.[81]

In the first 30 years of the 20th century enrolment in higher education accelerated driven by both population growth and increasing participation as a proportion of the population. 'By the end of the 1940s, college enrollment was surging. Large numbers of World War II veterans entered colleges assisted by such programs as the Service-men's Readjustment Act which provided education benefits.'[83] In fall 1949 there were about 2.4 million enrolled students.

80 T. D. Snyder, *120 Years of American Education: A Statistical Portrait*, technical report (National Center for Education Statistics, January 1993), p. 64.

81 Snyder, p. 64 & p. 77.

82 Data source: Snyder, pp. 76-77 (Table 24)

83 Snyder, p. 65.

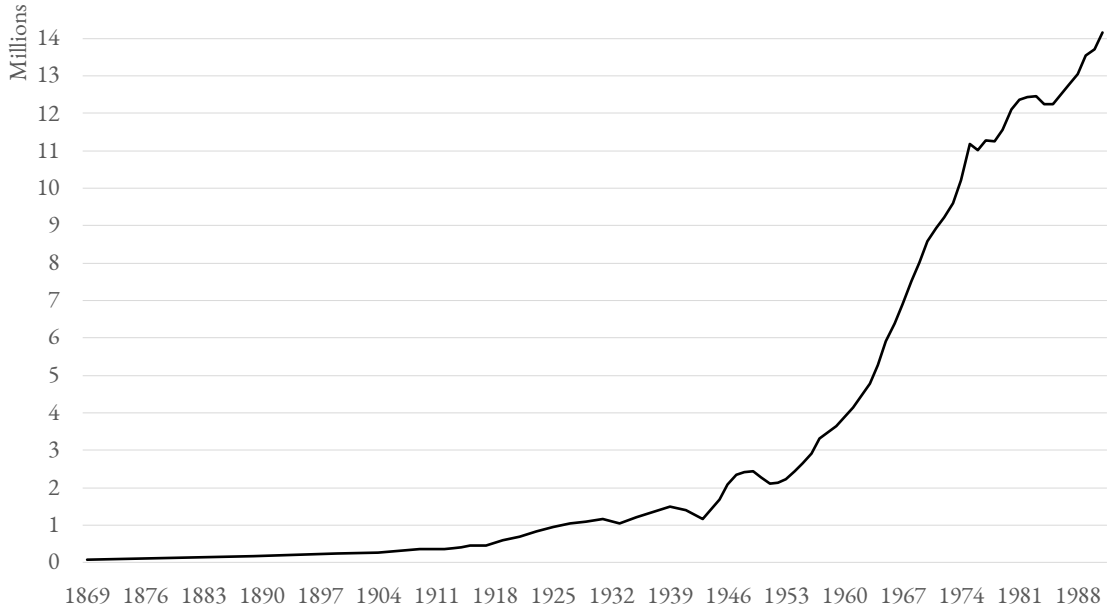


Figure 2.16: U.S. Student Enrolment [82]

From the end of the 1940s to the mid-1960s enrolments rose from 2.4 million to 5.9 million. 'The 1950s and 1960s marked two major developments. First, large numbers of young people entered college and second, public colleges expanded dramatically to meet the demand.'[84] This 20th century trend, commonly referred to as 'the massification of higher education,' continues now in the 21st century and is reflected in other countries besides the United States. The increase of enrolments over time shows the increasing demand for teaching.

During these years, younger academics trained in the tradition of Russell, Carnap, Reichenbach, Tarski and Quine joined the philosophy departments at UCLA and UCB and took up teaching. Mates took up a position in the Philosophy Department at Berkeley alongside Tarski in 1948, and Kalish joined the philosophy department at UCLA alongside Reichenbach in 1949. Kalish recalls with fondness how Reichenbach managed his recruitment, and his delight at having the opportunity to share an office with him.[85] After that, Kalish and Reichenbach taught the beginning course in symbolic logic at UCLA in alternate semesters with Kalish choosing to use Quine's *Methods of Logic* as the textbook for his courses rather than Reichenbach's *Elements of Symbolic Logic*.[86] Reichenbach died suddenly in 1953, and in 1954 his friend Carnap filled his place in the philosophy department at UCLA.

In 1948, around the time both Kalish and Mates took up teaching, Richard Montague joined UCB as a student. He studied mathematics, philosophy, and Semitic languages. His

84 Snyder, *120 Years of American Education: A Statistical Portrait*, p. 66.
85 Reichenbach, *Selected Writings 1909-1953*, pp. 45-46.
86 Reichenbach, p. 46.

teachers were Professors W. J. Fischel in Arabic, Paul Marhenke and Benson Mates in philosophy, and Alfred Tarski in mathematics. In 1955, two years before the completion of his dissertation at UCB, Montague began teaching alongside Kalish at UCLA.

In 1958 the English translation of Carnap's *Introduction to Symbolic Logic and Its Applications* was published. The German edition was written while he was working at the Institute for Advanced Study in Princeton in 1954. It was then translated while he was at UCLA. Carnap's teaching career ended in 1962 when he retired from UCLA.

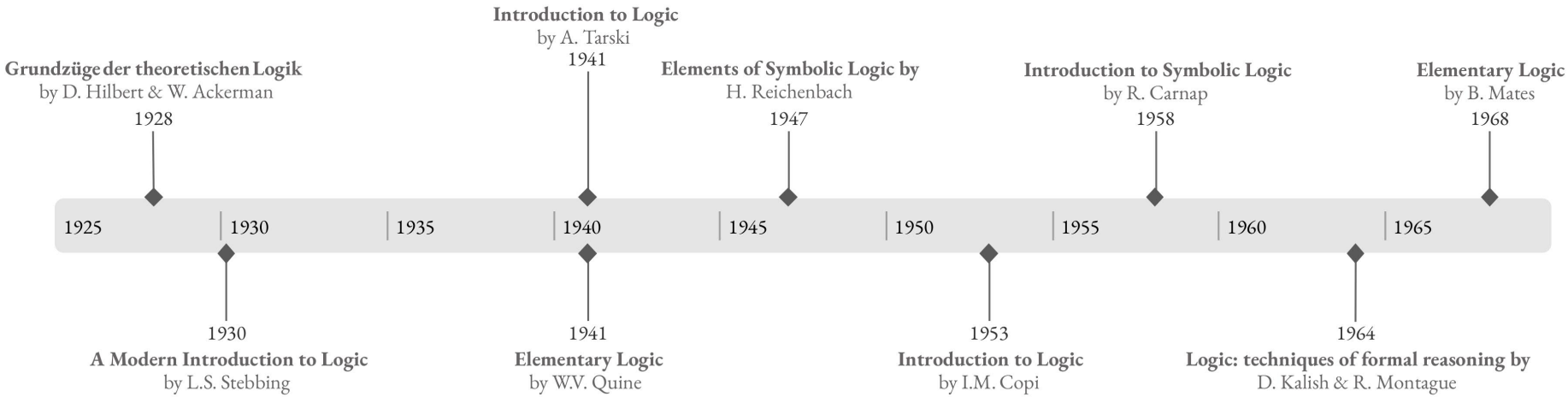
Figure 2.17: Richard Montague

Figure 2.18: Donald Kalish

In 1964 Kalish and Montague published *Logic: techniques of formal reasoning* and Mates published *Elementary Logic* in 1965. Kalish and Montague mention suggestions and discussions with Mates in the preface to their text. Likewise, Mates refers to encouragement and assistance received from Kalish and Montague. Mates also mentions the pedagogic tradition saying, 'The very considerable extent to which I have drawn from other textbooks, particularly V.W Quine, Alonzo Church, Patrick Suppes, and Irving Copi, will be evident to anyone familiar with these works.'[87]

87 Benson Mates, *Elementary Logic* (New York: Oxford University Press, 1965), p. viii.

Figure 2.19: Timeline of early modern logic textbooks



2.5 WHAT DO THE OLD TEXTBOOKS SAY ABOUT LOGIC?

I began my discussion of the history of introductory logic teaching with the hypothesis that introductory logic textbooks are produced by teachers copying from their own education. They do this hoping the texts that have stood the test of time are examples the best philosophic and pedagogic practice. It is also a way to avoid having to solve a hard pedagogic problem. I contend that like the boats of the Isle of Groix, introductory logic textbooks are a product of tradition. This hypothesis is not, strictly speaking, falsifiable, but its relevance could be disputed. If the tradition has indeed produced explanations of the subject which are well suited to the need – examples of the best philosophic and pedagogic practice – then the assertion that textbook explanations of logic are predominantly a product of tradition is not relevant.

There are eleven textbooks in the history I have discussed, summarised in table 2.1. Figure 2.19 is a timeline of the nine which introduce modern symbolic logic. In what follows, I discuss the psychological, normative, and other strategies the texts use to explain the subject matter of logic.

Table 2.1: Summary of historic textbooks

Author	**Title**	**First Published**
Richard Whateley	Elements of Logic	1826
John Stuart Mill	A System of Logic, Ratiocinative and Inductive	1843
David Hilbert, William Ackerman	Grundzüge der Theoretischen Logik	1928
L. Susan Stebbing	A Modern Introduction to Logic	1930
Alfred Tarski	Introduction to Logic and to the Methodology of Deductive Sciences	1941
W.V. Quine	Elementary Logic	1941
Han Reichenbach	Elements of Symbolic Logic	1947
Irving M Copi	Introduction to Logic	1953
Rudolf Carnap	Introduction to Symbolic Logic and its Applications	1958
Donald Kalish, Richard Montague	Logic: Techniques of Formal Reasoning	1964
Benson Mates	Elementary Logic	1965

The question is whether these texts supply a meaningful explanation of logic for the beginner in today's context. I argue that they do not. The sea may fashion boats, but time does not fashion teaching.

The earliest textbooks contain explanations of the subject matter of logic which refer to the reasoning or thinking process. Whately says:

> Logic, in the most extensive sense in which it is advisable to employ the name, may be considered as the Science, and also as the Art, of Reasoning. It investigates the principles on which argumentation is conducted, and furnishes such rules as may be derived from those principles, for guarding against erroneous deductions.[88]

88 Whately, *Elements of Logic*, p. 1.

Mill says:

> Logic has often been called the Art of Reasoning. A writer ...has adopted the above definition with an amendment; he has defined Logic to be the Science, as well as the Art of reasoning; meaning by the former term, the analysis of the mental process which takes place whenever we reason, and by the latter, the rules, grounded on that analysis, for conducting the process correctly. There can be no doubt as to the propriety of the emendation.[89]

Hilbert & Ackerman say:

> The purpose of the symbolic language in mathematical logic is to achieve in logic what it has achieved in mathematics, namely, an exact scientific treatment of its subject-matter. ... The transition from statements to their logical consequences, as occurs in the drawing of conclusions, is analysed into its primitive elements, and appears as a formal transformation of the initial formulas in accordance with certain rules, similar to the rules of algebra; logical thinking is reflected in a logical calculus.[90]

And Stebbing says:

> Logic, in the most usual and widest sense of the word, is concerned with reflective thinking.[91]

These explanations are psychological; they express ideas which connect the subject matter of logic to a psychological process, but they do not mean to make logic a sub-discipline of psychology.

While Whately uses psychological terminology, his focus is on the distinction between 'art' and 'science'. In his introduction, Whately challenges Aldrich's teaching and asserts his philosophy of logic: that logic is both an art and a science. It is a science when related to the theory of reasoning and art when associated with the practice of reasoning.

Mill adopts Whately's philosophy of logic completely, but there is a stronger psychological emphasis. Something else interesting is happening here; Mill has adopted Whately's philosophy of logic but rejected his logic. Whately's text is an introduction to Aristotelian syllogistic logic which Mill replaces this with his inductive system. Mill adopts Whately's philosophy of logic, but he also uses that philosophy of logic as part of an argument for the rejection of the logic that Whately was teaching. Mill says:

89 Mill, *A System of Logic, Ratiocinative and Inductive*, pp. 2-3.

90 David Hilbert and Wilhelm Ackermann, *Principles of Mathematical Logic*, ed. Robert E. Luce., trans. Lewis M. Hammond, George G. Leckie and F. Steinhardt (New York: Chelsea Publishing Co., 1950), p. 1.

91 Stebbing, *A Modern Introduction to Logic*, p. 1.

> Archbishop Whately has contended that syllogising, or reasoning from generals to particulars, is not, agreeable to the vulgar idea, a particular mode of reasoning, but the philosophical analysis of the mode in which all men reason, and must do so if they reason at all. With all deference due so high an authority, I cannot help thinking that the vulgar notion is, in this case, the more correct.[92]

Mill's reasoning appears to be this: logic depends on an analysis of reasoning, syllogising involves patterns of reasoning which move from premises which contain general (universal) statements to conclusions which contain statements about specific instances, but this is not how people actually reason. Mill argues that people actually reason inductively, that is they infer universal claims from observations of specific instances. So, since logic depends on an analysis of reasoning, and people reason inductively, the Aristotelian syllogistic logic should be rejected in favour of an inductive logic.

Between 1890 and 1914 logic was developing rapidly. At the same time, much of German-speaking philosophy was caught up in the psychologism dispute, an argument about whether logic and epistemology are part of psychology. Kusch gives examples of psychologistic argument, including one he attributes to Wilhelm Wundt, [93] known today as one of the founders of psychology. The argument is:

> 1. Normative-prescriptive disciplines – disciplines that tell us what we ought to do – must be based upon descriptive-explanatory sciences.
>
> 2. Logic is a normative-prescriptive discipline concerning human thinking.
>
> 3. There is only one science which qualifies as constituting the descriptive-explanatory foundation for logic: empirical psychology.
>
> *Ergo*, logic must be based upon psychology.[94]

The philosophy of logic exemplified in Whately and Mill's texts fuels Frege's anti-psychologism. While Frege attacked Mill for his psychologism, Godden argues that:

> Mill's position concerning the subject matter of logic is decidedly fractured. On the one hand, Mill held that the subject matter of logic is psychological processes – the operations of the understanding. On the other hand, Mill held that logic was inherently concerned with the truth and that, as such, the subject matter of logic is the objects of our thoughts, insofar as these determine the semantic and

92 Mill, *A System of Logic, Ratiocinative and Inductive*, Book II; Chapter III §3.

93 For a more detailed description of Wundt's position see Kusch (*Psychologism: A Case Study in the Sociology of Philosophical Knowledge*, pp. 128-137)

94 Kusch, 'Psychologism', §3, para. 1.

> evidentiary properties that obtain between the contents of our thoughts.[95]

In the end Godden does conclude that Mill's view is psychologistic because, for Mill, the principles of logic depend on psychology. However, his discussion shows that Mill's position is not clear-cut.

The psychologism dispute was brought to an end by World War I and the intellectual climate after the war.[96] Though the debate ended, in some sense it was never resolved. Frege and Husserl won the argument on psychologism in the sense that their position became the prevailing one, but they may not have won the dispute in terms of advancing the most convincing argument.

In the end, a much simpler argument may have prevailed. This argument is captured in Reichenbach's textbook:

> The actual process of thinking evades distinct analysis; it is in part logically determined, in part automatic, in part erratic; and what we observe as its constituents are isolated crystallizations of largely subconscious currents hidden below a haze of emotional processes. As far as there are any laws observable they are formulated in psychology; they include laws both of correct and of incorrect thinking, since the tendency to commit certain fallacies must be considered a psychological law in the same sense as the more fortunate habits of correct thinking.[97]

Copi offers a similar anti-psychological argument, saying:

> Logic has frequently been defined as the science of the laws of thought. But this definition, although it gives a clue to the nature of logic, is not accurate. In the first place, thinking is one of the processes studied by psychologists. Logic cannot be 'the' science of the laws of thought, because psychology is also a science which deals with laws of thought (among other things). And logic is not a branch of psychology; it is a separate and distinct field of study. In the second place, if 'thought' refers to any process that occurs in people's minds, not all thought is an object of study for the logician. All reasoning is thinking, but not all thinking is reasoning. ...There are many mental processes or kinds of thought that are different from reasoning.[98]

Essentially, the argument is that while people can reason well, they also do reason badly. So a formal system designed to describe reasoning must capture bad reasoning just as much as good

95 David Godden, 'Psychologism in the Logic of John Stuart Mill: Mill on the Subject Matter and Foundations of Ratiocinative Logic', *History and Philosophy of Logic* 26, no. 2 (2005): p. 132.
96 Kusch, 'Psychologism', §8, para. 1.
97 Reichenbach, *Elements of Symbolic Logic*, p. 1.
98 Copi, *Introduction to Logic*, pp. 4-5.

reasoning. The argument gives a reason to reject a strictly descriptive relationship between logic and reasoning. However, it opens the door wide for a normative connection between logic and reasoning – a connection which, for Frege, is just another form of psychologism.

Contra Frege, I argue that there is nothing fundamentally wrong with a psychological philosophy of logic. It would be a mistake to use this philosophy to define the whole discipline, but it can provide a good explanation of the aims of investigations that take place within the subject. While there is nothing wrong with making a connection between logic and thinking, a problem can arise: the question of adequacy.

To be good, a formal system must adequately describe its subject matter. A description might be inadequate because it is incomplete, or because it is inaccurate. We can show that a description is incomplete if there is a process which is not captured, and we can show that it is inaccurate if it does not produce the same result.

One way of interpreting Mill's argument against the Aristotelian syllogistic could be:

1. Logic describes how we reason.
2. We do not reason in the manner described by the Aristotelian syllogistic.
3. *Therefore*, the Aristotelian syllogistic is not correct.

A similar argument against classical logic could be made using the Wason Selection Task[99] as evidence for premise 2. Mill argues that the Aristotelian syllogistic is incomplete. The case made against classical logic with the Wason Selection Task is that classical logic is inaccurate.

The response which we see in Copi and Reichenbach's textbooks is to resist premise 1 in the argument above. I think this is unnecessary. The limits of simple logics like the Aristotelian syllogistic and classical logic are indeed apparent when applied to the task of modelling human reasoning. However, that does not make the modelling of human reasoning an illegitimate aim.

Modelling human reasoning is a fine aim for a formal system, nevertheless it makes for an unsatisfactory explanation of the subject matter of systems like the Aristotelian syllogistic or classical logic. A good explanation shouldn't undermine the logic which it aims to explain, and the claim that logic models reasoning leaves these systems open to objection based on their weakness for that purpose.

Switching to the other form of psychologism which Frege would recognise, Reichenbach, Copi, Mates, and Kalish & Montague all make statements that establish a normative relationship between logic and reasoning or argument. Reichenbach says:

> Logic has often been defined as the science that deals with the laws of thought. This is an ambiguous characterization unless we distinguish between psychological and logical laws of thought. ... If we want to say that logic deals with think-

99 See §8.1 for the details of this task.

> ing, we had better say that logic teaches us how thinking *should* proceed and not how it *does* proceed.[100]

Copi says:

> But the logician is not in the least concerned with the dark ways by which the mind arrives at its conclusions during the actual processes of reasoning. He is concerned only with the *correctness* of the completed process. His question is always: does the conclusion reached *follow* from the premisses used or assumed? If the conclusion does follow from the premisses, that is, if the premisses provide grounds or good evidence for the conclusion, so asserting the premisses to be true warrants asserting the conclusion to be true also, then the reasoning is correct. Otherwise it is incorrect. The distinction between correct and incorrect reasoning is the central problem with which logic deals.[101]

Mates says:

> Logic investigates the relation of *consequence* that holds between the premises and the conclusion of a sound argument. An argument is said to be *sound* (correct, valid) if its conclusion *follows from* or is a *consequence of* its premises; otherwise it is *unsound*.[102]

And Kalish and Montague say:

> Logic is concerned with arguments, good and bad. ...Virtue among arguments is known as validity. An argument is valid if it is impossible for its premises to be true and its conclusion false .[103]

These explanations are all normative – there is some sense in which logic tells us what we ought to do. Normative explanations are not inherently nonsensical, but they can lead to nonsense in the mind of the beginner, particularly when the normative notions are as closely associated with validity as they are in Mates and Kalish & Montague.

There are two good ways of looking at the normative situation. One is that you have some theory, about how truth works or about how proof works, from which you derive some norms by way of some hypothetical imperative – a rule which applies only in light of some desired outcome. You could say, as Priest does, that:

> We reason about all kinds of situations. We want to know what sorts of things hold in them, given we know other things; or what sorts of things don't hold given

100Reichenbach, *Elements of Symbolic Logic*, p. 1.
101Copi, *Introduction to Logic*, p. 6.
102Mates, *Elementary Logic*, p. 2.
103Donald Kalish and Richard Montague, *Logic: Techniques of Formal Reasoning* (New York: Harcourt, Brace & World, 1964), p. 3.

> that we know that other things don't. If we reason validly then, by the definition of validity, we can be assured that reasoning forward preserves the first property, and that reasoning backwards preserves the second. Validly is how one ought to reason if one wants to achieve these goals. The obligation is, then, hypothetical rather than categorical.[104]

In this case, norms are not the essential feature, and an explanation which centres them puts the reader's understanding of what is essential at risk.

The other good way of looking at the normative situation is as an attempt to construct models of the rules which govern behaviours like beliefs, assertions, and denials. As Steinberger says, 'we might advance the following interpretation: the normative connection between logic and thought consists in an agent's being committed to the logical consequences of her beliefs.'[105]

When we take the second way of looking at the normative situation and combine that with classical logic, we can produce arguments which show that classical logic is not adequate. It is this sort of idea which underlies Priest's earlier reasoning in which he said:

> ...the notion of validity that comes out of the orthodox account is a strangely perverse one according to which any rule whose conclusion is a logical truth is valid and, conversely, any rule whose premises contain a contradiction is valid. By a process that does not fall far short of indoctrination most logicians have now had their sensibilities dulled to these glaring anomalies. However, this is possible only because logicians have also forgotten that logic is a normative subject: it is supposed to provide an account of correct reasoning. When seen in this light the full force of these absurdities can be appreciated. Anyone who actually reasoned from an arbitrary premise to, e.g., the infinity of prime numbers, would not last long in an undergraduate mathematics course.[106]

This kind of argument parallels the psychologism debate. The case might be made:

1. Logic describes how we ought to reason.
2. We ought not reason in the manner described by classical logic.
3. *Therefore*, classical logic is not correct.

As with the psychological argument, we can resist the conclusion by rejecting the first premise. Here too, I think a rejecting premise 1 is unnecessary. Instead, I conclude that the normative explanation of the subject matter of logic is an inadequate explanation of the subject matter of systems like classical logic.

104 Graham Priest, 'Validity', in *The Nature of Logic*, ed. Achille C. Varzi, European review of philosophy (Stanford, Calif.: CSLI Publications, 1999), p. 202.

105 Florian Steinberger, 'Explosion and the Normativity of Logic', *Mind* 125, no. 498 (April 2016): p. 386.

106 Graham Priest, 'Two Dogmas of Quineanism', *The Philosophical Quarterly* 29, no. 117 (1979): p. 297.

The textbooks by Quine, Tarski, and Carnap use other strategies to introduce logic. Tarski opens with:

> Every scientific theory is a system of sentences which are accepted as true and which may be called laws or ASSERTED STATEMENTS or, for short, simply STATEMENTS. In mathematics, these statements follow one another in a definite order according to certain principles ...and they are, as a rule, accompanied by considerations intended to establish their validity. Considerations of this kind are referred to as PROOFS, and the statements established by them are called THEOREMS.[107]

Carnap says:

> This book presents a system of symbolic logic, together with illustrations of its use. Such a system is not a theory (i.e. a system of assertions about objects), but a language (i.e. a system of signs and of rules for their use). We will so construct this symbolic language that into it can be translated the sentences of any given theory about any objects whatever, provided only that some signs of the language have received determinate interpretations such that the signs serve to designate the basic concepts of the theory in question.[108]

And Quine says:

> logic studies the bearing of logical structure upon truth and falsehood. ...One statement *logically implies* another if from the truth of the one we can infer the other by virtue solely of the logical structure of the two statements.[109]

These are all quite good technical explanations. However, they are quite unhelpful unless you already know what's going on. Tarski and Carnap are using terms that they have developed special meaning for which is not communicated to the reader. Whereas Quine is effectively introducing a definition of validity which, to borrow Newton-Smith's phrase, just explains the obscure in terms of the equally obscure.[110]

The challenge of writing an introduction to an elementary logic textbook comes from the practical need to provide a sensible introduction to what the student will learn during the course which forces the teacher to say *something* about the subject matter of logic. That explanation of the subject matter of logic in the introduction should make sense of the logic introduced in the text. Ideally it should also help make sense of the discipline more broadly. Furthermore, it has to be accessible to the beginner. This is a complex challenge which requires careful consideration.

107 Tarski, *Introduction to Logic*, p. 3.

108 Rudolf. Carnap, *Introduction to Symbolic Logic and Its Applications*, trans. William H. Meyer and John Wilkinson (New York: Dover Publications, 1958), p. 1.

109 Willard Van Orman Quine, *Elementary Logic* (Boston: Ginn / Company, 1941), pp.1-2.

110 W. H. Newton-Smith, *Logic: An Introductory Course* (Taylor & Francis, 2003), p. 1.

Giving a decent account of the discipline requires a pluralistic approach simply to account for the variety of investigations which logicians undertake. Another kind of pluralistic virtue is worth considering. To return to the boat metaphor I began with, in a given context, one style of hull may be precisely the most perfect of all. Even assuming that is the case, a teacher still has a choice: instruct students in how to build just this style of hull, or educate students in the general principles of hull building exemplified by this style of hull. It is not necessary to go as far as introducing different styles of hull, especially not at an introductory level. It is also not necessary for students to fully grasp the principles of hull building. All they need to be better off is to grasp the concept that there *are* general principles.

If one took this kind of pluralistic approach, instruction in one logical system would be designed to teach students about the construction of logical systems generally and make room for the possibility of multiple logical systems. There is no one true boat, and there is no one true logic. These things depend very much on context and application. In some contexts, there is a uniquely best solution, but others could allow for multiple equally good solutions.

Rampant pluralism though, comes with the risk of overwhelming and confusing students. Simple concrete examples which can be worked through thoroughly are easiest for both teaching and learning. Pragmatically then, the logics introduced are simple, and their number limited. This places another constraint on the way the subject matter of logic is presented because some logics are too simple to represent some subject matters adequately. The teacher's choice of explanation is constrained because it doesn't make for a good learning experience to be given an explanation and then learn about some system which seems to fail to meet the objectives given in that explanation. The explanation offered has to accommodate the logic's simplicity.

The teacher's task is complicated by the dual need to explain the function of very simple logics while also giving insight into the many possible functions and pathways for development. None of the explanations of the subject matter of logic examined above are, strictly speaking, wrong. However, none of these examples achieve an ideal balance between telling students what the logics they are learning are good for, and what logics generally could be good for in simple, understandable terms.

A STUDY OF MODERN LOGIC TEXTBOOKS

> It can therefore be said, with all rigour, that it is the sea itself that fashions the boats... New boats are copied from those that come back ...progress is imperceptible; the craftsman is always copying, saying that nothing should be changed.
>
> Alain, Propos d'un Normand, September 1, 1908

3.1 CONCEPTS IN INTRODUCTORY EXPLANATIONS

Teachers are driven by the practical need to say something about what the student will learn during the course, and what the practical value of learning it is. As I talk to Gillian Russell, Johan van Benthem, and Dave Ripley in part II, they all show how much they care about teaching something valuable. The number of articles in the journal *Teaching Philosophy* on the value of logic in the humanities curriculum demonstrates how perennially relevant the question of value is. This kind of value however, is quite context dependent. For any given situation or programme, good teachers will ask themselves how their students will benefit from this instruction. The common motivation leaves its mark on the textbook.

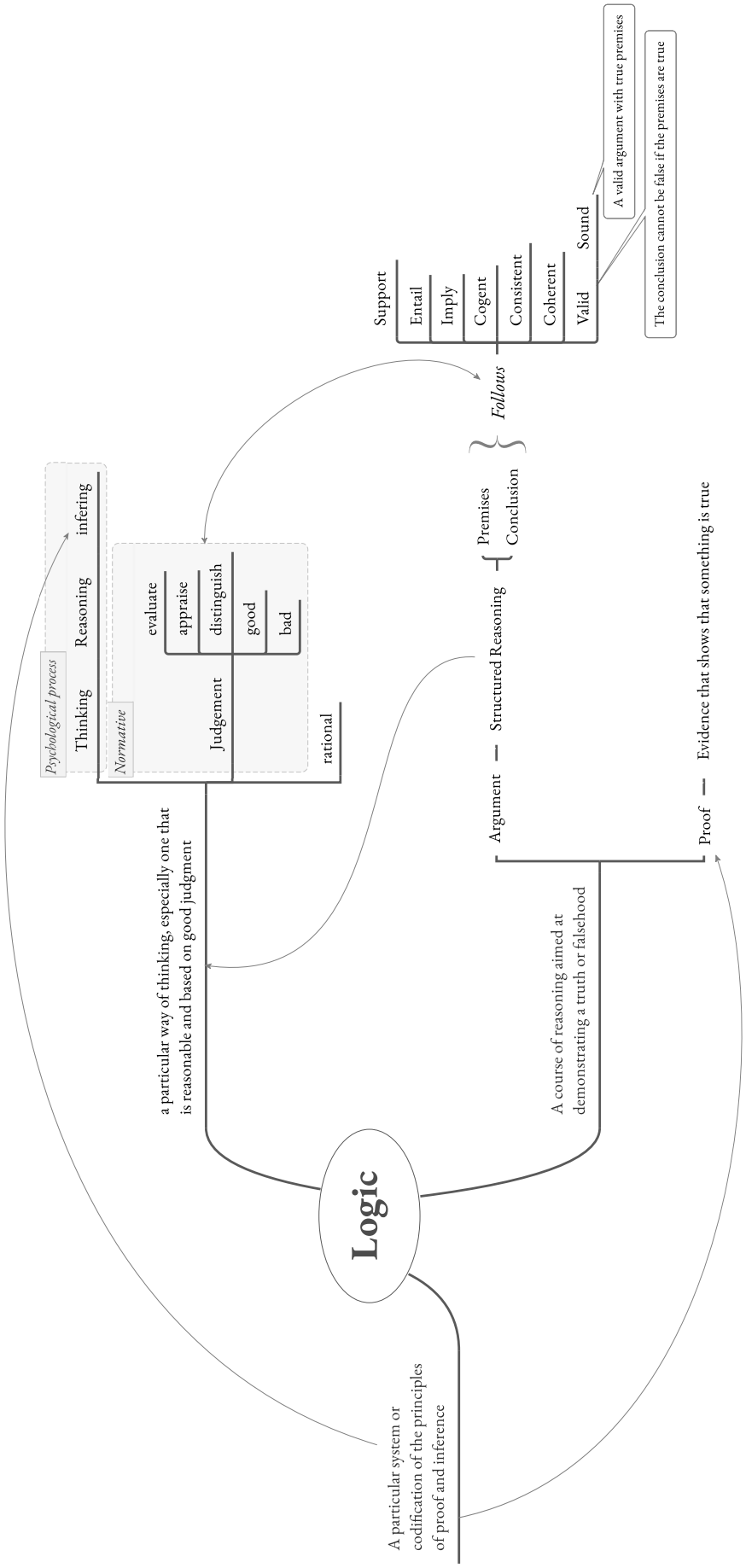
Prefixes and introductions are filled with attempts to respond to this very question. Responses to this question intertwine with the way teachers conceive the subject matter of logic, making introductions to logic texts fertile ground for an exploration of the ways in which the subject matter of logic can be conceived. I suspect that what each teacher learned from their teacher plays a significant role in how they conceive of and explain the subject matter of logic.

Tradition has an impact, but my aim is not to quantify the impact on that tradition. I point out the power of tradition mainly to make room for the idea that traditional ways of conceiving and expressing the subject matter of logic might not be optimal. In this chapter I explore what the options for explaining the subject matter of logic could be, and critically examine each of those options. This exploration is guided by the idea that explanations contain words which denote concepts – ideas which form in people's minds when they hear/read words – and that concepts combine to form understanding. Teachers combine concepts together to form an explanatory strategy meant to help students understand the subject matter of logic. Through

an understanding of the subject matter, students will know what they are meant to learn and the value of learning it. Many of the concepts used to explain the subject matter of logic are similar or inter-related. Figure 3.1 is a concept map which uses dictionary definitions of logic to show how these concepts and explanations relate.

The way I approach my analysis of the explanations of the subject matter of logic is superficial in the sense that it is not a critical examination of author's ideas about the subject matter of logic. My focus is on the words they have chosen and how those words link up with other words to form an impression. It is this sort of treatment which I used earlier to class Stebbing's explanation as psychological, even though a close reading of Stebbing's ideas about the subject matter of logic would reveal complexities which defy simple categorisation of this sort.

Figure 3.1: Logic: A Concept Map



The first task before me was compiling a list of introductory logic textbooks to examine, since examining every published textbook would not be feasible. As a way of limiting the possibilities, I decided to limit the search to textbooks used in tertiary institutions.

Still, the list of tertiary institutions is too long to examine them all. So as a means of cutting down the list to a manageable number, I turned to the list of top 200 philosophy departments form the QS World University Rankings by Subject 2018.[1] I do not consider the weaknesses of the QS ranking system relevant, as all I needed was a manageable number of universities. I give no weight to position on the list, and I don't infer anything about the quality of teaching from appearance or non-appearance on that list. While logic teaching can take place in a number of departments – as it sits at the intersection of philosophy, mathematics, and computer science – I limited my search to philosophy departments because I expected it would increase my chances of finding logic courses.

For each of the universities on that list, I searched for texts associated with introductory logic courses run within the last five years[2] either by the philosophy department, or by another department if accompanying material indicated that the course could be credited towards a philosophy degree.

I wasn't strict about the sources which indicated that a textbook had been used at one of these universities, accepting sources ranging from texts mentioned in syllabi, and on official university webpages, to credible amazon review comments. I reviewed each text mentioned and included only those designed as introductory logic textbooks. I confirmed that all the textbooks introduced classical logic (had any text only introduced the syllogistic and nothing more modern I would have excluded it), but beyond that made no further judgement on whether or not to include the textbook based on course content, choice of proof system, or the like. Finally, I excluded textbooks that had no English version.

The result was 60 universities where it was possible for me to identify at least one textbook. There are 38 unique textbooks.[3] There were 19 cases where more than one textbook was in use at a university, and some textbooks are used by more than one university, resulting in a combined total of 82 cases of textbook usage.[4]

This selection of textbooks is biased towards North American and European teaching. The bias starts with the QS top 200 list which over-represents European and North American universities. This compounds as I only include universities if I can identify textbooks via English pages on the internet. Table 3.1 is the regional break-down of the top 200 philosophy depart-

1 Starting at the University of Pittsburgh, which has occupied the top spot since the great Yale exodus of '63, and ending at... who cares which university was 200th?

2 I undertook this work in 2018, so the time period for text usage is between 2013 and 2018

3 The details of these 38 textbooks are given in appendix A.

4 Data on the textbooks and universities is available at https://doi.org/10.6084/m9.figshare.14361074.v1

ments form the QS World Rankings 2018, and table 3.2 shows the regional break-down of the 60 universities I included. Figures 3.2 and 3.3 give the same information more visually.

Table 3.1: Top 201 Universities by Region

Region	Universities	Proportion
Europe	80	40%
North America	66	33%
Asia	27	13%
South America	13	6.5%
Oceania	13	6.5%
West Asia	2	1%
Total	201	100%

Table 3.2: Included 60 Universities by Region

Region	Included	Proportion
North America	30	50%
Europe	18	30%
Oceania	7	12%
Asia	5	8%
Total	60	100%

Figure 3.2: Top 201 Universities by Region

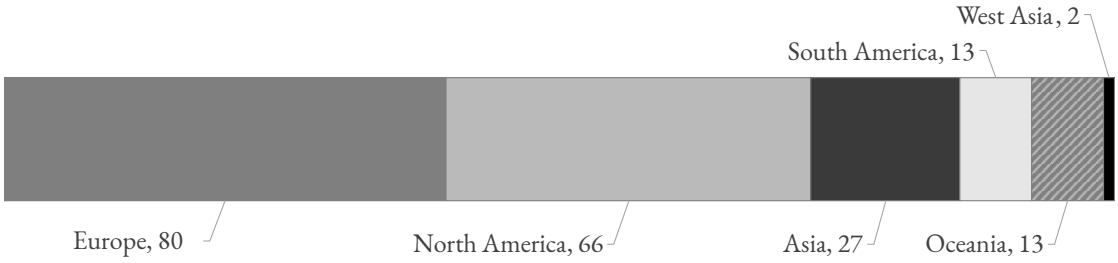
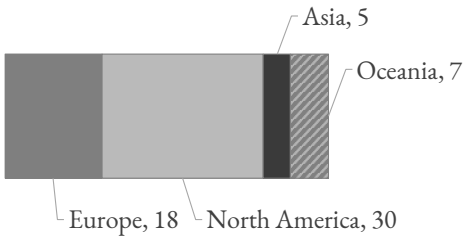


Figure 3.3: The 60 included Universities by Region

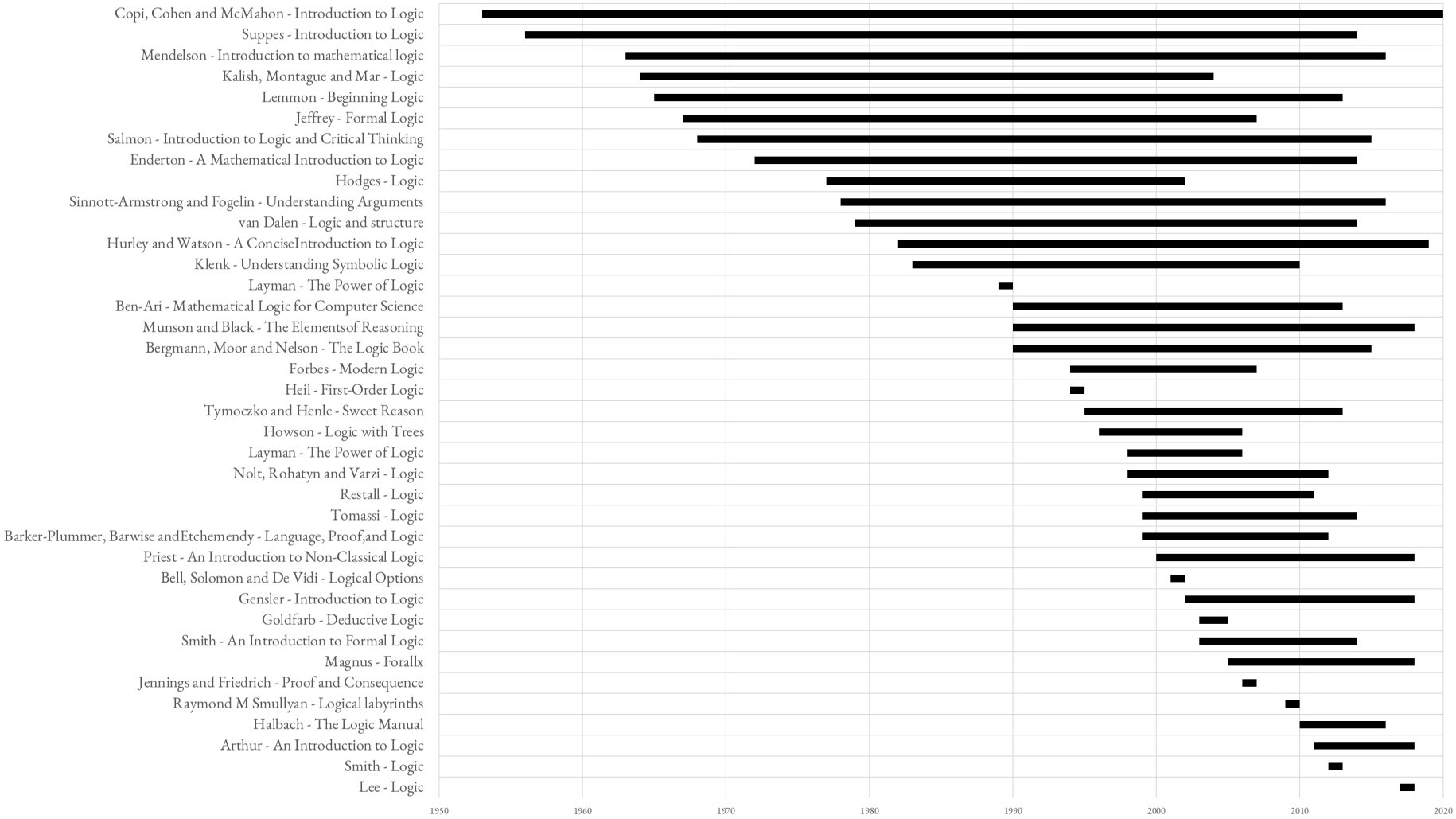


However, as table 3.3 shows, there is a reasonable spread across the QS University Rankings.

Table 3.3: Included Universities & Textbooks by Rank

University Rank	Universities	Text uses
1-50	20	29
51-100	17	21
101-150	9	12
151-200	14	20

Figure 3.4: A timeline of the production of these textbooks visually represented



Over half of the 38 included textbooks were first published in 1990 or later. Figure 3.4 is a timeline from the date the first edition was first published to the date the most recent edition was last published.

Most of the included textbooks were in use only once or twice – 19 of the 38 textbooks were used at only one university, and 28 textbooks are used at one or two universities. Two texts stand out at much more widely used: Barwise and Etchemendy's *Language, Proof and Logic* which was in use at 11 universities; and Hurley's *A Concise Introduction to Logic* which was in use at 7 universities.

Many authors use short memorable phrases to describe the subject matter of logic. Figure 3.5 presents a collection of such phrases from the 38 textbooks I studied.

Figure 3.5: Explanations of the subject matter of logic

Logic formalizes valid methods of reasoning
Symbolic logic is a mathematical model of deductive thought.
Logic provides a way of studying and classifying repeatable forms or patterns of reasoning
One of the popular definitions of logic is that it is the analysis of methods of reasoning
To study logic is to use the methods of rational inquiry on rationality itself
Logic is the science of good argument
Logic is the study of the methods and principles used to distinguish correct from incorrect reasoning.
Logic is the analysis and appraisal of arguments.
Logic is a useful tool to clarify and evaluate reasoning
Logic is the study of principles of reasoning.
Logic is concerned with what makes reasoning good and what makes arguments valid.
Logic is the study of reasoning and arguments
Logic may be defined as the organized body of knowledge, or science, that evaluates arguments.
Logic is concerned with arguments, good and bad.
The only thing logic is concerned with is whether arguments are good or bad, correct or incorrect.
Logic is a normative enterprise; its job is to evaluate arguments
Logic can be defined as the study of consistent sets of beliefs
Symbolic logic is usually described as the study of the difference between valid and invalid arguments
Logic's main concern is with the soundness and unsoundness of arguments
Logic is the study of arguments.
To study logic is to study argument
Logic is the study of methods for evaluating arguments.
Logic is the study of the methods and principles used to distinguish between good and bad reasoning.
Logic is the business of evaluating arguments, sorting good ones from bad ones.
Logic is the study of good reasoning, and in particular, what makes good reasoning good.
Logic is the field of study concerned with analyzing arguments and appraising their correctness or incorrectness
Logic is the science of truth.
Logic is the science of arguments
Logic is the theory of valid arguments or the theory of deductive inference
Formal logic is the science of deduction.
The point of logic is to give an account of the notion of validity: what follows from what.
The business of logic is the systematic evaluation of arguments for internal cogency.

To better understand the strategies used to explain the subject matter of logic in these 38 textbooks, I extracted a portion of text, from the first or second chapter, where the author explains the subject matter. I extracted whatever amount of each text I judged useful to understand the context of the author's explanation: sometimes the whole chapter, sometimes just a portion. How much text I extracted depended on how much of the text was devoted to discussing the subject matter of logic.

I used Nvivo 12 to search for occurrences of words and word forms and coded them as an occurrence of a concept in the extracted piece of text. In this way I was able to associate words with concepts regardless of the word form. For example, both 'good' and 'goodness' are coded as an occurrence of the concept 'Good'. I also coded close synonyms like 'good' and 'well' as an occurrence of the corresponding concept 'Good', while also being sensitive to contextual meaning (since not every use of the word 'well' means 'good'). I applied similar judgement in the case of 'reason', which I ignored when it was used in its justificatory sense (as in 'having a reason') as I was only interested in reasoning in the psychological sense.[5]

Table 3.4 shows the concepts, the number of text excerpts which contained that concept, and the number of times which that concept appeared in all the excerpts.

Table 3.4: Concepts Coded

Concept	**Texts**	**Instances**
Argument	32	992
Beliefs	18	106
Consistent	6	35
Normative	33	307
Appraise	2	3
Bad	16	30
Correct	19	56
Distinguish	13	21
Evaluate	15	36
Good	21	143
Incorrect	6	9
Ought	7	9
Psychological	36	409
Inference	18	116
Reasoning	31	252
Thinking	17	41
Truth	37	811
False	31	214
True	36	597
Validity	34	402
Consequence	5	25
Entail	4	7
follow from	20	80
Imply	12	24
Valid	29	266

5 The NVivo 12 project file counting the coding of these text excerpts is available at https://doi.org/10.6084/m9.figshare.14361083.v1

Table 3.5 shows the proportional concepts in each text.[6]

Table 3.5: Concepts as a proportion of all concepts in each text

Textbook	Argument	Beliefs	Consistent	Normative	Psychological	Truth	Validity
Arthur, *An Introduction to Logic* logic	0.27	0.00	0.00	0.05	0.32	0.29	0.07
Barker-Plummer, Barwise and Etchemendy, *Language, Proof, and Logic*	0.02	0.04	0.00	0.20	0.20	0.09	0.46
Bell, Solomon and De Vidi, *Logical Options*	0.32	0.03	0.03	0.09	0.10	0.19	0.24
Ben-Ari, *Mathematical Logic for Computer Science*	0.00	0.00	0.09	0.18	0.13	0.55	0.05
Bergmann, Moor and Nelson, *The Logic Book*	0.05	0.00	0.00	0.10	0.35	0.35	0.15
Copi, Cohen and McMahon, *Introduction to Logic*	0.42	0.03	0.00	0.10	0.18	0.22	0.04
Dalen, *Logic and Structure*	0.00	0.00	0.17	0.00	0.17	0.50	0.17
Enderton, *A Mathematical Introduction to Logic*	0.00	0.00	0.00	0.19	0.38	0.13	0.31
Forbes, *Modern Logic*	0.32	0.00	0.00	0.01	0.01	0.43	0.22
Gensler, *Introduction to Logic*	0.31	0.01	0.00	0.06	0.11	0.25	0.26
Goldfarb, *Deductive Logic*	0.24	0.00	0.00	0.09	0.26	0.33	0.09
Halbach, *The Logic Manual*	0.25	0.00	0.14	0.05	0.20	0.11	0.25
Heil, *First-Order Logic*	0.26	0.02	0.00	0.05	0.36	0.07	0.24
Hodges, *Logic*	0.02	0.44	0.28	0.06	0.02	0.16	0.02
Howson, *Logic with Trees*	0.00	0.01	0.00	0.00	0.15	0.65	0.19
Hurley and Watson, *A Concise Introduction to Logic*	0.48	0.01	0.00	0.13	0.11	0.19	0.08
Jeffrey, *Formal Logic*	0.21	0.00	0.00	0.00	0.00	0.47	0.32
Jennings and Friedrich, *Proof and Consequence*	0.48	0.00	0.00	0.37	0.13	0.02	0.00
Kalish, Montague and Mar, *Logic*	0.17	0.02	0.00	0.23	0.09	0.40	0.09
Klenk, *Understanding Symbolic Logic*	0.33	0.00	0.00	0.23	0.14	0.22	0.08
Layman, *The Power of Logic*	0.36	0.04	0.00	0.15	0.13	0.32	0.01
Lee, *Logic*	0.38	0.02	0.00	0.11	0.12	0.29	0.09
Lemmon, *Beginning Logic*	0.37	0.00	0.00	0.04	0.01	0.39	0.18
Magnus, *Forall* ***x***	0.38	0.02	0.01	0.04	0.01	0.42	0.11
Mendelson, *Introduction to Mathematical Logic*	0.19	0.00	0.00	0.00	0.15	0.54	0.12
Munson and Black, *The Elements of Reasoning*	0.57	0.00	0.00	0.10	0.22	0.02	0.09
Nolt, Rohatyn and Varzi, *Logic*	0.58	0.00	0.00	0.00	0.00	0.42	0.00
Priest, *An Introduction to Non-Classical Logic*	0.00	0.00	0.00	0.07	0.18	0.14	0.61
Restall, *Logic*	0.38	0.07	0.00	0.06	0.13	0.16	0.20
Salmon, *Introduction to Logic and Critical Thinking*	0.50	0.03	0.00	0.08	0.14	0.22	0.03

6 This data was extracted from the NVivo coding, but is available separately at https://doi.org/10.6084/m9.figshare.14361107.v2

Table 3.5: Concepts as a proportion of all concepts in each text

Textbook	Argument	Beliefs	Consistent	Normative	Psychological	Truth	Validity
Sinnott-Armstrong and Fogelin, *Understanding Arguments*	0.41	0.22	0.00	0.14	0.19	0.03	0.01
Smith, *An Introduction to Formal Logic*	0.22	0.02	0.00	0.11	0.25	0.29	0.11
Smith, *Logic*	0.02	0.00	0.00	0.10	0.33	0.56	0.00
Smullyan, *Logical Labyrinths*	0.00	0.04	0.00	0.13	0.09	0.74	0.00
Suppes, *Introduction to Logic*	0.11	0.00	0.00	0.14	0.62	0.00	0.14
Teller, *A Modern Formal Logic Primer*	0.46	0.07	0.00	0.13	0.10	0.16	0.09
Tomassi, *Logic*	0.37	0.00	0.00	0.07	0.01	0.34	0.20
Tymoczko and Henle, *Sweet Reason*	0.46	0.00	0.00	0.12	0.07	0.22	0.13

I gave each textbook a broad category based on the explanation of the subject matter of logic given in the textbook. The proportion of concepts informed, but did not determine, my categorisation. My categorisation was based on my judgement of where the conceptual emphasis lands. The table of included textbooks in appendix A indicates the category I assigned each text to. Of the 38 included textbooks, 33 contained statements which capture the essence of what the author says about logic. These are presented in appendix B.

While there is diversity in the strategies which authors use to explain the subject matter of logic, it is also clear that there is a dominant trend. Table 3.6 and Figure 3.6 illustrate the prevalence of each category of explanatory strategy in teaching, by showing how many times textbooks with that strategy are used.[7,8]

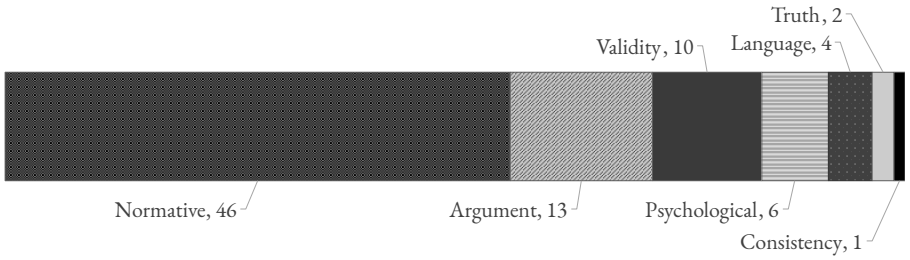
Table 3.6: Textbook usage by category

Explanatory Strategy	Uses
Normative	46
Argument	13
Validity	10
Psychological	6
Language	4
Truth	2
Consistency	1
Total	82

7 It is *not* the count of included texts by category – included texts that are used by more than one university are counted multiple times and multiple textbooks in use at the same university are each counted.

8 P.D Magnus' Forall*x* presents an unusual case because the text is available under a Creative Commons license. Magnus provides the book for free for use either as a text for a course or for self-directed study, and invites teachers to revise it to fit their needs. I have counted each instance of the use of this book, but I have only used Magnus' original explanation of the subject matter of logic in my analysis.

Figure 3.6: Textbook usage by category



When authors explain the subject matter of logic they are responding to the practical need to say something about what the student will learn and the value of learning it. They want to convey an idea about what to expect from the discipline as a whole, as well as to make sense of the instruction in the course. This is a mighty challenge for a few words.

The objective for my analysis is to explore the subject matter of logic: to discover what logic is about. I am driven to introductory texts rather than debates in the literature simply because that is where these points are discussed. However, from Hanson I have a warning about what I might find in conducting this examination. He says:

> If one wants to know whether there is unanimity or disagreement among the experts concerning the basic tenets of a particular subject, one way to find out is to look at what they say when introducing their subject to the uninitiated. Applying this approach to logic, one is tempted to conclude that the foundations of the subject must be in disarray. An examination of respected texts written by established practitioners reveals considerable disagreement about the pre-theoretic notion of logical consequence. Furthermore, these texts usually do not mention this disagreement. It is as though their authors either haven't noticed it or don't recognize its importance.[9]

While I have shown that there is a dominant trend, there is, as Hanson anticipates, considerable disagreement. He is also correct in claiming that the disagreement goes unmentioned. I found mention of the disagreement in Mates's textbook, but none of the contemporary texts which I examined mentioned it.

In the contemporary textbooks I examined, when the subject matter of logic is explained it is not presented as a mere formalism. The explanation of the subject matter extends beyond formalism, giving the formal system's context and application. The expression of subject matter explains what a formal system and definition of validity is for. That explanation becomes a benchmark against which those systems could be judged. Consequently, an expression of the

9 William H. Hanson, 'The Concept of Logical Consequence', *The Philosophical Review* 106, no. 3 (July 1997): p. 366.

subject matter of logic is an expression of a notion of inter-theoretic validity.

Much of the advanced literature assumes that there is a common notion of validity outside of the formal systems in which validity is defined – sensible given that instruction generally leads them to believe that this is the case. Debates are had regarding whether this or that formal definition matches up with validity proper, and disputants plumb their intuitions regarding whether some formal definition of validity accords with their intuitive understanding of 'genuine' validity. Experts argue about validity as if they are arguing about the same thing, not acknowledging that the fundamental notions which underpin what they mean by validity might be quite different. While I cannot prove it, I suspect that the words which teachers use to set the scene for logic instruction have a powerful shaping effect. I suspect that much modern dialogue is based on assumptions set up during early instruction. For these sorts of discussions where those involved may be talking past each other without knowing it, introducing the idea of inter-theoretic validity could be quite helpful.

Inter-theoretic validity is the notion of validity which stands apart from any given formal system. It is the explanation which gives the context and application for the formal systems in question, it explains what those systems and definitions are supposed to do. Or, using a logic-as-modelling view, it is what logical systems are supposed to model. If we are clear that we share a common notion of inter-theoretic validity, then we can be confident that we are talking about the same thing when we argue against competing logical theories. Where the notion of inter-theoretic validity differs, there can be no genuine rivalry between logical theories because the logical theories in question aren't talking about the same thing.

Notions of inter-theoretic validity also offer a way to understand how logical theories might be tested. A notion of inter-theoretic validity establishes a domain of investigation, clarifies the phenomena under investigation, and indicates what data would confirm or disconfirm rival theories. It lays a foundation for rigorous investigation. Notions of inter-theoretic validity also define the boundaries for legitimate theories and provide a way to quickly rule out some logics as non-contenders. If we want to know whether the trivial logic is legitimate or not we must look to the the notion of inter-theoretic validity being used.

In the introduction I explained that I assume from the beginning that it is possible for there to be multiple, equally legitimate, inter-theoretic validities. Despite the clear dominance of the normative inter-theoretic validity in introductory textbooks, this review demonstrates plurality in the strategies used to explain the subject matter of logic, opening the door for multiple notions of inter-theoretic validity.

Below I discuss the strategies used to explain the subject matter of logic, the words that are used to express different notions of inter-theoretic validity, and the consequences which follow from these different notions.

3.2 GOOD, BAD, CORRECT, AND INCORRECT

Since it is most dominant, and most strongly supported by recent tradition, I'll begin with the normative inter-theoretic validity. This is the notion present in the old textbooks by Reichenbach, Copi, Kalish & Montague, and Mates, and it is also the strategy used in the two most frequently used contemporary textbooks: Barwise and Etchemendy's *Language, Proof and Logic*,[10] and Hurley's *A Concise Introduction to Logic*.[11]

The textbooks which express normative notions vary in how evident that normativity is. Klenk and Lee make explicit claims in which the word 'normative' appears. Klenk says: 'Logic is a *normative* enterprise; its job is to *evaluate* arguments.'[12] Similarly, Lee says: '**Logic** is the study of the methods and principles used to distinguish between good and bad reasoning. It is a **normative** discipline, in the sense that it does not survey and describe *how* we actually reason (which is the job of the psychologist) but what we *should* do in reasoning.[13]' Other approaches, like Gensler's,[14] are less obvious. Here the approach often focuses on argument, but as Klenk says, argument is 'not enough for a definition of logic... it does not tell us what we are supposed to do with arguments. Just write them down? Count the number of words? Admire the calligraphy or typesetting?'[15] In many cases discussion of arguments leads to a straightforwardly normative interpretation like Hurley's that '**Logic** may be defined as the organized body of knowledge, or science, that evaluates arguments,'[16] so in cases where that interpretation was evident in the text I simply counted those too as cases of a normative approach. In between Gensler and Kenk's are explanations like Hurley's where the normativity is not present in an explicit claim nor buried in an interpretation, but presents itself through words like 'correct', 'good', and 'evaluate' being used to explain the role of logic. (See appendix B.3)

Textbooks which employ the normative strategy tend not to discuss beliefs, and vary in how much they talk about truth and validity. For the most part there's a split between texts which relate logic to reasoning and those which relate logic to argument. The overall impression that I'm left with is that the author's preference of 'argument' over 'reasoning' is more likely to be driven by their views on pedagogy than philosophic views. They're treated almost as synonyms,

10 Dave Barker-Plummer, Jon Barwise and John Etchemendy, *Language, Proof, and Logic*, 2nd ed. (Stanford, Calif.: CSLI Publications, 2011), ISBN: 9781575866321.

11 Patrick J. Hurley and Lori Watson, *A Concise Introduction to Logic*, 13th ed. (Boston, MA: Cengage Learning, 2017), ISBN: 9781305958098.

12 Virginia Klenk, *Understanding Symbolic Logic*, 3rd ed. (Upper Saddle River, N.J: Prentice Hall, 1994), p. 5, ISBN: 9780536632524.

13 Siu-Fan Lee, *Logic: A Complete Introduction* (Teach Yourself, 2017), p. 2, ISBN: 9781473608436.

14 Harry J. Gensler, *Introduction to Logic*, 3rd ed. (ProQuest Ebook Central, New York: Taylor & Francis, 2017), p. 1, ISBN: 9781317436119.

15 Klenk, *Understanding Symbolic Logic*, p. 4.

16 Hurley and Watson, *A Concise Introduction to Logic*, p. 1.

with some texts explaining their close relationship. For instance Klenk says, 'logic is concerned with the *verbal expression* of reasoning... The term that we will use for this verbal expression of reasoning is *argument*.'[17] Lee says, 'Reasoning can be formulated in the form of an argument, a structure comprised of exactly one conclusion and reasons to support it called premises.'[18] And Hurley says, 'An **inference**, in the narrow sense of the term, is the reasoning process expressed by an argument. In the broad sense of the term, "inference" is used interchangeably with "argument".'[19]

Usually what the normative strategy aims to convey is the idea that logic is more prescriptive than descriptive. Consider the example: in society there are general rules for reasoning. How do we think of logic in relation to those rules? Does logic describe what those rules are or does it prescribe those rules? If logic describes those rules, then the rules as they are in society form the data against which a logical theory is tested. If logic prescribes the rules, then it can never be wrong about what they are – it is their source.

There are two kinds of prescriptions which logic could give: hypothetical and categorical. Hypothetical prescriptions exist on the basis of some objective or aim, for example, if you want to achieve *x*, then do *y*. Categorical prescriptions on the other hand simply say what is required, permitted, or not permitted.

Hypothetical imperatives operate unproblematically. They say something like, if you want to achieve the reasoning goals that using this logical system would allow you to achieve, then you should use this logical system. There's nothing theoretically challenging to a hypothetical imperative. However, a hypothetical imperative does not explain the subject matter of logic. A hypothetical imperative tells you when to (or why you might) apply the subject matter, but it doesn't tell you what the subject matter is. Hypothetical imperatives offer, to use G. Russell's terminology, the weakest degree of normative entanglement.[20]

A weak degree of normative entanglement has no explanatory advantage. An analogy to mathematics helps show this lack. Russell's example of normative mathematical reasoning is: '67 + 58 = 125 perhaps means that you ought not to believe that you have 67 pence in one pocket, 58 pence in the other, and also believe that you have less than 125 pence in your pockets.'[21] With a weak degree of normative entanglement, the norm isn't taken as explaining anything about the meaning of '+' or '='. Strong normativity does the opposite; the norm becomes central to what '+' and '=' *mean*. The categorical imperative explains something about what it means to say '67 + 58 = 125'.

17 Klenk, *Understanding Symbolic Logic*, p. 4.
18 Lee, *Logic*, p. 25.
19 Hurley and Watson, *A Concise Introduction to Logic*, p. 5.
20 Russell, 'Logic Isn't Normative', p. 380.
21 Russell, p. 380.

It could be, as G. Russell says, that 'logic's apparent normative consequences are the result of widespread background norms about the relations between belief, reasoning, and truth, not logic's own normativity,'[22] but this is not a good interpretation in the context of receiving a message about the subject matter of logic. When the message is: 'Throughout almost the whole of its history, the primary interest of logicians has lain in the articulation of what could be called Canons of Correct Reasoning: the formulation of rules by which we could assess the arguments of others, and be guided in our own inferences.'[23] Normative notions are clearly central to the explanation of the subject matter of logic. Why would the speaker have placed so much emphasis on these notions if they were not central? The best interpretation here is that logic lays down the prescriptions for what is required, permitted, and not permitted in reasoning, inferring, and arguing.

Normativity is a notion of inter-theoretic validity in which normative notions are central to the meaning of validity. For Normativity, a distinction between a prescriptive and descriptive approach is troublesome because logic as subject matter is prescriptive while logic as a formal system is descriptive, giving logic both a prescriptive and a descriptive character which can be tricky to tease apart. Advocates for Normativity like Field point out the valuable role of Normativity in making sense of disagreements about logic. Field says:

> I don't want to make a big deal about definition or meaning: the point I'm making can be made in another way. It's that advocates of different logics presumably disagree about something – and something more than just how to use the term 'valid', if their disagreement is more than verbal. It would be nice to know what it is they disagree about. And they don't disagree about what's classically valid (as defined either model-theoretically or proof-theoretically); nor about what's intuitionistically valid, or LP-valid, or whatever. So what do they disagree about?[24]

Which G. Russell counters with the argument that there are other ways to make sense of disagreements about logic.[25] According to my theory, both options – taking normative notions as central to the meaning of validity, or not – are equally legitimate.

Taking normativity seriously is legitimate, but there are consequences.

The first consequence has to do with classical validity. Normativity as a notion of inter-theoretic validity explains what a definition of validity is supposed to do, it is a framework against which any subsequent definition may be tested. According to Normativity, good reas-

22 Russell, 'Logic Isn't Normative', p. 387.

23 R. E. Jennings and N. A. Friedrich, *Proof and Consequence: An Introduction to Classical Logic* (Peterborough, Ont.: Broadview Press, 2006), p. 4, ISBN: 9781551115474.

24 Hartry Field, 'What Is Logical Validity?', in *Foundations of Logical Consequence*, ed. Colin R. Caret and Ole Thomassen Hjortland (New York: Oxford University Press, 2015), p. 34.

25 Russell, 'Logic Isn't Normative', p. 386.

oning is central to what validity *means*; validity must tell us whether reasoning is good or not, that is its central function. If we accept that some argument form represents good reasoning, then it must be *valid*, conversely is some argument form does not represent good reasoning it cannot be *valid*.

Classical validity defines validity in terms of truth preservation. In a classically valid argument the premises cannot be true and the conclusion false. If we take normativity seriously then classical validity is inadequate if there are patterns of good reasoning which it fails to capture, and it is inaccurate if there are patterns of reasoning which it allows which do not represent good reasoning.

The first consequence of taking normativity seriously is that classical validity is both inadequate and inaccurate. The counterexample to classical validity's adequacy comes from accepting that inductive reasoning is good. One might deny that inductive reasoning is good, but inductive reasoning is so useful that this seems imprudent. Still, even if one insists that classical validity is adequate, there is the matter of its accuracy.

There are at least three counterexamples to the accuracy of classical logic, three forms of classical valid reasoning which are not good. Firstly, according to classical validity any argument where the conclusion is a logical truth is valid, but it doesn't seem right to say that reasoning from an irrelevant premise to a tautology is good reasoning. Secondly, any argument with contradictory premises is valid but similarly, doesn't seem right to say that reasoning from a contradiction to some arbitrary conclusion represents good reasoning. Thirdly, according to classical validity any statement implies itself, but this is begging the question and not an example of good reasoning. All three of these examples will be familiar territory to anyone who has taken an introductory logic course.

There are two other consequences to taking normativity seriously. Both are raised by Heathcote in the collection *Hume on Is and Ought* edited by Pigden. The first of these is that Hume's Law is false. The second is a rejection of inductive scepticism.[26]

What is Hume's Law? In *A Treatise of Human Nature* Hume says:

> In every system of morality, which I have hitherto met with, I have always remarked, that the author proceeds for some time in the ordinary way of reasoning, and establishes the being of a God, or makes observations concerning human affairs; when of a sudden I am surprised to find, that instead of the usual copulations of propositions, is, and is not, I meet with no proposition that is not connected with an ought, or an ought not. This change is imperceptible; but is, however, of the last consequence. For as this ought, or ought not, expresses some new relation

26 Rejecting inductive scepticism is essentially accepting that inductive reasoning is *valid* – a point already implied.

> or affirmation, 'tis necessary that it should be observed and explained; and at the same time that a reason should be given, for what seems altogether inconceivable, how this new relation can be a deduction from others, which are entirely different from it. But as authors do not commonly use this precaution, I shall presume to recommend it to the readers.[27]

Pigden explains how to this paragraph (referred to as No-Ought-From-Is) may be interpreted: 'At a minimum, the claim appears to be that it *seems* altogether inconceivable that propositions connected by an *ought* or an *ought not* can be deduced from premises involving the usual copulations of propositions, *is*, and *is not*, and at a maximum, that such deductions not only *seem* but actually *are* inconceivable.'[28]

In 'Hume's Master argument' Heathcote[29] argues that Hume uses similarly structured arguments to argue for both inductive scepticism and No-Ought-From-Is. Heathcote argues that Hume's argument for inductive scepticism fails and because it does, and the argument for No-Ought-From-Is significantly similar, we should expect a similar failure in the argument for No-Ought-From-Is. Many of the other contributors to *Hume on Is and Ought* criticise Heathcote's arguments, but his claim that there is a significant similarity between Hume's argument for inductive scepticism and Hume's argument for No-Ought-From-Is goes unchallenged, as does the assumption that if one of the arguments can be shown to be flawed we should expect the other to be flawed too.

An essential part of Heathcote's counter to the argument for inductive scepticism relies on the claim that inductive arguments can be *valid*. His argument relies on an intrinsic relationship between the meaning of validity and how you ought to, or at least may, reason – something which follows naturally from the normative notion of validity. The argument is that if you ought to, or at least may, reason inductively, inductive arguments must be *valid*.

Heathcote uses this same idea about validity to argue that Hume's Law is false – validity bridges the gap between ought and is. Pigden agrees that if Heathcote is granted his definition of validity, then Hume's Law is false. He says, 'If the bridging principle is analytic in something like the old-fashioned sense (that is true in virtue of the meanings of words) we may still have an inference to a substantive epistemic "ought" from substantively ought-free premises'[30] That is, if the normativity of logic is established by the definition of validity, it will generate instances of

27 David Hume, *A Treatise of Human Nature*, A New Edition, vol. 2 (London: Printed for Thomas & Joseph Allman, 1817), p. 172; book III, part I, section I, last para.

28 Charles R. Pigden, 'Introduction', in *Hume on Is and Ought*, ed. Charles R. Pigden (New York: Palgrave Macmillan, 2010), p. 6.

29 Adrian Heathcote, 'Hume's Master Argument', in *Hume on Is and Ought*, ed. Charles R. Pigden (New York: Palgrave Macmillan, 2010), 92–117.

30 Charles R. Pigden, 'Comments on 'Hume's Master Argument'', in *Hume on Is and Ought*, ed. Charles R. Pigden (New York: Palgrave Macmillan, 2010), p. 140.

inferences where the premises contain only statements of fact, but the conclusion is a statement about one's obligations.

On the other hand, Pigden argues that Hume derives No-Ought-From-Is from his understanding of what it means for an inference to be valid.[31] The difference of opinion comes from different positions on what it means for an argument to be valid. Heathcote too seems aware of this point. In his conclusion he argues:

> We cannot, and should not, trim our intuitions of validity, in Procrustean fashion, to suit our formal systems. Rather we must build formal systems to capture as much as we can of our intuitions of validity. Thus, the moral of this paper – in both the induction case and in the is-ought case – and the lesson to be learned from Hume's 'sceptical worries', is that Reason must expand until it fits the normative profile.[32]

What this debate between Heathcote and his critics shows is that Normativity and classical validity are not compatible as ideas about what it means for an argument to be valid.

While I do not argue the case for Normativity, my fundamental presumption is that it is a perfectly good inter-theoretic validity. It fulfils its role in telling us something about how well classical validity performs as a definition of validity. The verdict on classical validity that Normativity delivers seems clear – classical validity is not good.

Of course, this causes trouble for teaching classical logic when it is presented in the context of a normative explanation which is, as I demonstrated above, the dominant strategy.

Normativity causes trouble for teaching classical logic because when we use the normative notion of validity to test classical validity, classical validity comes up short. Classical logic fails to capture inductive reasoning, and it deems valid instances of obviously bad reasoning. Combining a normative explanation of the subject matter of logic with classical logic instruction is unforgivably contradictory. Accepting normativity leads naturally to the conclusion that the classical logic being introduced is inadequate or in need of revision – demanding an answer to why it is introduced at all.

Learning classical logic can be valuable. That's at least part of what the explanation of the subject matter of logic was supposed to provide: an explanation of the value of classical logic – something which this explanation of the subject matter of logic undermines. As I have said, normativity is a perfectly good theory. However, it is a bad explanation of classical logic. Just as the explanation of the subject matter of logic must be sensitive to the diversity in the whole

31 Charles R. Pigden, 'Snare's Puzzle/Hume's Purpose: Non-cognitivism and What Hume Was Really up to with No-ought-from-is', in *Hume on Is and Ought*, ed. Charles R. Pigden (New York: Palgrave Macmillan, 2010).

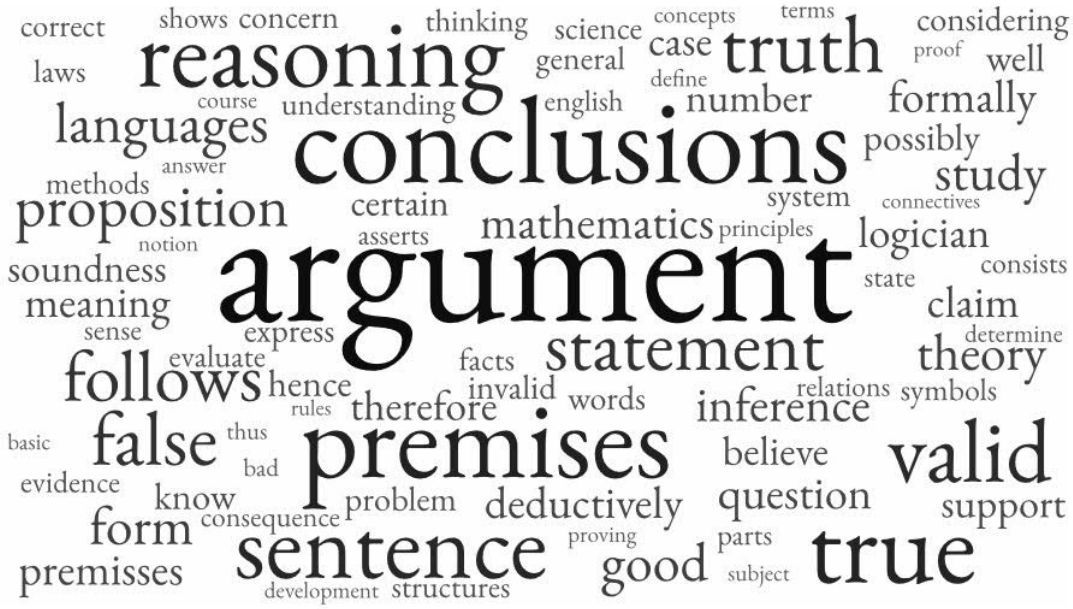
32 Heathcote, 'Hume's Master Argument', p. 115.

field of logic,[33] it must be sensitive to the logic that will be introduced.

3.3 ARGUMENT, REASONING, AND BELIEF

When we look at the words used to introduce logic one word looms large (as 3.7 illustrates). Of the 38 textbooks which I reviewed, seven emphasised 'argument' without using a clearly normative explanatory strategy.

Figure 3.7: Words frequently used in introductory textbooks



Tomassi introduces normative notions, saying, 'The central problem which worries the logician is just this: how, in general, can we tell good arguments from bad arguments?'[34] but explicitly argues against normativity. Tomassi ends up talking about *validity*. His discussion of validity goes through good argument, but he also discusses how validity and good argument come apart. He shows the value of validity in its ability to help with the task of distinguishing good arguments from bad ones but ensures that the reader does not come away thinking that the two are tightly related. Similarly, Forbes[35] and Lemmon[36] emphasise argument, but actually aim to characterise logic in terms of validity.[37] (See appendix B.1)

33 Since it does not do justice to one's fellow logicians to give an explanation of what logic is about which suggests that their area of study produces a logic which is inadequate or in need of revision.

34 Paul Tomassi, *Logic*, 1st ed. (ProQuest Ebook Central, Ann Arbor, MI: Routledge, Taylor & Francis Group, 2002), p. 2, ISBN: 9780203197035.

35 Graeme Forbes, *Modern Logic: A Text in Elementary Symbolic Logic* (New York: Oxford University Press, 1994), ISBN: 0195080297.

36 E. J. Lemmon, *Beginning Logic*, 2nd ed. (Boca Raton: Chapman & Hall/CRC, 1998), ISBN: 04123809000.

37 Lemmon says 'soundness' and 'unsoundness' but it's clear that he means validity. The use of 'sound' to mean 'valid' is also present in Mates' *Elementary Logic*.

While I believe that some of the textbooks which emphasise 'argument' actually explain logic in terms of validity, a minor difference in the presentation of these explanations creates an opportunity for the reader to come away with a very different interpretation. The difference in presentation is the difference between saying, 'arguments are valid or invalid,'[38] and saying '*valid* and *invalid* arguments.'[39] Dividing the two presentations is splitting hairs, but it does create an opportunity to learn a valuable lesson about communication.

Gopen & Swan discuss the importance of sentence endings. They say, 'It is a linguistic commonplace that readers naturally emphasize the material that arrives at the end of a sentence. We refer to that location as the "stress position".'[40] They warn:

> When the writer puts the emphatic material of a sentence in any place other than the stress position one of two things can happen; both are bad. First the reader might find the stress position occupied with material that clearly is not worthy of emphasis. In this case the reader must discern, without any structural clue, what else in the sentence may be the most likely candidate for emphasis. ... The second possibility is even worse: The reader may find the stress position occupied by something that does appear capable of receiving emphasis, even though the writer did not intend to give it any stress. In that case the reader is highly likely to emphasize this imposter material, and the writer will have lost an important opportunity to influence the reader's interpretive process.[41]

It is clear that Forbes intended to emphasise validity – he italicised it. Despite Forbes' intent, the sentence structure creates an opportunity to arrive at a different understanding. In this context 'argument' is clearly an impostor, but it is a troublesome impostor because it is a very reasonable option for receiving conceptual emphasis.

This misplaced emphasis also features in textbooks which emphasise a psychological concept like 'reasoning' in the explanation of logic. Enderton says, 'Symbolic logic is a mathematical model of deductive thought,'[42] but what he goes on to talk about is modelling and validity. Mendelson starts with, 'One of the popular definitions of logic is that it is the analysis of methods of reasoning,' but then goes on to say, 'The systematic formalization and cataloguing of

38 Richard Jeffrey, *Formal Logic: Its Scope and Limits*, 4th ed., ed. John P. Burgess (ProQuest Ebook Central, Indianapolis: Hackett Publishing Company, Inc, 2006), p. 1, ISBN: 9781624666063.

39 Forbes, *Modern Logic*, p. 3.

40 George D. Gopen and Judith A. Swan, 'The Science of Scientific Writing', *American Scientist* 78, no. 6 (1990): p. 552.

41 Gopen and Swan, p. 552.

42 Herbert B. Enderton, *A Mathematical Introduction to Logic*, 2nd ed. (San Diego: Harcourt, 2007), p. xi, ISBN: 0122384520.

valid methods of reasoning are a main task of logicians,'[43] suggesting that, again, the emphasis should be on validity. Ben-Ari says, 'Logic formalizes valid methods of reasoning,'[44] but spends more time talking about truth and proof than he does about reasoning. Finally, both exhibiting misplaced emphasis, and muddling 'argument' with 'reasoning'. Heil says: 'Arguments exhibit repeatable *patterns*. Some of these patterns represent valid reasoning – their premises imply their conclusions – and some do not. ... Logic provides a way of studying and classifying repeatable forms or patterns of reasoning. ...formal logic provides a powerful technique for assessing the validity and invalidity of arguments.'[45] Heil's sentences about logic end with 'reasoning' or 'argument', but what he is really trying to communicate is the concept of *form* through the discussion of patterns. (See appendix B.4)

Psychologism has fallen out of fashion, but formal methods can be used to model many things. One of those things could be psychological processes.[46] The same can be said of argument. The *Handbook of the Logic of Argument and Inference*[47] is an example of the sort of work which arises when 'argument' and 'inference' are taken as a serious candidates for the subject matter of logic. In the preface, the editors explain the three essential things which the handbook should do. Notably, one of those is to 'reflect the fact that there is much common agreement that since classical logic was not designed as a logic of practical reasoning, it is not an adequate general theory of it.'[48]

Framing logic as being essentially about 'argument' or 'inference' faces a similar trouble to Normativity: classical logic does not do this subject matter justice. When one commits to using the methods of formal analysis to model argument or reasoning there's an obvious dissonance with classical logic. Classical logic seems too limited to serve as an accurate model of these phenomena.

However, the fact that classical logic is not an adequate model of practical reasoning, does not undermine the legitimacy of attempting to construct formal models of reasoning. The conclusion to draw here is not that work of the sort undertaken in the handbook is not logic. The conclusion is that the explanation which motivates the type of work undertaken in the handbook makes for a poor explanation of classical logic. Yet these concepts are used time and again to explain classical logic. Frege's argument against psychologism (which recognises two forms

43 Elliott Mendelson, *Introduction to Mathematical Logic*, 4th ed. (1st reprint, New York: Chapman & Hall/CRC, 2001), p. 1, ISBN: 0412808307.

44 Mordechai Ben-Ari, *Mathematical Logic for Computer Science*, 3rd ed. (London: Springer, 2012), p. 1.

45 John Heil, *First-Order Logic: A Concise Introduction* (Boston: Jones / Bartlett Publishers, 1994), p. 2, ISBN: 9780867209570.

46 This doesn't have to be a model of every reasoning process, you might model just a subset. (Though if the target subset is those which are 'correct', the correctness must be externally justified somehow. Internal justification – appealing to the fact that the arguments are valid in the logical system – is circular.)

47 Dov M. Gabbay et al., eds., *Handbook of the Logic of Argument and Inference* (Amsterdam: Elsevier, 2002).

48 Gabbay et al., p. v.

of psychologism) is most useful when it is seen in the context of the explanation of classical logic: psychological concepts don't explain what classical logic is about.

A similar, but more generalisable insight than Frege's anti-psychologism comes from Gilbert Harman's distinction between inference and implication. He says, 'Inference and implication are very different things and the relation between them is rather obscure. Implication is a fairly abstract matter, a relation among propositions. Inference and reasoning are psychological processes, processes of reasoned change in view.'[49] He continues, 'similar remarks hold for consistency. Just as issues about implication have to be distinguished from issues about reasonable inference, issues about consistency have to be distinguished from issues about rationality and irrationality. Consistency and inconsistency are in the first instance relations among propositions, and only indirectly relations among propositional attitudes.'[50] Harman's distinction between inference and implication could also be seen as a distinction between 'belief' as the critical notion *vs.* 'truth' as the critical notion.

Belief is a propositional attitude – something a person can do with respect to a proposition. In his textbook Restall explains, 'logic concerns itself with reasons for *believing* something instead of something else. For beliefs are special. They function not only as the *outcome* of reasoning, but also as the *premises* in our reasoning.'[51] This psychological analysis is compatible with normativity. Indeed, Restall's explanation places conceptual emphasis on a normative notion: he says, 'Logic is the study of good reasoning, and in particular, what makes reasoning *good*.'[52] Despite being compatible with a normative approach, this psychological emphasis does not require it – a descriptive approach is equally legitimate.

In contrast, implication and consistency are relationships among propositions. Here, 'truth' is the critical notion. Harman explains, 'Propositions are consistent when and only when it is possible for them all to be *true* together [emphasis added].'[53] This analysis then makes it quite surprising when we find that though the conceptual emphasis in Hodges' textbook falls on the notion of consistency, beliefs feature strongly. He says, 'Logic can be defined as the study of consistent sets of beliefs... Some people prefer to define logic as the study of valid arguments. Between them and us there is no real disagreement... But consistency makes an easier beginning.'[54] He clarifies, 'The type of consistency which concerns logicians is...*compatibility of beliefs*. ...a set of beliefs is called *consistent* if these beliefs could all be true together in some possible

49 Gilbert Harman, 'Internal Critique', in *Handbook of the logic of argument and inference*, ed. Dov M. Gabbay et al. (Amsterdam: Elsevier, 2002), p. 171.

50 Harman, p. 174.

51 Greg Restall, *Logic: An Introduction*, Fundamentals of Philosophy (ProQuest Ebook Central, London: Routledge, Taylor & Francis Group, 2006), p. 6, ISBN: 9780415400671.

52 Restall, p. 6.

53 Harman, 'Internal Critique', p. 174.

54 Wilfrid Hodges, *Logic*, 2nd ed. (London: Penguin Books, 2001), p. 1, ISBN: 9780141003146.

situation.'[55,56] If we take Harman's distinction seriously, then the presence of 'belief' in Hodges' explanation of logic is misplaced, a distraction from the more important notion – 'truth'.

Harman's distinction gives us two projects. One which is about inference or reasoning, can be normative, and in which beliefs are central. The other is a study of the relationships among truth-bearers. This distinction does not exhaust the possibilities, but it does help draw boundaries around conceptual groupings. These boundaries can be very helpful in understanding which notions are important to introduce and which notions might cause trouble. Troublesome concepts represent both a missed opportunity to communicate an important concept, and an opportunity for some concept to appear more important than it is.

In many cases in these textbooks a discussion of reasoning, argument, or belief represents a missed opportunity to talk about formal methods, consequence relations, and truth. The reason that I say this is a missed opportunity is not because reasoning, argument, or belief are not legitimate components of the subject matter of logic; it's because the objective of these textbooks is to introduce classical logic. In the introduction to classical logic the focus must be on presenting an explanation which makes sense of classical logic. Introducing the broader possibilities in the field of logic must take a lower priority. That lower priority should be reflected in the words chosen as well as their order and frequency.

Before I move on there are two last points I want to make with respect to 'argument'. The first is that when I think about 'argument' as opposed to 'reasoning', what stands out to me is not that argument is externalised reasoning, but that it's *discursive*: it's multi-agent. Reasoning, as a psychological process, cannot involve more than a single reasoner. We can quite plausibly call some piece of externalised reasoning presented from a single point of view an argument. But often the point of externalising reasoning in that manner is to engage in dialogue. This dialogic aspect is important for the study of argumentation; but it does require a more complicated model than classical logic provides. Still there's no obvious reason to think that the study of argumentation should be excluded from the field of logic.

The second point I wish to discuss relates to the texts which focus on argument but which have not misplaced their emphasis. The texts of Nolt et al, Munson & Black, Arthur, and Sinnott-Armstrong & Fogelin focus on *argument analysis*.[57] Here argument is clearly a central notion. These are not so much cases of faulty explanations of classical logic, but cases in which teaching classical logic is questionable. Some teaching contexts call for instruction in the transferable skills associated with argument analysis. These skills do overlap with the skills developed by learning classical logic, and the propositional or predicate calculi can be used to

55 Hodges, *Logic*, p. 1.

56 See appendix B.2

57 With the texts of Nolt et al and Arthur, it seems possible to me that a student could end up in normative territory if they were really thinking about it, but the connection is not obvious in the text.

analyse arguments. However, classical logic might not be the best choice when this sort of skill development is the primary aim. One drawback to using classical logic for argument analysis is that it becomes cumbersome to work with for complex arguments. But the biggest drawback to using classical logic in these contexts is the time cost.

The risk in teaching classical logic in order to strengthen argument analysis skills is that the time spent learning classical logic is time spent not achieving the central objective of the course. For the kinds of university level courses which I am examining, there are baseline skills like reading and diagramming as well as background concepts like 'structure' and 'pattern' which teachers can draw on to teach argument analysis without having to introduce classical logic. Eemeren and Snoeck Henkemans[58] offer an alternative approach to argument analysis. Their textbook features the dialogic aspects of argumentation, contains an analysis of argument structure using diagramming which could be expanded upon in class, uses larger and more complex pieces of argumentation than would be possible when using classical logic, and provides a great way to introduce analytic thinking strategies. All without dragging in classical logic. It does equivocate between validity and soundness, so some caution in class about what's being said here is warranted. Still, I prefer this approach to the approaches which use propositional or predicate calculi to analyse arguments.

3.4 VALIDITY, LANGUAGE, AND TRUTH

The texts which do emphasise 'validity' offer little improvement in the explanation of the subject matter of logic over those which emphasise 'reasoning' or 'argument'. In all of these except Priest's textbook, despite placing conceptual emphasis on validity, validity is expressed as a property of something. In P. Smith[59] and Jeffrey[60] validity is a property of arguments. In Bergmann, Moor and Nelson[61] it's reasoning; and in Howson[62] it's inferences. In presenting validity as a property of something these textbooks construct the subject matter of logic – they say what validity applies to: reasoning, inferences, or arguments. (See appendix B.7)

The texts which emphasise validity use different supporting notions. Bergman, Moor & Nelson use truth preservation. They say: 'The hallmark of deductive logic is **truth-preservation**. Reasoning that is acceptable by the standards of deductive logic is always truth-preserving; that

58 Frans H. van Eemeren and Arnolda Francisca Snoeck Henkemans, *Argumentation: Analysis and Evaluation* (New York; London: Routledge, 2017).

59 Peter Smith, *An Introduction to Formal Logic* (7th printing with corrections, Cambridge, UK: Cambridge University Press, 2013), ISBN: 9780521008044.

60 Jeffrey, *Formal Logic*.

61 Merrie Bergmann, James Moor and Jack Nelson, *The Logic Book*, 6th ed. (Boston: McGraw-Hill, 2014), ISBN: 9780078038419.

62 Colin Howson, *Logic with Trees: An Introduction to Symbolic Logic*, 1st ed. (ProQuest Ebook Central, London: Routledge, Taylor & Francis Group, 1997), ISBN: 9780203976739.

is, it never takes one from truths to a falsehood. ... In deductive logic, reasoning that is truth-preserving is said to be "**valid**".'[63] P. Smith gets to validity via cogency, saying, 'The business of logic is the *systematic evaluation of arguments for internal cogency*. And the kind of internal cogency that will especially concern us is *deductive validity*.'[64] Priest and Jeffery use the notion of 'following from'. Priest says, 'The point of logic is to give an account of the notion of validity: what follows from what,'[65] and Jeffery says, 'Formal logic is the science of deduction. It aims to provide systematic means for telling whether or not given conclusions follow from given premises, i.e., whether arguments are valid or invalid.'[66] Finally Howson uses inference saying, 'There is much more to logic than the question of what makes *inferences* deductively valid or invalid, but to most people that is what logic is all about, so that is where we shall begin.'[67]

With regard to the concept of inference, I can do nothing more than reiterate the points I have already made. However, before I move on, let me briefly note that there is another option which is similar to 'reasoning' or 'argument': language. When language is taken as central one interpretation could be that logic models natural language. Again, taking this project seriously means abandoning classical logic, and even many non-classical logics. The study of language using formal methods gave rise to formal semantics. Formal semantics is a species of logic, but its character is quite different to what is studied in philosophy programmes. Later Ripley and I discuss this point.[68]

In the explanations of logic which emphasis validity there are two supporting notions which I have not yet discussed: 'follows from' and 'truth preserving'.

A way of centring language without turning logic into formal semantics, is to say that the role of logic is to provide an account of the natural language meaning of consequence. Here the focus is on whether '$\models$' is an accurate model of the meaning of the English 'is a consequence of' or 'follows from'. This understanding of logic is what Priest seems to have in mind when he says in his textbook, 'The point of logic is to give an account of the notion of validity: what follows from what.'[69] In the wider philosophic discourse Cook says:

> ...a logic (plus an identification and interpretation of logical vocabulary) is correct if and only if, for any way of interpreting the nonlogical vocabulary, the logic validates a particular argument if and only if the natural language statement corres-

63 Bergmann, Moor and Nelson, *The Logic Book*, pp. 1-2.
64 Smith, *An Introduction to Formal Logic*, p. 1.
65 Graham Priest, *An Introduction to Non-Classical Logic: From If to Is*, 2nd ed., Cambridge Introductions to Philosophy (ProQuest Ebook Central, Cambridge: Cambridge University Press, 2008), p. 3, ISBN: 9780511392306.
66 Jeffrey, *Formal Logic*, p. 1.
67 Howson, *Logic with Trees*, p. 1.
68 See §5.37
69 Priest, *An Introduction to Non-Classical Logic*.

> ponding to the conclusion of that argument is a logical consequence of the natural language statements corresponding to the premises of that argument.[70]

Priest has made similar statements.[71]

One intuitive way of developing the idea that the role of logic is to provide an account of the meaning of 'follows from' is that logic gives an account of conditional sentences in natural language. Here my argument is the same as it has been many times before, this is an acceptable account of the aim of some logical system, but it cannot be an account of classical logic. In his textbook Priest gives a thorough account of the ways in which the material conditional of classical logic fails as an account of the indicative English conditional.[72] In demonstrating his point he gives examples of arguments which would be valid if the indicative English conditional were material. One of these examples is:

> It is not the case that if there is a good god the prayers of evil people will be answered.
> Hence, there is a god.

Which is an example of the classically valid:

$$\neg(A \supset B) \vdash A$$

Another is:

> If John is in Paris he is in France, and if John is in London he is in England.
> Hence, it is either the case that if John is in Paris he is in England, or that if he is in London he is in France.

Which is an example of the classically valid:[73]

$$(A \supset B) \wedge (C \supset D) \vdash (A \supset D) \vee (C \supset B)$$

These examples show that classical logic cannot be an adequate account of the indicative English conditional.

Moving from 'follows from', let me pick up another notion I discussed earlier: 'truth preserving'. Two textbooks emphasise 'truth' in the explanation of logic. The first is Smullyan's[74] which is an idiosyncratic text which presents logic in the context of a series of puzzles of lying and truth-telling. The essential lesson that this creates is that logic is something that we

70 Cook, 'Let a Thousand Flowers Bloom: A Tour of Logical Pluralism', pp. 495-496.
71 Priest, 'Logical Pluralism', p. 196.
72 Priest, *An Introduction to Non-Classical Logic*, pp.11-15.
73 Priest, pp. 14-15.
74 Raymond M. Smullyan, *Logical Labyrinths* (Wellesley, MA: A. K. Peters, 2009), ISBN: 9781568814438.

use for figuring out the truth. The second is N.J. Smith's[75] which uses a more typical style of presentation. (See appendix B.5)

The main feature in N.J. Smith's introduction is an argument about the way in which logic is related to reasoning. He says: 'Logic is often described as the study of *reasoning*. Knowing basic logic is indeed essential to being able to reason well – yet it would be misleading to say that human reasoning is the primary subject matter of logic. Rather, logic stands to reasoning as mathematics stands to carpet cutting.'[76] He explains that mathematics is useful if you want to do a good job at carpet cutting, and similarly, logic is useful if you want to do a good job of reasoning. This is a presentation of the normativity via hypothetical imperative thesis, but Smith is clear that this doesn't say anything about the primary subject matter of logic. He says instead, 'logic is the science of *truth*,'[77] its subject matter is the laws of truth.

'The laws of truth' as a slogan for the subject matter of logic points the way to a certain sort of project to which classical logic could belong: a metaphysical and ontological project. When engaged in the metaphysical project the aim is to describe possible metaphysical structures for truth in much the same way as geometry describes abstract structures of space. When we engage in this pursuit we explore all the possible structures and configurations for truth because it is only after we have meaningful alternatives that we can ask which of those structures best represents the structure of truth in our universe. There are many potential questions inside of this metaphysical project. Two important questions are: how these metaphysical theories might be empirically verified; and whether it is possible for a relation of truth preservation to exist.

The laws of truth as the subject matter for logic has several advantages in the context of teaching classical logic. One of the most important advantages is that it is a subject matter in which classical logic is at least a live option. Another advantage is that it does provide some intuitive ways of explaining the value of logic, as N.J Smith demonstrates. Lastly, the application is clear, even though it is quite abstract.

However this framing comes with a risk of dogmatism: that students walk away thinking they know all there is to know about how truth works rather than thinking they've been introduced to merely one of many possible models. Rather like mistaking Euclidean geometry for an accurate model of all possible space, and of our space in particular.

Another disadvantage is that it might make it harder for students to grasp the generality of formal systems; that we might equally use logic to explore the laws of beauty or goodness. For instance, is conjunction a law of goodness? If 'A' is good and 'B' is good, then is 'A and B' good too?

75 Nicholas J. J. Smith, *Logic: The Laws of Truth* (ProQuest Ebook Central, Princeton, NJ: Princeton University Press, 2012), ISBN: 9781400842315.

76 Smith, p. 3.

77 Smith, p. 4.

I think the advantage of a presentation in which classical logic is at least a live option outweighs these disadvantages. Still the disadvantages are worth noting as it might be possible to compensate for these disadvantages during teaching.

Circling back to language, two of the textbooks which I examined actually talk about language, Halbach's[78] which is aimed at philosophy students, and Dalen's[79] which is aimed at mathematics students. Their target audience appears to affect their approach. Van Dalen speaks of the study of the language of mathematics by mathematical means,[80] whereas Halbach talks about formal languages and natural languages.[81] In both of these texts the essential points relate to language, though Halbach's remarks about logic in general look more psychological than linguistic. Van Dalen talks about the 'close connection between classes of mathematical structures and their syntactical description.'[82] Whereas Halbach mentions trying 'semantic theories on the highly regimented languages of logic.'[83] Both are alluding to proof-theory and model-theory. (See appendix B.6)

This represents another subject matter in which classical logic is a live option: semantic theorising. We can use logical systems to explore how the symbols used in formal systems get their meanings. Is it possible for meaning to come purely from the rules of the formal system, or must there be other structures to give the symbols their meaning? Could meaning come from the information conveyed by a proof?

It is something like this that G. Russell seems to have in mind when she argues:

> Semantic features of the language of an argument, including whether or not it admits of more than two truth statuses, the truth-conditions determined by the meaning of its logical expressions, etc. can determine a model theory which either will or won't admit of a counterexample to the argument. ... Getting the semantics, and eventually the model theory, correct is difficult – semantics is harder and both requires and admits of more theory and evidence than the layperson generally expects. To take just two examples, it looks as if the correct semantics for English must provide solutions to both the Sorites and Liar paradoxes. Still, this is the key to discovering the right set of models for a language, and thereby determining whether or not an argument form is valid.[84]

Ripley is similarly invested in the semantic investigation, though from a proof-theoretic per-

78 Volker Halbach, *The Logic Manual* (Oxford: Oxford University Press, 2010), ISBN: 9780199587841.
79 Dirk van Dalen, *Logic and Structure*, 5th ed. (London: Springer, 2013).
80 Dalen, p. 2.
81 Halbach, *The Logic Manual*, p. 2.
82 Dalen, *Logic and Structure*, p. 1.
83 Halbach, *The Logic Manual*, p. 3.
84 Russell, 'Logic Isn't Normative', p. 386.

spective. He also indicates that he thinks that semantic theories can be empirically tested and comments on the sort of data he thinks would be relevant.[85]

3.5 LOGIC AND ITS SUBJECT MATTER

Logic, the discipline, can be construed as containing broad subject matter groupings – logical projects. Each with a rough domain of investigation and source of relevant data. Broad subject matter groupings for logical theories provide the potential for contexts in which different logical theories can be compared; providing also a way to determine whether different logical theories aim to describe the same concept or not. Superficially, logical theories aim to describe the same concept: validity. However, differences in the subject matter alter what counts as relevant data which alters the meaning of validity.

The subject matter provides the framework for developing and testing logical theories. In this context, a logical theory is a theory about what the target phenomenon in a domain of investigation is like. For a given subject matter there can be multiple logical theories. The logical theory describes the notion of validity which the formal system aims to capture – the intertheoretic validity. Logical theories and the formal systems which describe them, produce predictions which can then be tested against the relevant data for the subject matter in question – this is a test of the adequacy of both the logical theory and the formal system.

There is a pedagogic problem for introductory logic textbooks which arises from the practical need to provide a sensible introduction to what the student will learn during the course. Teachers must say *something* about the subject matter of logic; about which there is a great deal of unstated disagreement. This disagreement is present in both the advanced literature and the way that the subject matter is explained to students.

My study of logic textbooks shows that while there is a dominant tradition of explaining the subject matter of logic in terms of correct reasoning, there are also some alternative approaches. From the pluralistic point of view, each of these approaches is equally legitimate in as far as the approach supplies a clear subject matter accompanied by an identification of the relevant data for that subject matter. However, in the context of teaching classical logic not all explanations of the subject matter are equal.

Explanations of the subject matter must make sense of the logics introduced in the course. Some explanations undermine attempts to understand the value of classical logic as it is not adequate to the proposed purpose. Explanations provided in introductory teaching must be aligned with the logics introduced. From the topics I have examined there are two options

85 See §5.66

which allow for the possibility that classical logic is correct: semantic theorising, and metaphysical analysis.

When the subject matter of logic is unpacked by exploring the many possible explanations it becomes obvious that the words chosen, the concepts they denote, and the order in which they are presented are all important. The subject matter emerges from they way in which the explanation is packaged. So, constructing an explanation is a delicate matter because a few words here or there can make a big difference.

Logic as a discipline is the study of several inter-related subjects (sometimes) using formal methods. Formal methods are subject matter independent – there's a general process of formalisation which is not tied to any given subject matter. Introductory logic teaching presents an opportunity to introduce and explain that process and draw attention to the way in which those methods could be applied to model many phenomena.

INTERVIEWS

INTERLUDE

The last two chapters have been dedicated to exploring the concepts that are embedded in introductory explanations of the subject matter of logic. On the way, I have discussed the practical challenge of producing such an explanation, as well as presenting a little of the history of the development of modern logic and the explanations of the subject matter that came with it. I explored the many ways in which the subject has been characterised in introductory texts and explored a variety of potential subject matters which belong within the broader discipline logic. My approach is fundamentally pluralistic, allowing that there are multiple possible subject matters for logic, each equally legitimate.

Now I turn to interviewing three logicians and teachers: Gillian Russell, Dave Ripley, and Johan van Benthem. I asked each of these logicians for an interview because the views they express seemed to me to entail different philosophies of logic. G. Russell's philosophy of logic comes from model theoretic perspective, but her writing also implies a strong interest in language. Ripley has a similar interest in language, but views logic from a proof theoretic perspective. On the other hand, what drew me to van Benthem, are the psychologistic claims he seems to make. Each logician expresses views which imply that they don't agree on the subject matter of logic.

I wanted to capture and explore these views, while retaining connection to the pedagogic questions which are fundamental to my research. These logicains are not only theorists, they are teachers. As teachers they can offer valuable insight into the reasons behind their choices when they teach. When examining textbooks alone these motivations are not available. None of these interviewees have authored textbooks included in my study, nevertheless their comments will be helpful for understanding the teacher's perspective.

Each interview is transcribed in its entirety, with only light editing. They are informal discussions which explore logic and the teaching of logic.

This thesis has two core threads: pedagogical and philosophical. The philosophical claim is that logic, the discipline, should be understood as having multiple subject matters. The pedagogic point is that some of these subject matters are better for making sense of the taught systems than others.

I have presented two pieces of evidence for the claim that logic has multiple subject matters. The first is an analysis of the words which teachers use to explain the subject matter of logic. The way the data for that analysis was collected means that we cannot generalise to all teaching from the sample I examined. For example, I cannot make a strong claim as to the prevalence of one way of explaining logic compared to another. Nevertheless, I have demonstrated the

plurality of explanatory strategies. There may be more strategies than I identified in my sample, but I can say for certain that there is more than one.

For the second piece of evidence, I interview three logicians. I selected these logicians because I suspected that they conceived logic differently to each other. This evidence is weaker because an interview is not a format that easily permits the presentation of a fully developed view on the subject matter of logic. So, it is hard to probe too deeply into the differences in how they each understand their subject matter. I said at the beginning that there is a void in the literature, meaning that there are no publications in which fully developed views on the subject matter of logic are presented.

So in each interview, I have two objectives. The first is to understand how each logician sees the subject matter of logic. The second is to strengthen my first piece of evidence. With the textbooks, I demonstrate that the subject matter of logic is explained in multiple ways. Textbooks represent what teachers think is the best way to explain the subject matter of logic, but do not necessarily reflect how teachers think about the subject matter of logic. It is not a good pedagogic strategy to deliver some complex analysis on what the subject matter of logic *really* is. Some level of simplification is necessary. So I want to explore to what extent we can take what teachers say about logic to students who know nothing about the subject as representative of their views.

1 **Bessie:** I'd like to start with some general background questions.

2 **Gillian:** OK, fire away.

3 **Bessie:** Where did you do your undergrad study?

4 **Gillian:** I was an undergrad at the University of St. Andrews in Scotland. That was a long time ago now. I started there in '94. I went there to study German and English and ended up switching into German and philosophy and never looked back.

5 **Bessie:** And for masters and PhD, where did you do those?

6 **Gillian:** So, Scotland is a little funny in that if you go to a good university in Scotland, your undergraduate degree is a master's degree. My first masters is from [University of] St Andrews. And then I did my graduate work at Princeton. They don't have a terminal masters, but you get a masters on the way to getting your PhD after you finish your coursework and then finished my PhD there. My advisor was Scott Soames, and I wrote about the analytic/synthetic distinction.

7 **Bessie:** And for logic do you remember your first logic course?

8 **Gillian:** Yes. It was also my first philosophy course. I was in my second year at St Andrews, and I was looking around for something more interesting than what I'd been studying so far, and I decided to do two philosophy courses, and one of them was called logic and epistemology. It was taught by two faculty members. Garret Cullity taught epistemology, and Stephen Read at St Andrew's taught logic. I remember finding the logic course incredibly hard. So I would do all the reading ahead of time before the lectures and then take copious notes during the lectures like desperately trying to understand and then go back to the reading and try to figure out what was going on and like the philosophy of logic we were reading. All this stuff about possible worlds: and I really really enjoyed it, I felt challenged and stimulated, but also there was this mathematical element to it that I missed from studying maths and physics when I was younger and it was really exciting to both be doing that kind of mathematical stuff that felt very clear cut and straightforward if not easy. And then combining that with dealing with really big questions was super exciting. Yeah, I loved that course. And then I went on to the second logic course at St Andrews which was taught by Stuart Shapiro; at the time he was visiting. After that, I took every logic course I could get my hands on. Definitely enjoyed it.

9 **Bessie:** Yeah, I would have done the same, but they only had the one logic course.

10 **Gillian:** Oh no! Doesn't Wellington... Is it Wellington in New Zealand that has a massive introductory logic course with a thousand students?

11 **Bessie:** Oh gosh. I think Auckland has a bigger introductory logic class. Rod Girle round here is known for running large logic classes. But when I did logic, it was smaller; I was at the

University of Otago. I did my undergrad and masters there, then took five years off to work. Now I've come back to study at Victoria University of Wellington. I'm still doing the same thing. I loved logic so much, and I was always really interested in the question of what logic is about. So I did it for the dissertation component of my undergrad, I also looked at what logic is about in relation to normativity for my master's. I'm still bothered by the question, so I want to address it in my PhD.

12 **Gillian:** That's a good question. I remember the first time I ever visited Australia. I went to a logic conference because Graham Priest said I could come visit the UQ department at the time. And then there was an AAL meeting at Noosa, and I remember asking everybody there what logic was. I was a first-year grad student, so I felt like I could ask the simple questions. I remember asking Greg and J.C. in the pre-logical pluralism days what logic was and all these other logicians were there, and I got totally different answers from people.

13 **Bessie:** Do you remember what Greg and JC said?

14 **Gillian:** I honestly don't. I do remember that some people were like, 'well we just we set up some axioms, and then we see what follows from them' and other people were more, 'well no it's about characterising validity'. But I don't remember all the answers I'm afraid. It was probably like 2000.

15 **Bessie:** I find it a generally fascinating question. The other thing that I found rather challenging is that it's not something that people are writing directly about. Which is part of why I wanted to do a series of interviews because I thought well that way, I could ask directly: what do you think logic is about?

16 **Gillian:** Yeah, okay. That makes sense. I mean I think one thing that happens with some formal topics is the subject matter kind of has this independence.

17 **Gillian:** And there are all these questions that you can ask and answer without having a very good grip on the foundations – the same way that you can be an excellent mathematician without having a good answer to 'what are numbers?'

18 **Bessie:** Sure.

19 **Gillian:** So you can do all kinds of important useful stuff in logic even if you don't know how to answer the question, or don't have a correct answer to the question about what logic is.

20 **Bessie:** Yeah. There's a lot to research out there in the discipline.

21 **Bessie:** It must have been quite interesting studying with Stephen in that introductory logic course. Mine didn't contain very much philosophy of logic, but since Stephen is so interested, I presume that yours was quite philosophical.

22 **Gillian:** Yeah, I mean I think one thing is that Stephen doesn't really pander to the fact that it's an introductory class. He tended just to get excited about things and start talking, so some of the reason it was hard to follow, but also some of the reason it was exciting, was that you had

the sense that there was this stuff out there that wasn't easy, but that was interesting. He used his textbook *Thinking About Logic*. It was the first time I'd encountered a lot of these things. But there was philosophy of logic as well as logic. Although he was teaching, I think, just first-order classical logic there was a proof system because I remember some of the questions on the exam – you had to prove some things. There was some translation; familiar things from a lot of people's logic class but then I think there were also essay questions that were... I remember we had to read Tarski's 'The Semantic Conception of Truth' in the third week which strikes me as funny now. But I loved that paper, and I was really excited about it.

23 **Bessie:** Well that does sound like an exciting introductory class, and it does sound more challenging than what we normally think of for that class. One of the things I have been wondering about is what the right sort of expectation is of a student coming into that first logic class.

24 **Gillian:** Do you mean what they should expect, or what we should expect of them?

25 **Bessie:** What we should expect of them. Or what we're establishing in the communication about what they're going to learn and what they're going to need to be able to do. I think the practical challenge of what they're ready to learn is different depending on your geographic region, your university and all of those external factors. But one of the things that I wonder about is the assumption that students won't be ready for *that*, so we won't teach it, and then we miss out some of the really exciting content.

26 **Gillian:** I guess... Two things that kind of – they don't answer the question, but I think they're useful guiding principles.

27 **Gillian:** One is that any time you get a chance to teach is this amazing opportunity to reach people that you might not reach normally. And it's incredibly important to think about the students you actually have in your class and what they need and what you can give them rather than sort of some idealised students that maybe you would want to work with or would be the perfect intro logic students, especially when you have a small class. I wouldn't be shy about changing my expectations as I went along, as I learned more about how they were doing, got more feedback from the grading and things like that. And if they seem to be particularly interested in some questions, I would be totally happy to take a little bit longer on one lecture to draw out some issues or to digress, to talk about something I hadn't planned to talk about or spend a little bit longer on something because they're interested.

28 **Gillian:** The other thing is... I remember one of my undergrad teachers, Peter Clark at St Andrews, saying that he believed in difficult courses and relatively easy grading. And just the way I was talking just now about my introductory logic course with Stephen Read in many ways it was way too hard for me. I just wasn't ready for it. I didn't know about any of these things. And maybe things should have gone more slowly. But on the other hand, you know,

people get more out of courses that they're excited about that they put a lot of work into. So, if you can challenge people, so they're not bored in class, but also give them some way to succeed. You don't want them all to fail, obviously. And you don't want to be giving them credit for something that's not a success. But there is a way to really ask a lot of people and then grade them appropriately for the level they're at.

29 **Bessie:** Yeah. I think that component of challenging logic students to read some philosophy of logic and write about it is an exciting way to bring some of that other content into the course. Otherwise, if you're just purely focused on the formal, as mine was, you can be left with a lot of questions. And then you're also without guidance on some of those harder readings in the philosophy of logic. I spent so long reading those 'Tonk' articles.

30 **Gillian:** Oh, 'The Runabout Inference Ticket'?

31 **Bessie:** Yes, 'The Runabout Inference Ticket'.

32 **Gillian:** Oh, it's gorgeous. But it's not exactly explanatory if you're not embedded in all those issues. I can see why it's important. Yeah.

33 **Bessie:** And finally, I think after about six months of reading and rereading, I was like oh – there are only 16 possible operators.

34 **Gillian:** Oh, OK. Like the binary truth-tables for... Okay.

35 **Bessie:** Yeah, exactly. It just hadn't dawned on me that that mathematical structure was there. And it was quite fun, in a recent logic course where I was talking about the operators and how they were structured, one of them said 'so that means there are only 16 possible ones?' and I was like: 'Yes'. But I think he was the only student in class who really got that step.

36 **Gillian:** There's at least one textbook that I teach from that asks students... I think it's Bostock's Intermediate Logic, it asks students... There's some proof of truth-functional completeness, and they're supposed to go through every one of the possible binary truth functors and figure out which ones are sufficient for expressing every truth function, and as part of that they have to figure out how many there are. Yeah that kind of exploratory question where they have a lot of stuff to figure out can be challenging but gives them a stronger grip on what's out there.

37 **Bessie:** I remember reading in, 'One True Logic?' where you spoke about the notion of fibbing to beginners. You're using it as an explanation for why, in general, people don't necessarily talk about some of the problems or issues in philosophy of logic. When you're in the advanced literature, it's all advanced; a lot of the beginning stuff is taken for granted. And when you're doing the intro stuff, you're assuming that the beginners aren't necessarily going to understand. I was just struck by that comment, although you laugh as if you don't like the feeling of fibbing to beginners?

38 **Gillian:** No. And then I do it all the time. In part because you need to think about how you

present information and what you want people to get out of it, and it can be really distracting if every single thing you say is hedged.

39 **Gillian:** And if you are teaching truth-functional classical logic, introducing them to truth tables and you want to introduce them to logical equivalents and being a tautology – It's not the time to say you suspect there aren't any tautologies really, right? First of all, you need to show then what that would be. And it's not the time to say, 'Actually I think there might be four truth values'. Or maybe even not the time to start talking about explosion. Or the law of excluded middle, maybe you don't think that's a law of logic or something like that. What you need them to do is have that experience of realising that not not P and P get the same truth value no matter what we value P gets, and seeing that modus ponens is valid, and that modus tollens is valid; seeing the patterns emerge. And you can distract from those patterns and make it harder for people to see them by trying to draw more a fine-grained picture. So yeah, I do think it's a pretty reasonable approach to teaching to sometimes say false things.

40 **Bessie:** Sure. I suppose it depends on your falsehood. If the falsehood is well developed to help them on a path to understanding or if the false thing is going to be difficult to understand, or introduce some additional challenge.

41 **Gillian:** Yeah. You can probably avoid a lot of it with careful planning and setup. I mean you can say things like, 'it might be the case that every sentence can only have one of two truth values true and false: suppose that's right'. If you set everything up as a conditional, I suspect they won't notice that's how it went, and you'll still be able to draw the simple patterns. So maybe that would be preferable.

42 **Bessie:** One of the other things that you've mentioned about intro logic courses is in the *Open Logic Project*[1], there's a comment of yours about the need for an introduction but the difficulty of creating an introduction which doesn't introduce bad philosophy of logic.

43 **Gillian:** Yeah, I think that's really hard.

44 **Gillian:** Partly, it has to do with the fact that we teach logic to adults. I think the comparison with arithmetic is quite informative. When you're teaching small children to do sums, they get a big sheet of sums, and they learn certain algorithms for getting the right answer. Like maybe they have blocks, and they take two blocks, and they put two more then they count them. It's not generally a part of that that you give them like a serious answer to the question of what numbers are, I don't think – I mean I'm not a primary school teacher, so maybe people do that these days. But I think you can get very far in mathematics without having serious answers

1 The *Open Logic Project* is a collection of teaching materials on mathematical logic aimed at a non-mathematical audience, intended for use in advanced logic courses as taught in many philosophy departments. There is a team of people is working on it, using the GitHub platform. It is written with configurability in mind and users are free to change it however they like, and share those changes. The LaTeX code is available for download. The project's website is https://openlogicproject.org/

to these questions and I don't think maths textbooks start with like a serious analysis of what numbers are. But logic we're teaching to college students and it's only reasonable that we start with some introduction that says what on earth this is supposed to be.

45 **Bessie:** There's the practical reason for an introduction as well. Which is to answer the question, 'what is this course about?' The problem for an introductory logic course is that the two questions are necessarily tangled together. What is this course about is also answering 'what is logic about, generally?' So, you have to say something about what logic is about generally.

46 **Gillian:** The way I actually do it in practice, when I think about my introductory logic classes, I start by presenting particular arguments. Things like 'All men are mortals, Socrates is a man; therefore, Socrates is mortal' and, 'All women are immortal, Gillian Russell is a woman; therefore Gillian Russell is immortal'. I present a bunch of arguments, some of them are valid, some of them are sound, some of them are neither. And then I asked the students 'Which ones are the good ones?' And they nearly always say well we like the one that's valid and sound and there'll be some explanation. But then other ones are, 'Yeah, it's good in some way and not good in another way'. And I guess the way I start is by trying to get them to recognise that validity in the wild in some arguments is the property they would try to capture and then offer a definition at least a rough informal definition. If I'm if I'm using a textbook that has a particular sort of informal approach like 'it's impossible for all the premises to be true and the conclusion false' then we'll use whatever definition they use in the textbook; but it might be just be something about the truth of the premises guarantees that of the conclusion or something like that. And then at the end of that class explain that logic's about capturing validity and you might have come to suspect by looking at some of the arguments that any argument of that form is going to be valid. And there's something in common between some of these valid arguments and that formal logic tries to capture that form and then as a motivation for using formal artificial languages that they have specific meanings that have been dictated by us and so there's less worry about ambiguity – we really know what something means. And then we're off...

47 **Bessie:** Yeah, cool. So, what sorts of things would you have in mind as bad philosophy of logic that you would not want to introduce in those courses?

48 **Gillian:** Oh. Saying that logic is the study of reasoning; it's about what we ought to believe, conditional and other things we believe; over psychologising logic; that kind of stuff. It's the sort of thing introductory logic textbooks say all the time it's not the worst sin in the world...

49 **Bessie:** Yes, it is the sort of thing intro logic textbooks say all the time. The other component of my thesis is analysing introductory logic textbooks. So I've got a pile of 38 textbooks, and an analysis of the different kinds of approaches that are taken in all of those and there's a strong trend towards explaining logic with a normative term like 'correct' or 'ought', and then something either about arguments or about reasoning. So yeah, that's the dominant trend –

something that's both normative and a little bit psychological. They use the word 'argument', but I think that argument in this context is used as a synonym for reasoning. Because they're not talking about a discursive argument, they're not talking about an attempt to convince or an interaction between two people. They're talking about an argument as representative of a chain of reasoning.

50 **Gillian:** It sounds like your dissertation is going to be interesting. I'd love to see a copy when it's done.

51 **Bessie:** So, in your opinion, what is the relationship between logic and psychology?

52 **Gillian:** The short answer is, 'I don't know'. The answer that maybe you're looking for is that I think some people think that logic is more psychological than it is. That it's about belief revision; it's about what we ought to believe, perhaps conditional on other things that we believe. I don't think logic is like that. One of the things I think people often don't pull apart very clearly – and historically I don't think these things have been pulled apart very much – the distinction between a subject that would be studying patterns of truth preservation on sentences and the subject that would be belief revision and changing belief. And I think once you have this clearly in mind; the way we actually do logics, I mean like formal logic with a kind of model-theoretic consequence relation, It doesn't look very well set up for studying belief revision. I mean, certainly not normative belief revision.

53 **Gillian:** When somebody presents a logic they say, 'here are my primitive terms, here is my syntax how you put them all together, here is some interpretation functions or models for interpreting this, and then in terms of those here's logical consequence defined in this language'. Right. And that gives you your relation of logical consequence, and it is rare and not essential in characterising a logic that somebody mentions beliefs or what you ought to believe. The words 'ought' and 'belief' just don't appear in a standard straightforward characterisation of logic. That doesn't mean that it's impossible that logic is supposed to tell you what you want to believe, but there's this puzzling question to start with since it doesn't seem to mention anything like that whereas it does mention something that looks a lot like truth. Maybe it's called one or zero in the logic, but it's the property that some sentence, that maybe contains a constant and a predicate, has if the referent of the constant satisfies the predicate. And it seems easy to understand a logic as trying to capture patterns of truth preservation on sentences. I think people sometimes think that you can nevertheless read off what you want to believe and how you change your beliefs from a logic. And then you get, initially, simple bridge principles proposed things like, 'if X entails Y, then anybody who believes every member of X ought to believe Y' and you say things like, 'well excess sentences, you know beliefs as a relation to propositions, what's going on there?' And then you say, 'what if Y is a contradiction?' So, the Gil

Harman type points[2] and they say, 'Oh no! Okay, so I guess it has to be more complicated'. But it seems to me that logics are not very well set up to be theories of how we ought to change our beliefs. One reason is that deduction is often expected to be monotonic – you'll get more consequences, you're not going to get fewer, and you're not going to lose the old ones. Whereas when we change our beliefs, when we learn something new, that sometimes means we should drop one of the old ones. So, the relation of logical consequence seems too austere and too meagre to be something that would help with belief revision. I also wonder about the formality of it. I guess it wouldn't be unreasonable to think that belief revision should be subject sensitive or that they might be different principles for different subject areas. And Logic's specifically set up to be very general, and possibly, as some people have thought, topic neutral. But why would reasoning have to be topic neutral?

54 **Bessie:** Yeah, I mean... It's certainly an interesting thought. I think once you tease the disciplines apart you get the idea of 'oh well I suppose we could maybe formally model that... But what would our system have to do and be like to capture the features that are part of belief revision?' So, monotonicity is one of the questions that you're asking. Another might be, who is the subject or speaker? Or, what objects are we modelling in there – is it propositions or beliefs? And then what does the formal language need to look like to support all those things? Which then has you looking at your old propositional calculus: giving it the side-eye and wondering, 'is that the right tool? Could this be reasonable?' It doesn't look like it's up to the task.

55 **Gillian:** Yes. To be clear, I think there is a subject matter that's the study of normative belief revision how we ought to change our beliefs in response to evidence, and I think that's important. I think there's also the subject matter of descriptive belief revision. You've sent me some notes and at some point mentioned that you think that all three of these things may be worth studying I don't remember quite how you put it, but that seems right to me, they're legitimate subject matters. And my view is not that there's anything wrong with studying belief revision. It just doesn't seem that doing it by trying to capture a relation of logical consequence promises to be fruitful. You could model it formally, and that doesn't sound ridiculous. We can model things mathematically...

56 **Bessie:** All sorts of things are open once you're using those kinds of mathematical methods. But yes, is it a relation of logical consequence that we're dealing with when we start to do that sort of thing?

57 **Gillian:** Yeah.

58 **Bessie:** My next question is, 'What does that kind of relationship mean for the definition of validity?' I mean by that question... When we do not concern ourselves with belief revision that

2 Gilbert Harman, *Change in View* (London; Cambridge, Mass.: The MIT Press, 1986), chap. 2.

leads us down a certain sort of path for understanding the definition of validity as a relationship of logical consequence...

59 **Gillian:** Again, the truth preservation path.

60 **Bessie:** Yeah. Once we go down that path, what is it that we're capturing?

61 **Gillian:** The answer that I'm happy with is something like patterns of truth preservation among sentences.

62 **Gillian:** It could be natural language sentences, but for idealisation reasons or clarity reasons, they're often artificial languages and artificial sentences. But any language will do. And then beyond that I think it gets harder and I'm not confident about the answer. It's clearly not sentences syntactically construed, because they have no truth value to be preserved or not, because you need to mean something to talk about being true or false. But... I think it turns out that meaning is much more complicated and much more interesting and much more systematic than you might have suspected. And it turns out there are all these different levels of meaning. Take contemporary philosophy of language, for example, thinking about Kaplanian character – there's the proposition expressed by a sentence as well as the referent. A set sentence clearly has to be a sentence with an interpretation. But what kind of interpretation is allowed? I think is a genuinely interesting and difficult question in the philosophy of logic. So, its patterns of truth preservation over sentences but that term 'sentence' needs a lot of filling in at some point.

63 **Gillian:** I do think of it as being a semantic property, broadly construed – Logical consequence; it's about meaning and truth type stuff.

64 **Bessie:** When we go down the line of truth preservation, we do end up in a very semantic territory. You talk about sentences and then with that semantic question, I start to wonder... What does that mean for the relationship between the semantics of natural language and the semantics of your formal system? I mean are we modelling Natural Languages? Or are we modelling truth preservation inside natural languages in our logic?

65 **Gillian:** Yeah, I think that's probably a reasonable way to think about it. I mean, people can mean a lot of different things by my modelling. It's idealisation. Like the same way that Newtonian mechanics is giving you models of physical reality by talking about point particles and positions and things. It makes certain assumptions that turn out to be false, but it does a pretty good job. It abstracts away from the noise of the real world. I think that analogy from physics is useful for thinking about the relationship between a truth-functional logic and natural language. And then with first-order logic where you bring in the quantifiers it shows you that even small steps into more detail makes things so much more difficult, so much more complicated. It's one thing to add modal operators, it's quite another to add modal operators and quantifiers together – you end up with so much more complexity and questions that are hard to answer – it's not trivial putting those things together. So, there were good reasons for the idealisation

because it's really hard when we get more accurate. I think you could think of different logics on that kind of vertical hierarchy where it's not like a first-order logic says that truth-functional logic is wrong. It just goes beyond it and says oh you know there are more logical consequences and more validities than were already recognised. And then modal logic, and higher-order logic, probably also context sensitive logics like Kaplan's you can think of those logics as being more or less kind of idealised than attempting for more accuracy. But then, when you get logics that are avowed rivals, like say relevant logic and classical logic, or relevant logic and LP or some paraconsistent logic, then I think they are different theories of the relation of logical consequence and it could really turn out that one of them is better than the other.

66 **Bessie:** I guess that sort of leads us to the discussion of pluralism and the question, 'if we think that there are multiple systems and they are competing then what are they competing to be about? What are they competing over?'

67 **Gillian:** Yeah. I think they're different theories of logical consequence. They're competing about the extension of the logical consequence relation.

68 **Bessie:** What do you think counts as evidence for deciding among them? Because some people do use normative evidence; arguing that we wouldn't reason in this way, so this logic doesn't seem to be the right one. What are your thoughts on that?

69 **Gillian:** That's a really good question. And it's something I want to think more about. I don't feel like I have a satisfactory answer to this question. But that said... Think about someone like Łukasiewicz who looks at classical truth-functional logic with two true values and says, 'I think we're missing something. Look at future contingents; I think it turns out that they can have a third truth value. It can be undefined, and now I can give you a more general logic. I can give you something that's supposed to go beyond classical logic so that it can make sense of sentences with this third truth value. It's a refinement of classical logic.' And then you get the truth tables and then at some point the truth... I don't know how well you know Łukasiewicz's three-valued logic, but there's a standard strong Kleene way of figuring out the truth values of complex expressions based on the truth values of their parts, allowing that some of the expressions could have no truth value or an undefined truth value. If you have a conjunction and one side is true, and the other side is undefined then say the conjunction is undefined and if it turns out that both sides are undefined the conjunction's undefined and that goes for all the binary connectives. So, if both sides are undefined then 'A or B' is undefined and the same with other things except Łukasiewicz can't quite accept that 'If P then P' could fail to be valid. And so, in the case of the conditional he says, 'If you have undefined on both sides then the conditional gets one'. So, you have this kind of weird truth table for the conditional.

70 **Gillian:** If we are thinking about what is the better logic – classical logic or Łukasiewicz's logic. You don't have to just look at the extension of the logical consequence relation. You can

also ask about the reasons for it coming out that way and whether they are correct. So, one question I think is, 'is it really possible for sentences to have this third truth value undefined?' Or 'is it actually the case that every sentence is true or false?' And if that's the case, then I think in some ways we've got to say that Łukasiewicz has gone wrong and his three-valued logic is not the right logic. It has too many bells and whistles, and it's mistaken about whether there are counterexamples to the law of excluded middle, for example. Its counterexamples all depend on there being a sentence that has no truth value. If it turns out there can't be such a sentence, then we know that that must be a mistake. So, I think I think we can get some evidence from semantics and from getting our semantic theories right. A lot of the time the way the relation of logical consequence turns out in some logic rests on these principles about meaning, about possible interpretations for the language and the other way round. If it turns out that sentences can have a third truth value and those sentences can enter into arguments, and we want to be able to assess the validity of arguments that contain such sentences then that would be a good reason for thinking that classical logic is insufficient on its own; that it doesn't capture all the evidence.

71 **Gillian:** And another thing, it's a really simple thing to say, but counterexamples are evidence in logic. So anytime you have an argument with two premises and a conclusion that is not true that tells you something about logical consequence. So, you know that it can't generally be the case that anything of that form has true premises and a true conclusion. So, the existence of certain patterns of truth values tells us something about the generalisations about those patterns of truth values and some people would be inclined to say that logic is a generalisation from our intuitions about logical consequence. Here's why I don't think that works. We have all the arguments laid out in front of us, and we look at them and we say, 'yeah that one feels valid' and 'oh, that one doesn't feel valid to me', and what I'm reporting on is not something out there, it's something in me, my intuition about the argument, and the job of the logician is to generalize about those intuitions. I don't think it's about that. I think sometimes we end up talking that way because we're not sure what else to count as evidence. But I think we get pushed that way in the physical sciences as well. One of the things about philosophy, and I think this is the sort of thing that Tim Williamson has said, I feel like I picked up this from him or maybe even Graham Priest, that someone can always say, 'but what about this possible case?' 'what about this possible scenario?' and the philosopher feels it's their job to respond to that. Whereas in some other subjects you would say, you know, 'to do physics, we have to assume that our eyes are working' and 'we have to assume that somebody didn't come into our lab overnight and set up all the equipment to work differently'. We have to assume a certain theory about how a voltmeter works to make the measurements that we count as our evidence. If you're a philosopher you can say things like, 'well how do you know that your voltmeter is

working?' 'how do you know that your eyes are working properly?' and in that kind of situation you end up saying, 'OK maybe my evidence isn't the reading on the meter, maybe it's like this sense data that I have access to, and that's what I have to explain. That's my data.' I think people get pushed to internalise the evidence by sceptical tendencies in philosophy. So, you get the logical positivists saying that the job of a scientific theory is to explain our sense data.

72 **Bessie:** Whereas I think the more interesting question is, 'how would you know that a voltmeter is working as expected?' There are expected parameters for a voltmeter. How would you test those to confirm that this one, right now, is working? I think that's the kind of question that should be focussed on.

73 **Gillian:** As a physicist or as a philosopher of science?

74 **Bessie:** In general. I believe that's the pragmatic solution – if anybody knocking around in everyday life is faced with these kinds of questions it's really that kind of thing.

75 **Gillian:** Go to Home Depot, get another one, see if you get the same readings.

76 **Bessie:** Yeah, and then as a physicist, you do that. To me it seems like the epistemically responsible thing to do. So, it's the thing that the physicist does, and it's the thing that a philosopher should do. And it's the thing that anybody in ordinary life would do – try to answer the more pragmatic question, 'how do I know how well this compares to the generally expected parameters of this sort of test?' One of the things that ends up happening if you make evidence too internal is that it becomes too hard to compare it to other instances.

77 **Gillian:** Yes, that seems right.

78 **Bessie:** So yes, I agree, the sceptical question motivates the internal move. But then you're probably better off redirecting that scepticism to the pragmatic problems that surround any search for evidence rather than internalising the evidence, rather than making it all subjective.

79 **Gillian:** Yes. I'm tempted to say that the evidence in logic is patterns of truth preservation or truth distributions across sentences. There's probably more to it than that because we're capable of entertaining counterfactuals and noticing that this could be true without this being true. And, I think, a lot of the time what we're preserving is the meaning of the sentence – in whatever sense of meaning is important for logic – keeping the meaning the same, this can hold without this holding.

80 **Bessie:** So, it seems like there are two components. One is that is the linguistic component – talking about natural language and natural language semantics; have we accurately captured that? But then, another component is the metaphysical stuff – the question of what's out there in the world and is the logic capturing that? What do you think the relationship between logic and metaphysics is?

81 **Gillian:** Gosh. I don't know. It seems to me that the metaphysics of meaning is very im-

portant to logic. So, figuring out what kind of meanings are possible is important. Yeah, I'm not sure what else to say.

82 **Bessie:** Yeah? I mean I think that your answer, talking about figuring out what meanings are possible, draws a connection between metaphysics and logic but it suggests that you'd be less likely to go down the path of saying that what we're trying to do in logic is to create metaphysical models of the way the world could be. Because there's a connection to linguistic meaning that you see as really important.

83 **Gillian:** So is this is close to that distinction that Etchemendy draws between representational and interpretational conceptions of models and consequence. Do you know the book *The Concept of Logical Consequence*?[3]

84 **Bessie:** I know of it, but I have never managed to get hold of a copy of myself and read it.

85 **Gillian:** OK. It is absolutely brilliant, and I think it's really important for anybody who's thinking about logical consequence – it will repay your time and I strongly recommend it to you. One of the things that he brings up is that there are two different ways of thinking about... Take first order Tarski models of a language; you've assigned extensions to the predicates and referents to the names, but what are those models supposed to represent?

86 **Gillian:** And when you changed a model – so you've changed the extension of some predicate or the referent of some name – what does that difference between the two models represent? And Etchemendy says, there are two ways of thinking about it. One is the models represent different ways the world could be. So, you could think of the different first-order models as different possible worlds. And that's what you were sketching there. And then another way, which is what Tarski had in mind originally, is that you're thinking about different possible meanings the predicates could have; and you're thinking about reinterpreting the non-logical parts of the language.

87 **Gillian:** Here's a problem for the different possible worlds way of thinking about it: we allow the models to change in such a way that the reference of names can change, and in particular in some model A equals B could be true, and another model A equals B could fail to be true (could be false). But if A and B co-referential names there's no possible world in which they fail to pick out the same objects and so you say, 'in what sense is that model some real possibility – a real possible world?'

88 **Gillian:** On the other hand, you can think about what's happening in these models as different possible meanings the expressions could have. You'd be saying, 'Well here's one possibility: A and B could both pick out this object' and 'here's another possibility: A might pick out this object and B might pick out a different object'. That makes perfect sense from the

3 John Etchemendy, *The Concept of Logical Consequence* (Stanford, CA: Center for the Study of Language & Information, 1999).

idea that what models represent is different interpretations the expressions could have. If you interpret A and B differently, of course, Hesperus is Phosphorus could turn out to be false, if Hesperus meant you and Phosphorus meant me, right?

89 **Gillian:** So one thing I think that goes wrong if you think of models as representing different possible worlds is that you really can't make sense of the logic of identity and what goes on with identity expressions.

90 **Bessie:** Right. Yeah, I can certainly see that happening. That's very interesting. What does that mean about identity then? Does that mean that identity is more linguistic? Something that would perhaps not be present if we were looking at the physical world, and trying to use logic to model it?

91 **Gillian:** I don't know. The thing that makes me resist the idea that identity is purely linguistic is usually thinking about personal identity and thinking about personal identity across time.

92 **Gillian:** The kind of thing that makes people say that identity is purely linguistic are the puzzles about the Ship of Theseus or fission and fusion of various objects. One option is always to say that it depends on what you mean by identity. If you mean the kind of identity that requires this to be the same, preservation of the same matter, then these things are identical. And if you mean preservation of the same function or something then these things are identical.

93 **Gillian:** But if we're talking about me, and me in the future, and two possible people in ten minutes time who could be me, and one of them is going to be tortured, I really want there to be an answer to the question of whether or not that person will be me. Independently of whether we made of the same stuff, or whether we have the same function. I think there is an answer to the question of whether that is me, and it's not dependent on our language and how we use identity. So, I'm inclined to resist the idea that identity might be purely linguistic. But it's a super deep question.

94 **Bessie:** Yes. Well, I think that's one of the things that's quite exciting about exploring logical puzzles is that you get led into these other kinds of questions.

95 **Bessie:** We talked a bit about what you do which, I think, indicated which logic you start with. But to be clear, which logic do you start with? Which is your first, when you teach?

96 **Gillian:** I start with classical truth-functional logic. I use the Barwise and Etchemendy's textbook *Language, Proof and Logic*. I like it a lot. I mean obviously whenever you have a textbook you probably don't like every single aspect of it. But I think it's a great textbook.

97 **Gillian:** One of the reasons that I start with classical logic is that it's the logic that most textbooks start with. But also, I'm not sure I know the answer to the question 'What is the correct logic?' I'm not sure that anybody knows the answer to that question. And if you're

going to start with one; classical logic is extremely unified, simple and elegant and I think it's a sensible place for anyone to begin independently of what their views on logic are.

98 **Bessie:** Yeah, I think there's a lot of good pragmatic reasons why you would start with that one. One, of course, you've alluded to is the practical value of there being such a commonality across people when they introduced to the same logic in the same universities, and everybody's doing the same thing so that when you move to another place there's a commonality of expectation about what you will know.

99 **Gillian:** Yeah, and it's not just in philosophy. If you are taking other courses at university, you're taking physics, or you're taking maths, there are things that you can learn in logic that will help you in those courses. Plenty of proofs of conditionals work by proving the contrapositive, or there'll be proofs the work by reductio and studying formal logic can help you learn to recognise and think about those patterns. It helps you with pattern recognition in other subjects. And so, it's incredibly powerful to give people. I don't want to say more than I believe, so here's an idea that might be right: if your language is simple enough – it's like the language of arithmetic or the language of Newtonian mechanics or something like this – it might really be that classical logic is its logic. And you have to introduce; I don't know, vague expressions or expressions about the future, or modal operators or something to get something else. And in that case, it's really useful for students in their education generally to get the basics.

100 **Bessie:** Because it's got those synergies with the other subjects.

101 **Gillian:** Yeah, like proof by cases and all that stuff.

102 **Bessie:** What definition of validity do you normally give? Or end up with?

103 **Gillian:** Oh, end up with? End up with; I would be talking about truth preservation across models – there's no model that makes all the premises true without making the conclusion true. I guess there are two distinctive features about that: one is the talk about models, exactly what that comes to is going to vary with the definition of model; but also the fact that I'm careful to say, 'if all the premises are true the conclusion has to be true as well'. I don't say, 'if all the premises are true the conclusion can't be false,' because I'm trying to allow for things like paraconsistent logics where the conclusion might be both true and false. That's usually where I think of myself as aiming. I nearly always start an intro logic with an informal characterisation in terms of the truth of the premises guaranteeing the truth of the conclusion. But as a matter of principle if the textbook has used a particular definition, I tell the students that if they use that definition, I'll count it as right. So, if the textbook says this and I ask them in the exam 'what's the definition of validity?' and they give the textbook's definition they'll get full marks. So sometimes, because it's in the textbooks that I'm using, I find myself saying it's impossible for all the premises to be true and the conclusion false. Even though I don't strictly believe that as a characterisation because it doesn't work for say indexical logics and it assumes that what you're

quantifying over is possible worlds rather than different ways the language could be so they're thoughts and assumptions that I don't hold. So, we're back to that: It's OK to fib to beginners to get them to see the bigger picture to help them appreciate some of the important patterns. And then I characterise validity in terms of truth tables – on any row of the truth table where all the premises are true; the conclusion is true as well. What changes is what I'm quantifying over: It starts with possible situations, and then it moves the rows of the truth table, and then it moves to models. So, the models get more sophisticated as you go on; and that's reasonable you're not going to start intro logic with a definition of a model.

104 **Bessie:** No. That's... I mean it's hard enough getting to a definition of validity that students can cope with...

105 **Gillian:** and distinguish from soundness. The most important thing on day one is distinguish between something being a vaguely satisfactory argument and being valid.

106 **Bessie:** Yes. And in that logic course what do you hope students come out with? Walk away with as the core thing they've learned?

107 **Gillian:** Ha. I want a mixture of things. I think it's normal that some students get more of one kind of thing than another and they'll be more talented in some areas than others. One of the things that I want is development of certain skills. That means like the ability to translate between English and some formal language. The ability to fill a truth table accurately and then read off various logical properties like being a tautology or being unsatisfiable – off that truth table. The ability to give the truth causes for the quantifiers or conjunction or other logical expressions. Proof abilities – in my course we use the natural deduction formal system, but I want intro students to learn some form of proof system. It isn't super important that they learn any particular proof system. But some of the things... I think this goes for education in general, some of the things you get out of education are proof of concept, that something is possible that something can be done well so that even when you forget the details later you know that there's something there that can be appealed to, or worked on, or you could go and do something more carefully, or get somebody else to do something more carefully, later on. It's almost like you develop this kind of respect for expertise by getting some of it yourself and then practising it and seeing how it can be better and how you can go wrong. So that, later on, you know that there's a good way to do a statistical study and a bad way and that it's important to get it right. So, some of it is just knowing that there is such a thing as a proof system. It helps you understand what it means to say something has a formal proof if you have spent some time with a formal proof system learning to produce these proofs, and learning what counts. And then some conceptual stuff. About what a logical consequence is, how it's related to provability, that they are not quite the same thing. The idea that not everything can be expressed in these formal languages and what kind of things get left out. Yeah so, some richer big picture stuff.

108 **Gillian:** On my intro logic exams I nearly always have one question at the end of the exam that presents an argument in English and then says, 'use one of the methods we studied in this course to say whether this document's valid or not'. It's a much more open-ended question than most of my questions on the exam. One thing you could do is to translate it into a formal language, but you could do that in ways that make the rest of it easier or harder. And then you could give a natural deduction proof, or you could draw truth table, or you could argue informally that the argument has to be valid. One thing I've found is some people are good at bringing all these things together; getting the right answer and using the tools we've learned in the course to do it. It's a non-trivial skill. Some people who are very good at giving natural deduction proofs are not that good at taking the English argument and applying the methods they learned when it's not just 'prove this conclusion from these premises in the formal language'. I'm looking for a variety of skills, even some memorisation stuff like learning some definitions.

109 **Bessie:** Yeah, well if you don't have like some of that foundational, 'just able to recall that *this* means *that* ' then it can be very hard to progress to the next step – to perform the right kind of communication. I mean test-taking is a communicative activity where you ask them questions, and you get their responses. So, if their definitions are not the same as yours then that communication breaks down.

110 **Gillian:** It's true, but it's amazing how many people's definitions are way off, and they can still get the right answer. They know how to do it. They literally can't describe it accurately. They're totally different things. It's amazing.

111 **Bessie:** Yeah. So, what are some of the things that you would not like people to walk away with as impressions about logic from a course?

112 **Gillian:** Gosh. I don't know that people usually think this, but I'm trying to think of things that I would hate my students to end up thinking... I once had a student who was teaching one-on-one. They were my class, but I think they missed a couple of classes. They were in office hours, and I was explaining something; they were very intelligent and confident– this was not somebody that suffered from self-esteem issues. I was trying to make them focus on the definitions of soundness and validity; to learn them and see how they applied in particular cases, and they wanted to talk about Hegel. They'd been reading Hegel on logic, but that didn't really relate to the thing they needed to learn for the class. And once they realised that I was trying to get them to learn this boring thing, which was just these definitions and how to apply them then they responded with something that really frustrated me, which was, 'Oh yeah, I guess it's fun to do semantics sometimes.' That suggested that logic was just about semantics, in the derogatory sense of semantics, as if we just *define* it this way... What I wanted them to see is that there are these properties out there: validity and soundness, and you need to be able to recognise when they appear in particular objects – these arguments. But it was as though

they thought I had said, 'I'm going to call these five arguments valid, and these five arguments sound, and you better learn that.' As if it was all down to how I was using the words. I don't them want to think we're just making it up. I think logic is about the world; and to some extent, the linguistic world, and I want them to see that.

113 **Gillian:** The other thing I would really hate if my students ended up thinking is... I wrote a chapter for a book recently. The book is called *Philosophy for Girls,* and it is aimed at young women from the ages of 16 to 20. It's supposed to be accessible, but especially something that might be interesting to young women. And so, what I wrote about is whether or not there's any such thing as feminist logic. And because I was writing about this topic, I started to look around at some views that were out there that people had about feminist logic, and I came across this book by Andrea Nye I think, *Words of Power*. Do you know about it?

114 **Bessie:** I know of Andrea Nye.

115 **Gillian:** Maybe you know more about it than I do. But she has this view on which logic is anti-feminist and in fact is this tool developed by men to exclude women and keep them out and keep them from power. So even without the gendered aspect right there's this possible view you could have of logic that it's somehow elitist as if it's 'you better learn these special rules; we're going to use them to see who gets into the right club'. And you could find yourself sitting in a logic class thinking 'I don't get these rules, I find them hard to remember, I'm never going to be in the club'. I don't want that. That would be a sign that I was not reaching my students.

116 **Gillian:** I want them to have the sense of discovery, that they're seeing into things and understanding things. I want them to have those sort of 'aha' moments where it's not like I listed a bunch of facts and now they know the facts they have to learn, it's that they've come to understand something and see something through being in the class; to own it for themselves. So I guess this is relevant to the teaching the people who are actually in your class, to find some way to reach them and get them interested and teach a class from which students you actually have can walk away with something special, something new.

117 **Bessie:** Allowing students to discover an experience for themselves rather than being instructed.

118 **Gillian:** And if you pitch your class too high, or too formal, or assume a background your students don't have, well that's it, right? If you're teaching it as if they're all computer science students when they're all philosophy students, or maybe you're teaching it as if they've all got strong backgrounds in set theory and they haven't. So, they don't understand what's going on; they don't have certain bits of vocabulary, or certain proof techniques that you're assuming. They're going to end up feeling that it is some sort of magic, and they don't have it – they're not wizards. And so, they're excluded by it.

119 **Gillian:** And so, I would consider myself to have failed if my students left with the feeling

that there's nothing there for them. Even if they don't want to go on with logic, I want them to have got something out of that class.

120 **Bessie:** Yeah.

121 **Bessie:** One of the things that worries me as an impression a student might walk away with from a logic class is a belief that they have learned the definition of validity.

122 **Gillian:** Oh okay.

123 **Bessie:** I believe it's such an open question, and yet they will be given a definition of validity – and they really must be given a definition – but I want them to understand that it's one of many options.

124 **Gillian:** Yeah. I even think that lots of philosophers think they did learn the definition of validity in logic and it was, 'it's impossible for the premises to be true and the conclusion false' and they've memorised that.

125 **Bessie:** Yeah, that's certainly my feeling, that the common belief is that there is one and only one definition of validity – it's that one you were introduced to in classical logic.

126 **Gillian:** On day one.

127 **Bessie:** Yeah. I feel like that's becoming less and less relevant as the general pluralistic movement picks up steam. I don't think there's a widely adopted commitment to a strong form of pluralism. But as far as I understand only Graham Priest is strongly advocating for monism.

128 **Gillian:** I think monism's been the default assumption through much of logical history.

129 **Gillian:** And then recently there's been this pluralist movement, and some people have responded to it. But I think there are plenty of monists out there who haven't bothered to articulate their views. They're just not convinced by the pluralist arguments. Tim Williamson seems like somebody I would be inclined to say is a monist. I don't know whether he'd endorse that. But yeah, I think he thinks classical logic – higher-order modal logic – is the One True Logic and it's a matter of getting the right one.

130 **Bessie:** My impression of Tim Williamson's work is that he's taken a very metaphysical approach and he's not interested in a linguistic component at all. He doesn't think that what we're doing in logic is exploring different interpretations the expressions could have. He is interested in the metaphysics of *the* world that we're in and that implies monism. That's my understanding of the position that he's coming from. Is that the same as yours?

131 **Gillian:** Yeah. That seems about right. I mean he does say, I think, that it's not the only possible conception of logic.

132 **Bessie:** But that certainly seems to be the project that he's interested in. There are many things that you can do using formal mathematical methods, so it doesn't mean that there's exclusively one project.

133 **Gillian:** There are boring forms of pluralism, or trivial forms of pluralism, that I take it

everybody is going to endorse. But when it comes down to multiple non-compatible characterisations of logical consequence that are correct. Not everybody thinks that.

134 **Bessie:** Well thank you, this has been an amazingly satisfying and interesting conversation.

135 **Gillian:** Yeah. I really enjoyed it. Good luck with the project.

136 **Bessie:** Thank you so much. Goodbye.

A CONVERSATION WITH DAVE RIPLEY

1 **Bessie:** So, I normally start with a few background questions.

2 **Dave:** Sure. Lead away.

3 **Bessie:** So where did you study philosophy? Well, how did you get into philosophy first?

4 **Dave:** I started my undergrad as a music major. I was a music theory major and I took a philosophy class first or second semester of undergrad. One of these breadth requirements, the US is full of breadth requirements. I took a philosophy class and I absolutely hated it. And I said, I want nothing to do with philosophy ever again. It was all scepticism. Is there really a table here? How do you know? It was that kind of thing. I said, okay, I see what philosophy is and I want neither hide nor hair of it. And I went back to being a music major. And then I didn't learn to play the piano very well, so they threw me out of being a music major. They said, 'you can't be a major anymore because you don't know how to play the piano. We've given you years, but you still don't know how to play the piano. So, you're out'. Then I had to find something else to do. So, I went to study poetry and I became a literature major and that's where I spent the bulk of my undergrad, as a literature major studying poetry. But then I ran into the same problem. Just like I knew I had to learn to play piano to stay a music major, but I just didn't do it because I didn't want to. Similarly, I knew that I had to read a lot of novels to be a literature major, but I didn't want to read any novels. I wanted to read poetry. So, I just didn't read the novels. So, they threw me out. So, I had to figure out what I was going to do. I'd spent years as an undergrad, and I needed a degree to show for it. And so, I thought, OK, well, I studied poetry, and I studied the history of the English language and found that very fascinating, I had also done some mechanics in the English department and I thought that was good. So, I thought I'd try to make this individualised major about the mechanics of language or something. But when I went and talked to the individualised major office, they said, 'you know about linguistics, right?' I said, 'no, I never heard of linguistics. What's that?' So, I became a linguistics major. Then, because of the institution I was in which had only a small linguistics department, we were required to supplement linguistics either with philosophy or with psychology. And because I was on the more theoretical side and the less experimental side, I supplemented it with philosophy. That was the path back to philosophy, it was via formal semantics, through doing formal modelling of the English language. And then by the time I went to graduate school I knew that I wanted to pursue that kind of logical approach, so I went to grad school in philosophy. But my undergrad degree is not in philosophy. It was logic that brought me to philosophy rather than the other way around.

5 **Bessie:** That's an interesting pathway in. So, I suppose by the time you came back to philo-

sophy, you were actually pursuing subjects that were a lot more interesting to you as opposed to the initial stuff that was so dull.

6 **Dave:** Yeah. There was there was a semester late my undergrad which was the semester where I thought that I could get into this philosophy thing. It was two different courses that I took that semester at the same time, one right after another in the same room. My Thursday afternoon was Logical Approaches to Paradoxes, With JC Beall followed by feminist theory with Diana Meyers. And it was those two courses that showed me that there was something in philosophy that I valued. Whereas the metaphysical, 'does a table exist?' I still don't care. I didn't care then; I don't care now. Not interested. But logic and feminist theory both were addressing the sorts of things that I cared about. The sorts of things that I was interested in.

7 **Bessie:** And I suppose that interest in language is something that stays with you and that's part of why you're interested in logic?

8 **Dave:** Yes. And in trying to get employed, I've moved through various institutions, been in different environments, and exposed to different ideas. And I am someone who takes on a lot from my environment, I think more or less everyone is, but certainly I am. My first postdoc was in a cognitive science department where I was doing much more empirical work, working with linguists trying to build formal models that directly model some aspects of the way we talk, and building models for formal pragmatics in some ways. It was quite empirically engaged stuff. My later postdocs were at Melbourne Uni, where the connection between the logical community and linguistics is more tenuous. So, during that time I started pursuing more purely formal work and more work that connected to philosophy of language more directly than it connected to linguistics. Whereas earlier I'd been doing work that was more connected to linguistics than it would be the philosophy of language. And I think that's because of the kind of philosophical and logical environments I found myself in. It's been some years now since I've run any experiments. I think that's because I like talking to people and I like working with people so I work on things that the people around me are working on, because that's what I find the most satisfying.

9 **Bessie:** So, riffing off that influence of environment, let's talk about where you have been. Which university did you start your undergraduate?

10 **Dave:** I studied my undergraduate at University of Connecticut. I'm from the state of Connecticut, so I went to the state school. From there, I went to do my Master's and PhD at university in North Carolina at Chapel Hill. There was very little logic going on there. I went there in part because I didn't get into a lot of places I applied to, and in part because I thought that it would be a good opportunity to learn a lot about different areas of philosophy, to broaden my horizons, because I didn't have an undergraduate background in philosophy. I thought, well, what if I turned out to be excited by some other area of philosophy? I should

go to a place that offers me a range of different topics. And as a result, I went to a place where there was there was not a lot of logic going on. And so, a lot of the logic I did there, I did on my own. My supervisor, Keith Simmons, was a logician, still is a logician. He was a wonderful supervisor. But in terms of logic in the department, it was basically him and me. Then I did a postdoc at Institut Jean Nicod, which is in Paris. It's a cognitive science institute largely. There's a lot of philosophy there, but it's at institution where the philosophy department is devoted to French philosophy. In that philosophy department, there's not much being done in English, and there's not much that's engaging with the kind of philosophy that I had a background in. The philosophy that I have a background in is instead at the Institut, which is in the cognitive science department. (Institut Jean Nicod has a complicated institutional position that isn't worth going into, but to a first approximation, it's inside the cognitive science department there). I spent a year there. Then I did three years of postdocs at University of Melbourne with Greg Restall and Graham Priest. Then I went back to the University of Connecticut. Suzy and I solved the two-body problem there[1]. We got employed at the University of Connecticut where I had done my undergrad. We worked there for, I think, four years. And now we're here at Monash.

11 **Bessie:** Lovely, thank you for that.

12 **Bessie:** Can you tell me what your first logic course was?

13 **Dave:** Yeah, to some extent, it's a long time ago.

14 **Dave:** It was taught by Scott Lehmann, who's now emeritus at the University of Connecticut. I remember relatively little of the content. I got into it because I had a friend (a very good friend of mine who later became my best man) who took the class, and said, 'you've got to take this class, it's super easy and the teacher's fun'. And I thought, super easy class and the teacher's fun? That sounds like a good one, I'll sign up. He was right. It was an easy class and the teacher was fun. Scott taught it. He would start every class by reading a poem. He would bring a poem in every class meeting.

15 **Bessie:** Well that would be a great way to connect with you.

16 **Dave:** It was. For me, it was wonderful. I knew it was technically a philosophy class, but Scott brought poetry into the room, so I felt like it was my kind of space, I felt at home. He taught it from his own notes; as everyone does. I imagine it was probably a pretty standard classical propositional and classical predicate class, but really, I don't remember the content at all. And that wasn't what pulled me into logic. This was before I came back around to philosophy the long way. This was just filling up credits because my friend recommended the course. I did

1 The two-body problem is a dilemma for life partners in academia, relating to the difficulty of both partners obtaining jobs at the same university or within a reasonable commuting distance from each other

enjoy it, and I did well in it. But that wasn't what pulled me in. It was just something that gave me a bit of background for later when I did get pulled in; I had something to draw on.

17 **Bessie:** So, he taught it from his own notes?

18 **Dave:** Yeah. That's standard in my experience in logic. Maybe less so...

19 **Bessie:** It's mixed. There's plenty of people out there using textbooks. But then, I was taught from course notes rather than a specific textbook.

20 **Dave:** I think a lot of textbooks, not counting the big glossy ones, but the ones that are written by logicians, have their origin in these kinds of notes. You know, they're notes that someone's been using for a decade and then they think, well, why not publish?

21 **Bessie:** Well, the first modern logic textbook, Hilbert and Ackerman's textbook published in nineteen twenty-eight, was a compilation of notes over the decades of teaching. I was very much drawn from that source. And I think there are quite a few textbooks that are developed out of the experience of teaching and then turned into textbooks. And in many, the motivation for writing textbooks, often articulated in the preface, comes from some dissatisfaction with other textbooks.

22 **Dave:** And finally doing it right after all the years of everyone else screwing it up, I'll be the one who sets it right. I think that's an impulse a lot of people have shared.

23 **Bessie:** Yeah, I think so.

24 **Bessie:** So, you probably won't then recall exactly what Scott said about logic. Whether he gave a broad characterisation of it, like a sentence beginning, 'Logic is about...'

25 **Dave:** No, I wouldn't have the foggiest idea what he said about that. And that wouldn't have been something that particularly fascinated me or drew me in. And I definitely don't remember now. I probably did that course in two thousand or two thousand one, so it's almost 20 years ago. I could hardly tell you what I had for breakfast this morning.

26 **Bessie:** Yeah. So likewise, for that class, did you form much of an impression about what logic was about?

27 **Dave:** I don't remember it well enough to tell you.

28 **Dave:** I have a better memory of the class I did with JC about paradoxes, which was where I started to really get pulled into studying logic. I have some impression of what I thought at the time about that class. But the prior class is a dim memory. I did later do a Gödel's incompleteness theorems class, also with Scott, and I have a better memory of that as well. But my memories of that first class are lost to the sands of time.

29 **Bessie:** Well, let's talk about the class with JC. Do you remember if he was using a book for that class?

30 **Dave:** It was Sainsbury's book; Paradoxes[2] The second edition. It's now in its third edition,

but we were using the second. It's a slim book, it's got like five pages on the Sorites and five pages on the Ravens and five pages on the Liar... That sort of thing. I think it was the textbook for the entire semester. We went through a very slim book very, very slowly.

31 **Bessie:** Well, that makes sense. You said you remember the impression of logic that you formed at that time, as in what you thought logic was about?

32 **Dave:** Yeah. And I think it was probably a pretty strongly shaped by the formal semantics that I was also doing at the time in the linguistics department.

33 **Dave:** Vagueness and the liar paradox grabbed me very early on, they are the two paradoxes that I've done the most work on since. They both seem to me to have the following feature: they arise in our natural language. We have a sentence that says of itself that it is false, not because of any complicated arithmetic construction, but because there is this string of English words: 'this sentence is false'. I had to study Gödelian and Tarskian constructions and so on to learn about using and applying formal tools. But the fundamental phenomenon that drew me in was in natural language. We have these puzzling sentences and our best theories of natural language meaning, or at least what I took, and take, to be our best series of natural language meaning, struggle to understand how these sentences can mean what they obviously do mean. This sentence says of itself that it's false. It just does, that's obvious. And yet it creates trouble for understanding how that can be so. Similarly, for vagueness; if you open any formal semantics textbook, it's two valued (with maybe a third value for presupposition failure) ,and so, is maybe implicitly committed, I take it, to epistemicism or contextualism about vagueness. But I just don't take epistemicism or contextualism about vagueness to be remotely plausible. And so, this question arises. It might not be something which matters to linguists who can be satisfied with the approximation of a two valued semantics. And fair enough, they've got other things to do. But, if you get concerned about how we have these fuzzy boundaries or lack of boundaries that characterise vagueness, then you start to see that the formal models we have don't sit well with that. So, can we build better models? How can we develop the formal tools that enable us to understand our own language better? That, for me was the initial motivation that drew me in. And it's in many ways still how I conceive of what's going on. I become less and less empirical as the years go on. But what motivates me is that I want to understand how human language and human communication works. The paradoxes don't take logic to state. You can see the problem with vagueness before you ever encounter a propositional language. The problems are not, in the first instance, logical problems. It's rather that they are problems for understanding our language. And then logic helps systematise and coordinate attempts to answer these problems.

34 **Bessie:** Right. That's very fascinating.

2 Richard Mark Sainsbury, *Paradoxes*, 2nd ed. (Cambridge: Cambridge University Press, 1999).

35 **Bessie:** So, I think my next question is maybe going to throw us off this line of development. But I'll come back to it in a minute. You said to me in conversation before that you thought logic was about proof, am I recalling that correctly?

36 **Dave:** I think there's an important sense of which I don't really think logic is about anything. But as I teach it, I do teach it as centrally about proof.

37 **Dave:** For the kinds of logic we typically teach to beginners in logic; propositional languages, maybe first order languages, whether classical or non-classical. The relationship between those things and natural languages (English and so on) is indirect. The kinds of formal systems that I really cut my teeth on; formal semantic systems, don't look anything like propositional or first order logic. Nowadays I can see a sense in which they do look alike, but it took me a long time to see that sense. If you look at the formal systems that you would see in like the Heim and Kratzer textbook,[3] or various sorts of type theoretic formalisations of natural language and categorical grammars, there's just a different shape between those and the kinds of things we often teach in logic class, at least in philosophy. When I look at the kinds of logic that we tend to introduce people to in philosophy, the propositional languages classical or non-classical. I think, well, what is this language immediately good for? What is what's its real use? Many logic textbooks say that the purpose is to evaluate natural language arguments and see whether they're any good. But I don't think that it's all that useful for that purpose. I don't think what makes natural language arguments good, when they're good, can be explained by a theory of classical or even non-classical validity. So, when I think about the purpose of these sorts of logics I think about where they came from. Where did we get them in the first place? We got it out of the Hilbertian and pre-Hilbertian moment where people were trying to formalise the notion of proof. We had the practice of mathematical proof starting to reflect on itself. And there is a reason they ended up at something like classical logic. It's not a coincidence; they didn't do a bad job of it. It's a good answer to a question in that area. And so, when I'm introducing students to logic, I usually start there. So that I can teach them very simple logic, so that I don't have to like launch straight into real grammars and so on. Although that is how I was introduced, I have found it difficult to start students there. Whereas starting by introducing atomic sentences and some connectives is much more manageable. But then I want to say, well, what is this good for? And I think the story that we're evaluating whether philosophical arguments are any good is not a story that I find plausible. It's not a story that I could sell well, because I just wouldn't believe it. But that's more pedagogical. If I had to say logic was about anything, I would probably say proof. But first I would try to resist saying that logic was about anything.

38 **Bessie:** That makes sense to me, in two ways. First, I agree with you that logic isn't about

3 Irene Heim and Angelika Kratzer, *Semantics in Generative Grammar* (Malden, Mass: Blackwell, 1998).

saying whether a philosophical argument is good or bad. Secondly, the pedagogical approach of not teaching what you don't agree with.

39 **Dave:** *laughs*

40 **Dave:** Yeah. I mean, you know I teach students lots of things that I don't agree with. I teach them classical logic.

41 **Dave:** But for the first course in logic, it's just hard to see what logic is... In the sense of, what I do when I'm being a logician. If my students were to walk into my office and see me doing logic, they would just see me pacing furiously and writing strange things on the whiteboard and occasionally coming over the computer to type something or read something and then go back to pacing and writing strange things on the whiteboard. If they were to just watch me do logic, they wouldn't know what it is that I'm doing. I want to treat an introduction to logic as an introduction to what it means to do logic. Well that's too much to do in one unit. So instead, I try to show them what it is to do logic the way I do logic. To invite them in to a way to do logic. I want them to see the thing that I love doing. And that's my main goal. And the thing I love doing is not taking sloppy philosophical arguments and then trying to regiment them in some language that fits them very awkwardly and then declaring, this one's good, this one's bad. I don't do that, and I don't want to do that. If I did that in class, I wouldn't be showing them the thing that fascinates me about logic, I wouldn't be showing them any of the things that fascinate me about logic.

42 **Bessie:** Thinking about teaching, and the logic that you're teaching, which formal system do you introduce first, and why that one?

43 **Dave:** Essentially, it's Gentzen's NJ. If I'm remembering the details of NJ exactly as Gentzen does it, I believe the only difference other than, which symbols I use, is that I add a verum constant to it. But otherwise, I believe it's exactly Gentzen's NJ. Anyway, a simple Gentzen style natural deduction system for intuitionistic logic with introduction and elimination, rules for conjunction, disjunction, implication, negation, verum and falsum. That's the system that I start them with.

44 **Dave:** Why that one? Essentially, I think two broad families of reasons. The first has to do with why I've selected a natural deduction system. The second has to do with why I've selected an intuitionistic system.

45 **Dave:** The proof systems I began with were all tree systems; that was my introduction to logic. So, when I started teaching logic, I started teaching with tree systems. That seemed sensible to me: I like this stuff, I learned it that way, I'll teach it that way. But one thing that I remember struggling with, and then saw my own students struggle with, is seeing the value of proof systems at all. My original education was heavily model theoretic, and I taught it that way. But when I was learning, there was a sense in which I didn't know how to see the trees as a proof

system because I didn't really know what a proof system was. I thought we were just thinking about models in this strange, systematic way. Nowadays, I feel much more comfortable looking at trees as a proof system. Sometimes people say that trees are semantically polluted, or that they're not a real proof system. I think that's all wrong. I'm a strong defender of trees as proof system; they're real proofs in every sense you could ask for. But it took me a long time to get there. I had to learn other natural deduction and sequent systems to be able to see what a proof is on about.

46 **Dave:** The first natural deduction system I ever taught was the awful Copi system. It was in the Hurley textbook, and using that textbook is the worst teaching experience I've ever had; of logic or anything else. That's more about the publisher than it is about the book. But I hate Copi's natural deduction system. But the fact that it's a natural deduction system means that students can't just turn the crank and generate the answer. It's not as simple as 'follow these rules and you'll do it correctly' the way that it is with trees. It demanded a bit of creativity from them and they really responded to it and the students enjoyed it much more than students had enjoyed it when I taught trees. It's like a puzzle solving exercise for them, rather than the rote, 'why isn't a machine doing this for me?', way that trees can sometimes be. So, I abandoned the textbook and that awful proof system, but decided to stick with natural deduction.

47 **Dave:** Now, as to why *intuitionistic* natural deduction. I think of logic as strongly interdisciplinary. You've got philosophers, mathematicians, linguists, computer scientists, even electrical engineers doing logic. Logic is all over the place. And lots of different systems are used in all these different areas. And when you think about what's at the heart of logic, by which I mean what sorts of logics are of the broadest use; of use to the logicians in philosophy departments, but also to logicians in computer science departments and in linguistics departments, and so on. I think intuitionistic propositional logic is a good candidate for being near the heart of what's going on. It's maybe slightly less useful to philosophers and mathematicians than classical logic is, but it compensates for that by being much more useful to linguists and computer scientists. And it's also of notable use to philosophers and to a lesser extent, mathematicians. So, I think intuitionistic logic is a good starting point. It's simple, it's approachable, it's workable, it's valuable to know something about if you go on to study logic from almost any angle. And so that's why intuitionistic logic as the starting place. The natural deduction, that's mostly about the pedagogy. It makes it a little more interesting.

48 **Bessie:** Could you talk about the experience of connecting with students and their experience in the classroom. How do you find that? How do you find the experience of teaching that way? How many of them do you manage to reach?

49 **Dave:** I love teaching logic. I love teaching in general, and I love teaching logic more than I love teaching anything else. In terms of how many students I reach? It's hard to say, right. How

does a student end up enrolled in a logic unit taught under a philosophy code? I mean, logic is part of philosophy, I don't put an asterisk after it when I call myself a philosopher, but there is something that students are expecting from philosophy that they maybe don't see when they walk into my logic classroom. And maybe it's deliberate that it's absent. I'm exposing them to formal systems and way of thinking that is maybe a more usual way of thinking in a mathematics course. It's something they wouldn't see in a lot of their other philosophy courses. And so, I feel that there's this initial bump. It's as if we're all on the bus and we all hit the bump together and that's the point at which the students seem to be thinking, 'wait a minute, I thought this was the philosophy course'. I want to say, 'no, no, it is philosophy, but here's a case of philosophy that you maybe haven't seen before or you maybe haven't seen in this way before'. Flipside of that is, some students will come in who are not philosophy students, but they do have a sense of what logic is – maths students, computer science students. They've talked to the people in their own fields who've recommended learning some logic. And they end up enrolled in a philosophy class wondering, 'what's this going to be about?'

50 **Dave:** The extent to which philosophy that isn't itself logic is in my first course in logic is largely in reflecting on Platonism versus Intuitionism as philosophies of mathematics and connecting these to intuitionistic logic and classical logic. We don't do much philosophy of mathematics. It's handwaving sketches of the worldviews that these logics came out of so they're not just bare, but it's more evocative than it is systematic argumentation. So, students who don't usually take philosophy classes get exposed to a few ideas that way, that maybe they don't expect because they're there for the formalism. But I think often the philosophy students have a much more serious bump. You know, what is it, to think formally? What is it to be asked to prove something in a homework assignment? And what am I then expecting them to hand me? These are things that math students don't particularly struggle with, and philosophy students often do. So, there's that early bump. And not everyone makes it through that bump. For example, last semester I started this semester with 60 enrolled and 15 of them dropped along the way, which is a larger than average number of drops for a philosophy unit. And I think that's just the nature of the beast. It's hard for students to see what logic is before they take the class, there just is going to be that surprise. So my own goals with it anyway are you just get as many people through the door as I can, so that the people who will be interested in doing it can find their way in and remain. Logic is what it is. I love it. I don't think everyone has to love it. I just want to give it its best face. To show it to students in a way that enables them to see something of what there is to love about it, so that if they love it, they can discover it. If they don't love it, hopefully we will have a fun semester and then go our separate ways. But I definitely feel like around week two or three there is that moment where students realize that this is what we're going to be doing all semester and I'm explaining that there will be homework exercises, and there

are correct and incorrect answers. It's is that kind of class, and there are definitely students who are in philosophy because they don't want to be in that kind of class. And they drop out at the point they realise what it is.

51 **Bessie:** It sounds like you've got quite a diverse population of students and that you're drawing from disciplines that aren't necessarily philosophy as well.

52 **Dave:** I've only taught this class at Monash once, so it's a bit early for me to have a firm sense of how it will or won't play out here, but that's certainly my goal. It's what I'm trying to do. And it was the case at the University of Connecticut, which is the place where I really developed the course I'm currently teaching. But when I did it here it drew on that broad base. When I do it again next semester, hopefully a similar thing will happen.

53 **Bessie:** Cool. So, is there something that you could say is your main objective with introductory teaching, perhaps the main thing you want students to walk away with?

54 **Dave:** The way I think about it to myself is that I want them to see what it is that gets me excited about logic. Whether they get excited about it or not. Some of them may come away thinking, 'oh, that's it? Dave's a weirdo.' That's fine. They're not wrong. But I want them to see what gets me excited. The biggest thing I do in service of that, I think, is that there is a lot of practicing really basic careful 'is this a proof? Is that not a proof? Find the error in this proof. Construct a proof of this thing from that thing within these natural deduction systems' or when we get to the models, which I do later in the semester, 'build a counterexample to this thing. Is this thing valid? Is that thing not valid? Make a truth table.' All of this sort of stuff. It's not the point, but you have to do it to be able to see the point. I have to do a lot of those exercises and homework things just to build the right ways of thinking in the students. But every week, have between one and three exercises that are much higher level, seriously conceptual, things that I genuinely find interesting myself, that are approachable to some extent, given what the students know. At least they know enough that they ought to be able to understand what I'm asking them to do in the exercise; and then they learn something, hopefully, by doing it. So, for example when we do the natural deduction, we don't just build the proofs, but we also talk about normalisation and reduction relations and reducing proofs. So, when you introduce a connective and immediately eliminate that connective, how you can rearrange the proof to avoid the round-about – the detour. So, one of the earliest exercises I give them is in implication week, which is week three. The first week is 'here's the language.' The second week is, 'here's conjunction' And so in week three of the first course in logic I say, 'Suppose we use a different language. Suppose we used a language where we were allowed to have a conditional that was its own antecedent.' So, we have a sentence that is a conditional sentence, but it is its own antecedent. 'Now look at the following proof...' And this is basically Curry's paradox. But there's not a truth predicate, it's a weird language. Rather than introducing the truth predicates,

because we don't know predicates yet, all we have are propositions and connectives. So now I give them a proof of the consequent of that conditional. With no open assumptions. Which you do, just because it's Curry's paradox. And I hand them that proof. I say, 'note that this proof is not in normal form. Reduce the proof until you see what happens. And give all the proofs you arrive at along the way and tell me what happens.' And what happens is as you reduced the proof, it just goes into a cycle. And in fact, the proof I give them reduces to itself in a single step. So, they reduced the proof. They get that proof itself back. And of course, it's ready for reduction again. So, they reduce it and get it back. And that's that. And this is something that I find endlessly fascinating. The topic of these looping reductions is something that I am doing research on, and it's something that I think is genuinely exciting about what we're doing here. But I give it to them in week three of a first course in logic. That way they can see something of it. They don't yet know about interpreting reducing proofs as executing a computation, there's lots of perspective on this they don't have. They don't know about connections to truth because I haven't said anything about truth. I just said, suppose there's this weird sentence, but they can see the machinery of it already. And I try to fill the course with exercises like that. That give students an opportunity to see something that I take to be genuinely interesting and not just an exercise to build capacity so that five years from now they can do something interesting. It's not that interesting in week 2, but in week three, we start to get to the real interesting stuff, and I try to sustain that throughout. And that's probably driven by this goal I have of getting them to see what it is that interests me. Which will not be obvious to them as they walk through the door and they just see this weird guy who is very happy to be doing logic, but what's he doing?

55 **Bessie:** You've said a number of things that I think are interesting in the context of the amount of textbooks that I've reviewed because I haven't seen any textbooks that take an intuitionistic approach. They also tend to have a model theoretic heavy start. Where they're introducing model theoretic meaning concepts: truth tables. And they usually do that well before they get to any kind of proof system.

56 **Dave:** I agree. I think that's very common. Before I sarcastically said you've got all these logicians who think everyone else has done it wrong, so they're going to do it right. But I am, of course, one of those logicians that I'm sarcastically slagging off. Yeah, I don't know of a published textbook that takes this order at all. I'm doing this all from my own notes.

57 **Bessie:** Well, it sounds like a very exciting and novel approach and I wish I was taking that class; it sounds good.

58 **Dave:** Well I hope it is. I can see that some students really connect with it and really get excited about it and really like it. And that's good. The thing I want to do is make it as accessible as I can. I try to maintain that interest but make that interest available to a large number of

people.

59 **Bessie:** Here's a question that takes us back to where we started. What do you think the relationship between the logic and natural language is?

60 **Dave:** To the extent that I have any idea what logic is, I suppose I think about it as a weirdish corner of mathematics. We've got these formal structures, they bear some family resemblance to each other. You know, you can say what a group is or what a ring is or something very precisely. I don't know that there's much that's useful to say at that level of precision about what a logic is. You might offer some precise understanding for some particular purpose, but it's more of a family resemblance thing, I think. And these things, these formal things, they are just whatever formal things are, man. I'm no philosopher of mathematics. I'm no philosopher of logic. I don't really have a dog in that race, but whatever, whatever rings and groups and numbers and sets are logics are probably things like that.

61 **Dave:** Now, in the study of any particular real world phenomena out there, you might find that you have use of various mathematical structures, and often, because those mathematical structures are instantiating structures that have something to do with the real world structures that you're looking at and studying. And I think so it goes with logics. In studying natural language there are these sorts of structures that look a lot like some of the structures that logicians study as pure formalisms. So, I think about it is as just another instance of the usefulness of mathematics and natural science. There are all kinds of mathematical structures. I don't know what they are, but they're sure useful to think about if you're thinking about the real world. I'm not saying they're not part of the real world. I suspect they are part of the real world. That's probably why they're useful to think about. I can see how many objects are on the left side of my desk. I can see how many objects there are on the right side of my desk. I want to know how many objects there are on my desk. There's this handy thing addition I can do. I think of it basically like that.

62 **Bessie:** Yeah. That makes me think of the concept of modelling something, constructing a mathematical model, broadly speaking, of some situation.

63 **Dave:** Absolutely. And I find that a very congenial way to go when I'm talking to extended family who wonder what I do. I go straight to the modelling explanation, that's my aeroplane conversation explanation. That's my, I don't actually want to get into the details, but I want to give someone a picture of the broad area. Building mathematical models of language, that's absolutely what I go to. I don't do it to end the conversation, I do it because I actually think that is the right way to see what I'm doing, from a very, very high level before you get to any of the details.

64 **Bessie:** One of the things that I don't understand very well is the details of proof theory. How would a proof theorist see that idea of modelling a language? Is that still compatible with

proof theory?

65 **Dave:** Yeah, it's just a coincidence that it's the same word. The word model in talking about modelling a language and the word model in the sense of like we've got models and proofs, and these are different tools we use in logic are just totally different senses. There's a number of different ways that you could apply proof theoretic tools to studying language. There's a whole project that travels under the name proof theoretic semantics, which has a Prawitz influenced, Schroeder-Heister influenced, Dummett influenced way of thinking about language, where often the proof theoretic structures they have in mind really are these natural deduction proofs, and their reduction relations and these are meant to teach us something important about how meaning in natural language operates. These are often theories of natural language that base meaning not around truth conditions, but around conditions under which something is warranted, or conditions under which we have evidence for something, or something as verified, or something like that, or assertion conditions, but the conditions are conditions under which an assertion is warranted. Things of that nature. And there are these notions of canonical warrant that some of those folks think are useful in actually studying... If you think semantics and epistemology are closely tied together, then there is a proof theoretic semantics-y tradition that uses a lot of the tools of proof theory to explore natural language semantics.

66 **Dave:** A lot of that stuff is not heavily empirical, but some of it is. Here's an example: this really cool paper 'Proof-theoretic reconstruction of generalized quantifiers' by Francez and Ben-Avi from a few years back. It's in the Journal of Semantics.[4] There's a property that quantifiers can have or lack called conservativity. If we use Q as a variable for quantifiers. So, Q could be, 'all', 'many', 'most', 'few', 'none', 'some', and so on. Across all natural languages, Q [A's are B's] is equivalent to Q [A's] are A and B. Right. So 'all cats are red' is equivalent to 'all cats are red cats'. 'No cats are red' is equivalent to 'no cats are red cats'. 'Few cats are red' is equivalent to 'few cats are red cats'. There are lots of other inferential properties that 'all', 'no', and 'few' differ from each other very strongly, but this one seems to be something of a semantic universal across a very wide range of languages, if not all languages. But it's trivial to define a quantifier that doesn't obey it; there's no conceptual problem for that sort of quantifier. It's just somehow the world's languages don't seem to realise quantifiers that are exceptions to this rule. And there was this interesting article a few years back, as I said, by Francez and Ben-Avi, that was able to derive this constraint from a proof theoretic treatment of what quantifies are in the first place. So, it's something with some real empirical bite. It's hard to get evidence that bites at deep theoretic frameworks. But if model theoretic frameworks simply propose this constraint and proof theoretic ones can derive it, well, now there's some interesting empirical bite. So,

4 Nissim Francez and Gilad Ben-Avi, 'Proof-theoretic Reconstruction of Generalized Quantifiers', *Journal of Semantics* 32, no. 3 (August 2015): 313–371.

there is that whole tradition in natural language semantics. I don't really work in that tradition myself, but I am an adjacent fellow traveller. I'm interested. I work downstream from what Greg Restall does, where rather than using the proof theoretic tools of natural deduction, it's more using the proof theoretic tools of the sequent calculus. And rather than thinking about epistemology – warrant – it's more thinking about coherence and the conditions under which collections of speech acts (assertions and denials) do or don't fit together. Does this collection clash with itself? Is that collection coherent? Modelling those sorts of relations, the structures that you're driven to immediately, at least to me, look a lot like sequent proofs. That's the area that I work in. But both of these, either the mainstream proof theoretic semantics tradition or the Restall influenced sequent coherency stuff that I do, give you good pictures where the formal tools you're using are the tools of proof theory, but you're using them to give models of various natural language phenomena, often around speech acts. So, both will be about assertion and maybe denial.

67 **Bessie:** That's a very interesting discussion which I think raises the question about normativity and how normativity relates to those different areas. Would they be inclined to say that logic is normative, given that approach?

68 **Dave:** I work within that latter approach, and to the extent that I understand the question, whether logic is normative, which is not a huge extent, I doubt that it is. I think semantics is normative, I think natural language meaning is normative, and I want to give theories of these norms and the formal tools I'm going to use to give theories of these norms are the formal tools that I'm learning from logicians, and taking from logicians, and developing as a logician. But the logics themselves are whatever abstracta are. They're as normative as the number five is. Five is the correct number, if I'm engaged in the process of counting the fingers on my left hand. But that's not for the number five to be normative. I was engaged in a project that brought with it its own norms, and the number five answered to those norms in a way that other things didn't. That's how I think of it. So, I don't think there's inherent to these projects of using the tools of proof theory to explore natural language, any commitment to logic itself being normative. Although I'm sure some people in these projects do think of logic itself as normative. I don't, though I am engaged in a project along those lines.

69 **Bessie:** So, I have another tricky question for you.

70 **Dave:** Uh Oh.

71 **Bessie:** Definition of validity.

72 **Dave:** Oh, wow. Whatever. Who cares? That's not a tricky question.

laughs.

73 **Dave:** If I'm reading someone's book or paper and they start using the word validity, it's on them to tell me what they mean. And if I start using the word, it's on me to explain what

I mean in that context. Often in the stuff that I read, and the stuff that I write, validity is used with reference to some implicit formal system. So given the context, valid will mean classically valid, or intuitionistically valid, or LP valid, or valid in TW, or whatever. Validity just in some formal system where that's fixed by context. As I said earlier, I'm not much of a philosopher of logic, and one of the central reasons that I'm not much of philosopher of logic is I really don't see what they're arguing about most of the time. I take it, among the things they're arguing about is this exact question of, not validity of this formal system or that formal system, but validity as such, what is it?

74 **Bessie:** And to you there may not be such a thing, and if there is, you don't care? What's important to you is the definition of validity in the specific formal system that you're dealing with. So, it's not so much the definition of validity as the many definitions of validity.

75 **Dave:** Yeah. I might even go stronger. I strongly suspect that there is nothing there. Officially, I don't care whether there is anything there or not. If there is something there, I never talk about it so, it's fine. But there's a reason I'm never talking about it; that's because I really think there's nothing there.

76 **Bessie:** Given that approach to the definition of validity, presumably you still use a definition of validity. When you introduce your students to that, how specific are you about saying that this is the definition of validity in this formal system and that there are other definitions of validity you could have?

77 **Dave:** Very careful. The point at which I introduce validity is about halfway through the semester. We get natural deduction for intuitionistic logic on the table. The next thing we do is truth tables for classical logic. And so right at this point, we have two different ways of determining validity. One, by the existence of a proof, and the other by the non-existence of a countermodel. And two different extensions for the validity predicate. When we're doing intuitionistic proofs there's not even a need for the word validity and I don't bother teaching it. It's a word I introduce precisely when we have two systems to compare. So, I can talk about things being intuitionistically valid or classically valid. And that's the first time that they meet the word validity in the logic unit I run. They meet it as qualified by intuitionistically or classically, so that we can have something to compare. Then we can look at a particular argument and say this is classically valid, but not intuitionistically valid. And then we can look around and ask, 'are there any arguments that are intuitionistically valid but not classically valid?' And see no, there aren't any like that. These kinds of questions are the kinds of questions that we need something like the word validity in order to ask in the first place. And these are questions that I want to ask. They are the questions about comparison of different logical systems, and questions about different ways of determining validity, whether proof theoretic or model theoretic. There's a lot in a first course of logic that I would love to tell them, but I feel like I have

to leave to one side and there's the pedagogical lies and the pedagogical oversimplifications and all of that. But this is one I try to be reasonably careful about.

78 **Bessie:** Also, your approach gives you time to ease them in, so that by the time you introduce validity, they're more likely to understand what you're talking about, have a bit more context for it. Whereas in many other courses it's the first thing that's introduced.

79 **Dave:** And I find that validity as the existence of a proof is conceptually so much easier for them to grapple with than absence of counterexample. I ran into this myself teaching logic a lot, and I've talked to a number of other people who have run into this teaching logic, I'm sure you've seen this too. Where over and over again, you run into confusion of the sort that this this has to be valid because the premises are true, or this has to be invalid because the premises are false. Just the conceptual articulation of a truth table definition of validity is conceptually quite difficult. This idea of there being no way for this specific combination of T's and F's to come up... It's difficult to explain, right?

80 **Bessie:** Yes.

81 **Dave:** I find it much easier to just show them what a proof is, and say, 'it's valid if you got one of those.' It's a lot easier to get their heads around. So, I start with the easier one and introduce the truth tabular definition when we get to truth tables. But by delaying it like that they're more comfortable with thinking formally by the time they reach something as difficult as the truth table definition of validity, which I think is one of the more difficult things I include in the unit.

82 **Dave:** And as you say, a lot of textbooks and courses start with it. And that to me is starting at the deep end.

83 **Bessie:** Yes, I think you're right. I think there's something very challenging about the definition of validity. And then, even if you were to be quite pluralistic about it at the start and say that this is one of many ways of defining validity, they wouldn't understand what you mean because you haven't done anything that would allow you to compare and contrast definitions.

84 **Dave:** Yeah, that's definitely the case. And that's the risk of all of this. You've got to start somewhere, and wherever you start is intimately related to eighty other things that you can't introduce because you have to start somewhere. And that's frustrating, but also the joy of teaching is having this opportunity to pick a path through some of this stuff.

85 **Bessie:** So, what would you really dislike if students walked away from your logic class thinking? What do you not want as an outcome of your logic teaching?

86 **Dave:** There a thing that you sometimes see students who have had a first course in logic do or say, sometimes in other philosophy areas, or sometimes even non-philosophical contexts. They'll say, 'logic says blah' as though that establishes 'blah'. And if my students come away from my class and they think that 'logic says blah', then that is a sign that I have not achieved

my goal. I have failed to communicate something fundamentally important. So that to me is the thing I'm most trying to avoid. I want them to come away knowing never to say, 'logic says blah' as a way of insisting on blah. If they want to insist on 'blah' they have to take responsibility for it, they can't blame it on logic.

87 **Bessie:** That certainly agrees with my sentiment. I'm quite against the idea that you can use your logical proof to insist upon a conclusion in some way. There's just so much more to it than that, and yet people seem to really think that it's the function of logic to tell you what you ought to believe.

88 **Dave:** Like oh, I drew these fancy squiggles on the wall. So now you have to agree with me. That's not how it works.

laughs

1 **Bessie:** Well I'd like to start with some background questions about yourself. So, for instance where did you study logic or what university did you go to for undergraduate?

2 **Johan:** The University of Amsterdam. I was a student of physics. The philosophy department occupied a floor in the physics building and that got me interested and ended up taking my first logic course. Eventually I switched to mathematics and philosophy. I thought those were the two best subjects to take if you're interested in logic.

3 **Bessie** What year did you start your studies?

4 **Johan:** I started Amsterdam in '67. I studied there until '72. I think I did my master's in philosophy in '72 and in mathematics in '73 and then I did my PhD in Amsterdam in '77. That was in mathematics.

5 **Johan:** The very first logic book I read was a textbook by Stanley Jevons. I bought it because I always lost debates in our student circles. One of these students took pity on me and said, 'Johan, you always start by claiming "A" and at the end you're claiming "not-A", you should really study some logic.' This intrigued me; I thought I could both improve my actual performance and understand something interesting. Nowadays though, if my students want to know if logic will help them win debates. I say, 'no, winning debates is a talent'. Logic can help a little bit, but a lot of debating is experience.

6 **Bessie:** And did you read Jevons before you took your first logic course?

7 **Johan:** Yes. I thought it was indeed very interesting. That there are these patterns; it's a bit similar to learning grammar. You use a language all the time, but you've never realised that there are these stable recurring patterns and laws. And I discovered that the same was true of reasoning.

8 **Bessie:** It is interesting that you read Jevons, because it's such an old textbook.

9 **Johan:** Yes. When I was a student in Holland there was a cheap series of paperbacks that were Dutch translations of a variety of foreign books. And I guess the choice of those books was dictated by which ones were out of copyright. But despite its age, Jevon's textbook is nice because it's written in an engaging style, at least I found it so, and it gives an impression of what logic is about.

10 **Bessie:** What sort of characterisation does Jevons give of logic? What's the impression that you walk away with having looked at that book? In terms of what logic is about?

11 **Johan:** I must admit that I don't remember any details.[1]

12 **Johan:** What struck me most in the book was the discovery that I just gave, of patterns in reasoning. Exactly how logic was characterised, I don't remember.

13 **Johan:** One way to think about it comes from Boole's *Laws of Thought*, which I only read later. One of my students quite recently told me that it was interesting to read what Boole actually says about the laws of thought. It didn't stand out as very important to me at the time because I was trying to understand his system, but Boole says that his system represents only certain aspects of our natural reasoning. He remarks that in the way people actually reason there are deviations from his propositional logic, and then he makes this intriguing statement that it doesn't mean that people make mistakes. What the deviation means is that there are probably more subtle psychological laws of reasoning beyond logic that we don't yet understand. Isn't that interesting? [2]

14 **Bessie:** Yes. He's saying that there is greater complexity out there to be captured; not that the logical laws don't work, but that there's a greater field to be discovered.

15 **Johan:** From that point of view I was also intrigued by work in the psychology of reasoning. If you look at some very simplistic prediction of what propositional logic applied in a blind mechanical fashion would predict, you can see that people don't do that. Then your options are to say one of two things: if you're Wason and Johnson-Laird you say logic is wrong; and if you believe in logic you say people are wrong. But there is also literature that argues that often when people are make 'mistakes', it just means that there is a more subtle problem that they are dealing with. I have a colleague in Amsterdam, Michiel van Lambalgen, a logician who's done a lot of mathematical work. Michiel has written a book with Keith Stenning who is a cognitive psychologist from Edinburgh, and they have a much more systematic view on this. They say that when people reason, or solve problems, there's two processes you should distinguish. There's what they call reasoning towards an interpretation – where people are trying to figure out what the problem is that they're supposed to solve – and then there's reasoning from an interpretation. In the book they give a description of these processes, but they also show how confusions arise if you don't make the distinction. That you seriously misunderstand what

1 Jevons' text opens, 'Logic may be most briefly defined as the **Science of Reasoning**. It is more commonly defined, however, as the **Science of the Laws of Thought**. ... By a **Law of Thought** we mean a certain uniformity or agreement which exists and must exist in the modes in which all persons think and reason, so long as they do not make what we call mistakes, or fall into self-contradiction and fallacy. The laws of thought are natural laws with which we have no power to interfere'

2 From Boole's Laws of Thought: Let it be granted that the laws of valid reasoning, such as they are determined to be in this work, or, to speak more generally, such as they would finally appear in the conclusions of an exhaustive analysis, form but a part of the system of laws by which the actual processes of reasoning, whether right or wrong, are governed. Let it be granted that if that system were known to us in its completeness, we should perceive that the whole intellectual procedure was necessary, even as the movements of the inorganic world are necessary. And let it finally, as a consequence of this hypothesis, be granted that the phenomena of incorrect reasoning or error, wheresoever presented, are due to the interference of other laws with those laws of which right reasoning is the product.

your experimental subjects are doing; or trying to do.

16 **Bessie:** Yes indeed. Well, going back to the questions about your start, what do you remember of your first logic course?

17 **Johan:** It was an introduction to propositional and predicate logic taught by Else Barth, a philosophical logician in Amsterdam. The textbook for the course was Rescher's *Introduction to Logic*.[3] She taught in a very inspiring way; she was one of those hard line Russellian/Geach type philosophers who thought that using logic you can do away with millennia of bad philosophy. A strong memory I have is that Rescher's book has errors in the proof rules for predicate logic. This sloppiness was not uncommon at the time, as you know, with Copi being the most famous example. This is good because it means that students can spot mistakes, and one of the first things I did was spot one: a proof rule that wasn't sound! It's stimulating because you think: there's something in logic for me to do!

18 **Bessie:** Yeah, it gives you that opportunity to explore and see how it works for yourself by being able to find the mistakes in the textbook.

19 **Johan:** One thing I did want to say, before we go on; on the one hand, I very much like the idea of probing your point, asking 'what do they say about logic?' on the other hand, you could also wonder how important it is to know. To me it's close to the question 'what is logic?' and sometimes those questions are not well motivated. Let me give a rhetorical objection. I had a colleague, a philosopher of science, who wrote a paper saying, 'if you ask, "what is physics?" you probably won't get any satisfactory answer from physicists even though physics is the most successful science around'. In fact, it's so successful that nobody cares, right?

20 **Johan:** This colleague would also say: suppose that your wife wakes you up in the morning and says, 'I have a question, what's the nature of our relationship?' He says this question just signals that 'your marriage is in serious trouble!' In the same way, it may be very hard to answer this sort of 'what is' question about logic, but that doesn't mean that the field is not respectable; like physics.

21 **Johan:** One further answer that you might give is that of a friend of mine Frans Zwarts, a prominent Dutch linguist. Frans was a leader of a movement which started using logic to analyse natural language, around 1980. He was attacked by fellow linguists who questioned whether what he was doing was still linguistics. He responded with what I now call the Zwarts syllogism. He said, 'linguistics is that subject which is done by the best linguists. I am one of the best linguists in this country. Therefore...

3 Rescher says, 'Logic, then is the study of (1) the techniques of formulating information in language (broadly understood to include symbolic "languages" such as mathematics), and (2) the methods of extracting information from linguistic formulations' [page 11]. Later he also says, 'Logic is thus concerned primarily with the informative use of language' [page 12] (rather than the evaluative or directive functions of language he mentioned on page 1).

22 **Bessie:** ...what I do is linguistics.'

23 **Johan:** The serious point in this reply is that a subject is, in some sense, the sum total of all topics that its practitioners have found interesting.

24 **Bessie:** I think that's a good point. It also helps make sense of Richard Montague's work. Because he was doing that grammatical work, and when I look at that I wonder, 'is that linguistics or logic?' Well Montague was a logician, so what he was doing was logic.

25 **Johan:** I find your dissertation topic interesting because there's quite a mismatch between what logicians write in textbooks, and what they may even say when they give public lectures about logic, and the actual practice of the field. And that is very much worth probing. To give an example, taking a cue from your questions, when I give a public talk I fall back on things like 'logic is the science of reasoning' or something like that. Why is that? Because it follows a groove of thought. It seems to resonate with people. Even though when you teach the actual logic course there's a lot of other topics that are very different. To be more concrete; is a first logic course only about reasoning and inference? Especially if you look at the amount of time devoted to each topic? A lot of time is spent on introducing formal languages and their semantics (unless you get a very proof theoretical introduction to logic). But that language, like any language, can be used for lots of things. It can be used to define things; it can be used to communicate. The language of predicate logic, because that's usually the language introduced, is not necessarily geared towards inference, right? I can use it, for instance, if you want me to describe a situation. Now you may say that's sort of reasoning, but it's not exactly that. I'm not yet drawing any conclusions. I'm just using the language to maybe state a fact, a hypothesis, or a conjecture, or something like that.

26 **Johan:** So here's one general point, I think Wilfrid Hodges makes it somewhere. He says, 'if you look at the actual history of logic there are two themes that are equally important – deduction and definition'. Deduction stands for proof and inference. Definition stands for the use of language. I find this very congenial, and it has nothing to do with any of my new-fangled views on logic that we may still come to. If you just look at the actual practice of the field, even around nineteen hundred, proof and definition are equally important. Or if you wish, inference and language. Now add to that a paper I like a lot by Beth, my predecessor in Amsterdam. Beth wrote an article in the early 1960s called 'Constants of mathematical thought'.[4] Beth claims that there are three main topics in the actual pursuit of logic – proof, (so again inference and consequence), definition, so that's like what we had with Hodges; and the third aspect he adds is algorithm or computation. Some logicians would maybe want to say that proof is the same as computation. But intuitively Beth's distinction in three parts makes a lot of sense if only

4 A copy is available in Evert Willem Beth, *Aspects of Modern Logic*, ed. E. M. Barth and J. J. A. Mooij, trans. D. H. J. de Jongh and S. De Jongh-Kearl (Springer Netherlands, 1970)

because it's backed up by some main branches of mathematical logic. Proof is proof theory, definition is model theory, and computation is recursion theory: those are the pillars of the field.

27 **Johan:** If you look at what do logicians research on, what do they write on, what's in their mind...

28 **Bessie:** It does seem to be those three things, yes.

29 **Johan:** And then when you say in your first logic course, 'logic is about the science of reasoning' you've suppressed definition and computation even though they're included in the actual topics that you do.

30 **Bessie:** Although, to comment on what people actually say in introductory logic textbooks, they moved away from saying that logic is about reasoning. The older textbooks, like Richard Whateley's or J.S. Mill's will say that logic is about reasoning, but it seems that there was a change at some point. I attribute it to the anti-psychologistic movement. But whatever the cause, what emerges in textbooks is that instead of talking about reasoning in a descriptive sense, they're very clear that logic is about correct reasoning – they give it this clear normative character.

31 **Johan:** Very good point. Yes, that's an interesting change. I haven't looked at that myself, but it rings true to me. Well OK, I see several stages of retreat in the history of logic, first from reasoning to inference, then I've also noticed that many of my colleagues don't like inference because even that sounds too psychologistic to them because inference is something that you do. So, then they say consequence relations because they are agent-free, right? In that way you've taken any human activity out of logic and it's just about some human-free universe. And then you could say that a lot of the research that I'm interested in brings back some features of this human activity into the picture of what logic is about.

32 **Johan:** Actually I've had one historical question about this 19th century formative period; to me there is a curious historical mismatch. Frege writes at the same time as Wilhelm Wundt, the father of modern cognitive psychology. So, at the same time that *Begriffsschrift* appears[5], Wundt[6] writes his magnum opus on human reasoning and that's the birth of psychology. It's very curious that anti-psychologism becomes a term exactly at the time that psychology is a serious arrival as an academic field.[7]

33 **Johan:** With Frege I don't think you will find direct references to psychologists of the day.

5 Frege's *Begriffsschrift* was published in German in 1879

6 Wundt's *Principles of Physiological Psychology* was published in German 1874; In 1879 Wundt founded the first formal laboratory for psychological research at the University of Leipzig. The creation of this laboratory marks the establishment of psychology as a separate discipline.

7 In *Psychologism: a Case Study in the Sociology of Philosophical knowledge* Kusch argues that 'arguments against psychologism often were, at the same time, arguments against the appointment of experimental psychologists to philosophical chairs' (p. 211).

But it's almost as if, if there's competition you become more extremist. The other thing is that it's asymmetric. Wundt doesn't discuss Frege, but he did discuss Boole. He knew Boole's system of propositional logic. He's not very negative about it, but there's a few things which he finds map very badly onto human reasoning. One example he gives is the commutativity of conjunction. He says that almost all conjunctive constructions in natural language are not commutative – the order makes an enormous difference in the meaning. He gives a sort of bad example, it still makes the point, he discusses white sheep and sheep white. He says these things are totally different: a white sheep is a sort of sheep, but sheep white is a shade of white. So even though in some sense you might say there's a conjunction going on, that conjunction is not commutative. And maybe that's not a deep objection to logic, but he says the psychology of reasoning should understand these order effects; how it is that we say and mean one thing rather than another.

34 **Bessie:** I think that also connects back to the options when faced with a mismatch between reasoning and the logical system that you're dealing with. Where you either say that this reasoning is wrong, in which case you developed a normative theory where your system is right and people ought to conform their reasoning to the system, or you say the system is either wrong, or in need of further development so that it can capture these features that it's at present not capturing. The popularity of anti-psychologism as a doctrine has, I think, negatively impacted the potential research program of applying logics to the study of human reasoning.

35 **Johan:** I agree. Still, it's useful to realise that this is not just a problem for logic. At Stanford I occasionally speak frankly with colleagues in Philosophy of Language and Epistemology where they have similar problems. In epistemology for instance, what you may write in your paper is that you are going to analyse what it means to say, 'I know that...'. But then the moment one brings in some facts about usage in English... what they shift to talking about is philosophical intuitions. So I say to them, 'you study the sub-dialect of English spoken in the top ten universities in the United States. Why is that dialect of English more interesting than the English spoken by billions of people on this planet?' Other philosophers like to appeal to 'what people do'. For example, a good friend of mine, John Perry was lecturing on philosophy of language and meaning, and at one point I stopped focusing on what he was saying and just counted the number of times he said 'what do we do'. I commented, 'You've said this about 50 times now in the last half hour, so it's got to be relevant what the psychologists and the linguists and others think people really do' and in response he retreated to the claim that what philosophers are studying is folk psychology. Folk psychology is a mysterious thing not studied by psychologists or linguists, and for some strange reason, probably through a mutation in their DNA, philosophers' minds have unique access to folk psychology.

36 **Bessie:** *Laughs* Yeah.

37 **Johan:** In other words, what I want to say is that this ambivalent relationship between theory and some empirical practice out there is not just in logic; epistemology and philosophy of language face a similar challenge.

38 **Bessie:** So connecting back to this idea of the pluralities in the field of logic. One of the things that I thought was very novel about *Logic in Action* as compared with many other introductory courses or textbooks was the strong commitment that you give to showing the kinds of options that are out there – where you can take logic next – in those outlook chapters which connect new areas of study to what has been introduced in the text.

39 **Johan:** This is indeed the way I do it. I also wanted to convey to the students, that I see logic as giving you a view of connections between lots of different things. I see logic as lying at a crossroads – you can go to very different places: to computer science, psychology, mathematics, philosophy... Though I should note that this works well for some students and not for others. Student populations are diverse. Sometimes we speak of students as if they are a homogeneous group, but they are not. A good example comes from the work of Keith Stenning (whom I mentioned in connection with Michiel van Lambalgen). Keith once did psychological research connected with the course 'Language, Proof and Logic' that Barwise and Etchemendy were teaching at Stanford in the early '90s. They added graphical methods to illustrate logic – their thinking was that making things visual helps. And here's what Keith found. It helps some of the students and it makes everything more difficult for the others, because people fall very roughly into two kinds: those who actually do think graphically, the ones helped by geometry, graph theory, and things like that, and those who think algebraically and computationally, for whom it's actually much better to give them rules of manipulation, because that's how their mind works. The study included a pre-test of the students with reasoning questions to determine which types they belong to; graphic vs. symbolic. This is of course a very rough description. Then they looked at the results of those students after they had taken the course and what they found was that the graphic types did much better. But the algebraic/symbolic types did worse after instruction because they'd been taught in a style which didn't resonate with the way their mind works. In my experience, I also find another diversity with students of *Logic in Action*: there's open types and closed types. Many students are open in the sense that everything becomes more interesting to them if they see connections. But there's also students for whom everything becomes more threatening and disorienting when they see those connections. So, I also have students who hate the course. They want to be trained in, say, just a natural deduction system, and don't want to be challenged to think about the many things that the system relates to. I should say that these types need not be for life. I work a lot with Alexandru Baltag, a colleague of mine who is one of the most brilliant minds I know. And Alexandru likes to say that as a beginning student you belong to the closed type. But he says, 'I found around age 35

there was something changing in my brain. I began to like the broad perspective on logic'. So now he's definitely an open type. Maybe once on the secure basis that there is something you know well, you can now afford the luxury of looking outside.

40 **Johan:** But in any case, the course is based on this premise of giving these various connections and it works for a lot of students. Let's put it like that.

41 **Bessie:** In 'An old discipline with a new twist' you say that in the course *Logic in Action* you want to export your revolutionary ideas by by-passing academic colleagues and influencing the youth instead. Can you talk a bit about that?

42 **Johan:** Yes, that's another aspect of the course. In that paper I said, just to dramatise it a bit, that you have two types of modern logic courses. Some want to change the agenda, they have sort of activist purpose, like mine, and some almost have the opposite: they want to stick to the old agenda but make it look more modern. Like these on-line courses that mainly teach you natural deduction, but now with all these wonderful computational tools. My analogy was of classical Mass with rock guitars. Basically, you don't change the old dogmas of the religion, but you're willing to sell it with whatever state of the art technology is available. My course is more like Barwise and Etchemendy's who also tried to change the agenda in their course *Language, Proof and Logic* – and I'm upfront about it. You might ask, 'why do it for students at that level?' and it's because the first logic course that people get is so important because it sets their mind and expectations. If you want to change something, it's going to be very hard after the first logic course because their mind has been set, both explicitly and implicitly in the methodology.

43 **Bessie:** In everything that you do.

44 **Johan:** Yeah. I find this especially true with philosophical logicians, mathematical logicians are often easier to influence. And why is this? They're mathematicians, they like to play with new ideas and they will just say, 'I don't have a particular view Johan, maybe observation and communication are logical topics as you claim, maybe not, but just show me something new and interesting to think about'.

45 **Johan:** You had one question that I should ask you about. You say, 'in *Logic in Action* you say that logic will be presented as a key element in a general study of reasoning information and communication'. Then you say, 'that gives us an idea about what you think logic can be used for, but what do you think logic is about?' What do you mean by that question?

46 **Bessie:** I'll go back and set the scene a bit more for that question. One of the things that I thought was exceptional, and very exciting, about *Logic in Action* is just how often you use the word 'information' when you're talking about logic. And I think that's novel and is crucial to establishing the pluralist view that's available in that course – you can make those connections to the other disciplines because of that way that you've characterized logic. But it makes logic look quite instrumental; you're able to see logic as a tool and how you might use that tool but...

47 **Johan:** I think I see where you're heading. *Logic in Action* is not a considered view, but if you were to push me on this, I might want to say that the topic behind *Logic in Action* is information. I'm not the first to say so, because in some sense that was also what the situation theorists said. That was the view of logic with Barwise and Perry. So, then logic would not be the science of reasoning but the science of information. And information in a broad sense. And if you're interested in information then there's two main things to study: namely what's the structure and how does it get modified, made, and so on.

48 **Johan:** I give the famous Mohist example of the three sources of knowledge, and I really do think they are the three core topics for logic. First, inference is one way of dealing with information; but equally important is observation, this includes a lot: simple observation, experiment and so on; and finally communication. I claim this is a natural unity because if you see people solving problems like in my example of the waiter in the restaurant, that's exactly what you see. Pure deduction is not going to help the waiter solve his problem. He needs some further input. Well that's what his questions do, but the questions and the answers alone would not be enough. You must put them together. In science it's the same. Mathematics is a bit unusual maybe in that it's only about reasoning. But take physics, that's a combination of mathematics with observation and experiment. And communication is also important because modern science includes figuring out what others have done or what others think and so on. So, I think inference, observation and communication are core topics in logic. Now again I'm not the first to say this. If you want to look for some historical roots, I found quotes as early as Ajdukiewicz in the 1930s who said that asking a question is just as logical as drawing a conclusion because asking a question is an equally fundamental informative process. You see what I mean? Logic is about all rational information seeking activities. And another person who has said that long before me is Hintikka. Also since the course appeared, I have corresponded with people, and a Spanish historian of logic told me that the history of information as a notion in logic is also pretty old. He has found 18th century logic texts where the word information already plays a role.

49 **Johan:** I hope that helps. I'd be inclined to go for for information as the main topic and information handling because it has these two aspects. It's not just what information is like statically. It's how you can get it and share it.

50 **Bessie:** So that's not just about the information itself but it's about your processing of the information; that includes the reasoning process, the psychological stuff, but also the communicative process and the interaction with the other agents who are also reasoners.

51 **Johan:** Yes, there's that multi-agent aspect. Now of course you could say argumentation is really sort of reasoning, but the point is that it's multi-agent, right?

52 **Bessie:** It is. One of the things I noted as a peculiarity when I was looking at introductory

logic textbooks was that, 'argument' is often presented as something that's synonymous with 'reasoning' and the discursive aspect of argument which we know is there is just omitted from these textbooks. And that really surprised me when I stopped to reflect on it.

53 **Johan:** Yeah. That is surprising. I could tie this to one of the questions that you had about the definition of validity. Look here's what I would say even before this *Logic in Action* course. There are several major takes on validity and logic. The semantic take, where it's explained in terms of truth. This still covers a lot of different senses, it could be standard Tarski, it could be the much earlier Bolzano which is more or less like Tarski, but Bolzano thought that the premises had to be consistent. Then there's the proof theoretic view. Which if you take it very purely you could even think of as an alternative to semantics. Like if you're a Gentzen-style proof theorist and you think that the meaning comes from the proof rules. But, there's a third view of validity which you often don't get to see in textbooks, which was brought out by Lorenzen namely the pragmatic or game theoretic intuition.

54 **Johan:** Lorenzen in his work on dialogue games in the nineteen fifties said that if you look at the history of logic it's likely that the patterns that were considered valid had something to do with what was successful in argumentation. Lorenzen's made this view precise. He said here's another intuition of validity: a conclusion C follows from premises P, if in argument or dialogue a person who claims the conclusion against an opponent who is willing to grant the premises has a winning strategy. So, you might say here that the intuition is that validity is something that's compelling. In the sense that in an argumentation setting you can never go wrong...

55 **Bessie:** If you present a valid argument. I love that.

56 **Johan:** It's a very interesting view, though of course it doesn't make the other views obsolete. In fact, there are theorems relating this game theoretic approach to semantic or proof theoretic approaches, but it's another intuition. Because now you could say validity has to be something which is so strong that you can maintain it. But what's the best chance of maintaining? Well against someone else, so in a multi-agent setting.

57 **Bessie:** Yeah, I like it. It also helps make sense of the pre-logical intuition that something is valid when it has this kind of power. That somebody else will have to accept what you've said as a consequence of your argument. I think it helps make sense of some of the intuitive notions about what validity means which we sometimes encounter as teachers and then we have to persuade students to set those intuitions aside. I like having diverse ways in which we can characterise validity which will allow us to show students that their intuitions are not wrong; but that sometimes different kinds of rules apply.

58 **Johan:** In *Logic in Action* I have my own arrangement, but it's not as if I have cut myself off from the history of logic. I'm picking up themes that were maybe minority views or approaches,

but they were there.

59 **Johan:** I'm looking at your list of questions again. So, you read my paper 'Logic and Psychology: do the facts matter?' You'll notice it has a lot of running around, and that's basically for two reasons. One is that my own views have been in flux. As a student I started out as a strict normativist – logical validity is what it is and everybody else who does something differently is wrong. I guess it comes with the radicalism of youth.

60 **Johan:** Over time I've noticed, and not just with me because this is true in my environment as well, that there's some sort of public opinion change. A lot of logicians that I know nowadays think that the facts should be more relevant. The only thing is, it doesn't lead to a precise idea of how. This change is not limited to logic. My institute had a meeting some ten years ago with the formal semanticists of natural language. Formal semantics has an interesting interface with logic because you're looking at the meanings of expressions in natural language with logical techniques, this was done by Montague and the tradition after him. The discussion was about 'what's the nature of the evidence?' In the 1970s you would just have an author write a sentence and it's correct English or another one which is incorrect English, then you see a little star. The question is what's the nature of these stars? Well, these stars were supposed to represent linguistic intuitions. The trained linguist says that this sentence is not correct and that one is correct. It wouldn't have occurred to anyone to check in corpora of spoken English or something like that as to whether people use it, or whether it has problems, like whether the starred sentence truly has problems when you use it in communication. What our discussion revealed was that while that answer seemed fine in the 70s, somehow it didn't sound as good as it sounded in the 70s. People had lost that faith in themselves. If the English language were actually used completely differently from what our stars say, shouldn't we have a problem? Wouldn't we prefer a theory that's at least roughly in line with actual usage, even though, of course, we're willing to occasionally say, 'well people are wrong'.

61 **Johan:** Even in mathematical logic which is often considered the greatest stronghold of this purely normative view of logic, opinions are shifting. I was at a lecture some years ago where someone said that what they do is model the actual reasoning of mathematicians. So I replied, 'let's give up natural deduction systems because there's nothing in a mathematical seminar that corresponds to that particular formalism. Instead, let's take a fresh look at what mathematicians say about proof and what they actually do.' Well they wouldn't go that far yet... But it was very interesting to me because I think that you should be willing to give up received ideas about logical systems if mathematical practice doesn't correspond.

62 **Johan:** Anyway, my views are in flux. And I don't think the community has arrived at a new view, but they've become more sensitive to empirical relevance. As I say, I don't have a fully considered opinion. I do think that the connection between what logical systems do and what

you see in practice is a serious research topic which should be high on the agenda. That has a lot of aspects that I haven't even charted out. For a start, maybe you should make the distinction which van Lambalgen and Stenning make about the process of interpreting the problem and reasoning from your interpretation. And there's other things that I think might be important. As you said, what we're probably going to find is that there's a lot of aspects to actual reasoning that we don't yet have in our logical systems, but that we could incorporate.

63 **Johan:** I'm not sure where this will go but for example, Thomas Icard, a colleague of mine at Stanford, argued in his dissertation that reasoning – just reasoning now, I'm not talking about communication or other things – is usually driven by goals. There's a sort of 'why?' question. There's a topic and there's something you want to achieve. Thomas feels we should also model that structure. A formal proof is an impoverished rendering of reasoning because you don't see what it is for. I don't see such points as a threat to logic. There may be a lot of interesting structure that we've just never looked at. And if you were to go to your mathematics seminar, I think a lot of what you hear has to do with goal-oriented reasoning. The speaker keeps reminding the audience why they were doing this, what they were trying to get at and so on, keeping track of the overall purpose.

64 **Johan:** Let me mention two more topics that I think are extremely interesting and we haven't yet come to terms with. One is learning. I think a typical feature of expert practice and daily life as well is learning and becoming better at things. Well that raises issues of learnability of logical systems. So what is the logical system actually modelling? Only the expert's final stable performance?

65 **Johan:** Now there's a lot of learning theory and you would think that it's important. In Amsterdam to annoy my colleagues I sometimes claim that weaker logics represent earlier stages of learning. So then, I say, 'OK intuitionistic logic which is weaker than classical logic – that's what children do, they are constructivists because they want an example of every existential statement. Once you are mature you can use classical logic because you're even willing to accept arguments that show that some X exists with a certain property even though you can't give a particular example. That might even give you a thrill, like say some complex argument which shows that there must be life on Mars even though you can't give any specific form that it might actually take. And then I add that intuitionistic logic wins in the end because as we get old and our mental abilities decline, we lose certain inference rules! So, intuitionism describes our happy youth and our old age.' Now, of course, I'm not serious. Learning could be very different, it's an area for logical research.

66 **Johan:** Here's the other thing I want to mention, it's from my Institute in Amsterdam, but I haven't worked on myself, though it sounds right to me. It also fits with the broader view in *Logic in Action*. It's is an idea from psychology and artificial intelligence research. In actual

problem solving, what do you do? You have some data and some problem you want to solve, maybe a question you want to answer, a conclusion you want to reach. The reality is that you will not approach this as a purely deductive problem because that disregards the crucial role of memory. For almost any problem that you encounter you have a stored memory of similar problems that you solved in the past. So, what may happen first in your mind is that you try to see a similar problem that you've already solved. Is there some pattern in the current problem, provided by your memory, that perhaps you only need to fill in. If it's complete recognition, then there's no reasoning involved it's just recall. If there's no complete recall, some additional reasoning may be needed to get there and in extreme scenarios maybe all you can do is reason. And this takes me back to my earlier example; when you're young, you don't have this experience of millions of stored similar problems in your memory; and when you're old, you don't either, because your memory begins to fail. Now, this sounds very reasonable that this is how it works. Mathematicians like this picture of how research works. When you have a mathematical problem what you're definitely not going to do is remember: these are my possible actions, these are my rules, and I'm trying to prove this theorem. The first thing you do is consciously or unconsciously draw on your memory. You have this vast database; that's why you're an expert. You think to yourself, 'what are similar problems?' Well let's work there, and so on and... Again, this is not a threat to logic. But if you want to model that you'd need a richer notion of a logical system. The system would have rules and principles and so on, but it would also have a component of memory storage. And it's even dynamic because every time you solve a problem it goes into this memory storage.

67 **Johan:** So, I don't have a very good philosophical position on logic and reality, but what I'm suggesting is that maybe logical theory should be closer to reality. There are a lot of interesting structures in reality, which could be good for logic. I don't see any problem with that. Just imagine a wonderful new theory of formal systems which consist of rules plus memory. And think of new Gödel theorems about what you can and cannot achieve with such an architecture. That's how I view that contact.

68 **Johan:** Of course all this is too early for an introduction to logic. This would definitely confuse students.

69 **Bessie:** Absolutely. Yes.

70 **Johan:** So the topics I mentioned just now I would teach at a later stage. *Logic in Action* is, I think, as much as one can do that early level. Another thing is that, for these ideas I've mentioned, we don't have fixed tools and results yet. And presumably what we're going to need there, which is also not crystallized out in research yet, is it's going to be both logic and probability. This is very likely especially with memory because the brain is a probabilistic machine, and at the moment there is no standard joint theory of logic and probability that will do that

for us. This would even go back to Boole whose book is about the laws of logic and probability. So, we could say those 19th century logicians were on the right track.

71 **Bessie:** Yeah. They were in very early stages of tool development and they were struggling with this problem of what they had the tools to deal with and what they thought the overall problem was. And it seems that at some point we developed this amazingly powerful tool[8] and became really enamoured with that tool and some of the aspects of the broader program fell by the wayside over this excitement with how powerful and amazing this system was.

72 **Johan:** Yeah. I think this is true and you can also back that up a bit historically. If you take our standard history it works like this: Boole, Frege, let's say Hilbert or Gödel or something like that. In this story the probabilistic aspects of Boole's thinking are downplayed, Boole just invented propositional logic because that fits best, and then Frege adds predicate logic, then Gödel adds this whole sort of theoretical depth to the whole enterprise. But you can write a richer parallel history with contemporaries. Boole can be put alongside Bolzano. Bolzano had a very different much broader view of what logic was about, as a study of styles of reasoning. Bolzano does not assume that there is only one notion of logical consequence: he thinks it depends on the task you're working on. He has different notions of logical consequence and points out different properties that they have. Next, Frege can be put side by side with Peirce. Peirce is around the same time. In our traditional story we also suppress that fact that Peirce discovered predicate logic independently because it doesn't fit; and we now think that salvation came through German universities and not through Harvard. I think it's pretty uncontroversial that Peirce developed a predicate logic, but Peirce's programme was broader containing varieties of reasoning: abduction, deduction, and induction, and so on. Maybe the following is a less convincing parallel, but in a sense, you could put Gödel side by side with his contemporary Carnap. Not that Carnap was as deep as Gödel. But Carnap was one of the few people who understood Gödel's theorem and he even made some contributions. That's been neglected because people think Carnap is a philosopher of science. But Carnap of course also did inductive logic and other things connecting with probability. So, in addition to the standard textbook lineage, there is this history of broader role models. And if you go to modern times it's clear enough. I consider people like David Lewis as logicians. But look at what he wrote about: a lot about probability, causality, knowledge, and games. I think it will be good for logic to go on this broader path.

73 **Johan:** I'm not thinking of this in terms of what's right or wrong. If you took a history of science point of view you could say that this contraction to foundations of mathematics and some satellite philosophy which happened with Frege, and Russell and Whitehead, and Gödel

8 Classical logic i.e. the propositional and predicate calculi

was enormously successful. It's quite imaginable that if Peirce had won the day, we wouldn't have had the completeness theorem and the incompleteness theorem.

74 **Bessie:** Fair enough. Something that's beautiful about the Russell Whitehead's logic is how simple it is. And that's its beauty and its power. It's quite easy to learn; quite easy to work with. I think this is what makes it such a good logic to teach and learn as the first one. That's why we teach it in introductory logic courses. And your *Logic in Action* is no deviation from that – you start there.

75 **Johan:** Yes. *Logic in Action* combines these two aspects because I start with propositional logic, predicate logic, and the syllogistic, and that's relatively standard. That's to show 'this is how we work'. The rest of the story is about a broadening of the agenda. But the broadening of the agenda doesn't mean that we give up the established style of working. Because it's a historical achievement, and we're not going to throw that away. That's how I would think about it.

76 **Johan:** You have this question, 'How do you explain what logic is about?'[9] Well, I think that is answered by the design of the course. But other motives for doing things this way are not so deep. First, I like this broad view, that's just a sort of intellectual temperament. I told you I was a student of physics. Then I did mathematics and philosophy because I couldn't choose, but I thought that it would be good to be able to think in both these ways. And later on, I picked up linguistics and computer science. But also, when I think about the future of logic, I personally think it would be better to take this broader point of view. This is controversial. If you think about the survival of your subject, there's a lot of different views. And there are some I call not foundationalists, but fundamentalists. Some people seem to think that we should stop doing all these broad topics and go back to the golden 30s! This to me is like fundamentalists who wish to recreate, or invent, some mythical golden age for its simplicity and clarity. Personally, I don't believe that logic's is going to survive on that. Not because I think all that stuff is wrong. I said that already, right? We have it but it's not enough. I think a broader approach gives us at least the opportunity to make our agenda larger, have interesting things to do, and maybe become more relevant to other areas. Though I must say, I mostly listen to people who tend to reinforce this belief. I gave a talk about the broad view of logic at the Helsinki meeting of this big Congress, *Logic Methodology and Philosophy of Science* in 2015. This was the opening lecture for both the ASL logic colloquium and this big conference. I explained all my deviant views, and afterwards one distinguished colleague came up to me and said, 'well Johan, I've been convinced for a long time that logic is dead, but your lecture gave me the first glimmer of hope that there might be something for us to do in the next century.'

77 **Bessie:** *Laughs* Yes, well I think your approach which makes connections and opens areas

9 When you teach introductory logic how do you explain what logic is about?

for study in ways not typically opened in the standard approach to logic. I find that approach very exciting.

78 **Johan:** And that brings us back to this issue of 'what is logic about?' Well, I am enamoured of the Zwarts syllogism. So, I say, if enough logicians think this way then that's what logic is about; or will be about.

79 **Johan:** There's one more thing I want to say about standard introductions. We've been talking about mine all the time. But traditional introductions are not necessarily wrong, they're just another point of entry. But one question that one should ask oneself very seriously in relation to the students is, 'what are we teaching, and why?' This question is of great relevance to me right now because in Holland there may be an opportunity to make logic a subject in high schools. But the question is, 'What?' I find this a very dangerous gift. Suppose that they give us the opportunity to instruct the whole Dutch youth. What is it that we're going to teach them? Now we're forced to think because we shouldn't teach them our hobbies. We should teach them something which is useful for life. You know, it's almost a question of morality. So, what are those generally useful things that you'd put into the course? Logic instruction traditionally features this training in proof systems, and I totally believe in training and reasoning. But there is a serious question of to what extent training in logical proof systems is actually good as a training in reasoning.

80 **Bessie:** Yes, and it's an empirical question.

81 **Johan:** When I was a student, I think about '73 I read an article by some American philosopher pointing out research indicating that skills in logical systems reasoning don't seem to transfer very well. People who are good at that are not better problem solvers in other areas. So, if you were to teach your high school course, what topics would you choose, and what claim about benefits would you make?

82 **Bessie:** That formal proof systems improve your reasoning is a claim that lots of logic textbooks make. I've seen it frequently in the introductions I reviewed. They say that you should learn this stuff because it will teach you to be better at reasoning. But reflecting on the research I'm aware of, well, there's no evidence to support that claim.

83 **Johan:** And that is just one example. Another example is the translation drill. There's all this stuff about translating natural language sentences into first order logic. The status of that to me has become very unclear. What are we training people in? Becoming little computational linguistics engines? What's that good for? Because if you formalise without any particular purpose...

84 **Johan:** Let me specify what I want students to be able to do. Suppose that they have a real problem; they're studying something. I would like them to be able to formalise that to some extent. I would want them to be able to write down a few essential things about it in formulas.

But there's this illusion that they could take the text of the problem and turn that into a set at or first order formulas, though we know that in many problems that's not helpful at all. So, I've sometimes thought that we should teach not so much translation as formal language assisted paraphrase; for real problems.

85 **Bessie:** My last question deals with the pragmatics of introductory teaching; what you're aiming to teach students and why are you doing that?

86 **Johan:** For a start I'm always deeply aware of the diversity of the student population. If I set a list of aims, I'm aware that they're not going to work equally well for all students. But with a compulsory logic course, I do ask myself 'Why am I teaching this?'. I think it's only fair if the aim is not job opportunities for research logicians, but to do something good for the whole group, you must ask yourself 'why?' I want students to become sensitive to some abstract notions like the difference between syntax and semantics. I see it as opening minds to a set of intellectual notions and tools. Next would be formal skills to some extent because I think it's important to understand how the logical machinery works, but I don't overdo it. I don't want them to get the impression that when they're working through long formal proofs that they've now understood the essence of problem solving or mathematics, or something like that. So, I do just enough to make them appreciate what it is. Then the third point is the modelling aspect as I mentioned, how do we get from real problems to representations of essential features in a logical language. Next, at the meta-level, I want them to appreciate the notion of a logical system. But again, I try not to overdo it because I don't believe that logic is the science of formal systems and over-emphasising formal systems would give them the wrong impression. And the final thing is indeed this interdisciplinary crossroads view that we have discussed. I try to make them see that with the sort of things that are taught to them, they can see many wider connections and analogies, so their life becomes a bit richer.

87 **Bessie:** I think that's a very good set of aims and one that would offer a lot of things for a broad audience. A follow up to my final question... What sort of thing would you want students to not walk away with the impression of? What sort of things do you want to avoid?

88 **Johan:** Okay. This is a very good question. First, many colleagues I know believe in the following fallacy: if someone gets to know logic they're going to like it. So, in teaching logic, we're going to make friends for life. So, I tell them a logic course is also an excellent way of making enemies. I was once in a national committee for funding cognitive science in Holland and there was a nice proposal on the table about logic and reasoning. But the project was killed by a distinguished professor who said, 'I took an introduction to logic course as a student, and the shock that this sort of irrelevant nonsense could actually be taught at universities was so great it stayed with me through life.' There was nothing I could say to overcome that opposition. He'd had a logic course of the wrong sort, and that makes enemies, sometimes powerful enemies

who can do us harm. Fortunately, most people who hate a logic course don't talk about it in later life. So many of our enemies are just silent enemies. In short, I think your question is very well taken. We should think about what we want to achieve and what we don't want to achieve. And I do think that my course is a way of avoiding misconceptions because students get a training in the formal mindset, but in a way that puts that in perspective.

SUBJECT MATTER PLURALISM

UNDERSTANDING LOGIC

> In some ways we feel that we are as confused as ever, but we think we are confused on a higher level and about more important things.
>
> E. C. Kelley[1]

7.1 WHAT IS THE NATURE OF LOGIC?

The objectives of this thesis are split between the philosophical and the pedagogical. The philosophical objective is to propose a way of understanding logic. The pedagogic questions follow from this philosophical objective. They are about how to teach logic so that it can be better understood. In §3.2–3.4, I discussed some of the options for the subject matter of logic. There, my argument was primarily pedagogical and focused on the potential for giving an adequate account of classical logic. In this chapter, I will deal with pedagogic questions, but the focus here is philosophical. In chapter 8, I return to the pedagogical discussion. However, here I focus on the philosophical point that there are multiple ways of conceiving the subject matter of logic.

This philosophical point is significant because there is a noticeable lack of clarity in the fundamental theories that theorists are using when talking to each other about logics and logical systems. Theorists typically assume that they are using the same theory in logical discussions. However, since no one is clear about what they are discussing, this is not a safe assumption. When logicians talk about validity, particularly when there is a question of rivalry between different logical systems, they talk as if they are discussing the same thing. They show no awareness that this might not be true.

Perhaps this lack of clarity has its roots in teaching. The textbooks I examine in chapter 3 all use an assertive style when introducing logic, declaring it to study this or that. With no indication that there may be more than one option. This error in thinking maybe further encouraged when *the* definition of validity is introduced. Again, there is little to indicate that there are multiple approaches to giving technical definitions of validity in these textbooks. This error in thinking might be corrected as students progress through the discipline, though the way theorists argue about which systems are *correct* seems to show that this is not the case. Systems often cannot be correct or incorrect in the way that they expect because their understanding of correctness is too absolute. For example, Priest argues that monism is viable in the context

1 Earl C. Kelley, *The Workshop Way of Learning* (New York: Harper & Brothers, 1951), p. 2

of the 'canonical application of logic' – which is the 'application of a logic in the analysis of reasoning.'[2] However, he considers only one way to interpret 'the analysis of reasoning' almost implying that this is the *only* way to understand it. This implied homogeneity in the study of reasoning is exactly the sort of argument that made it necessary for Harman[3] to argue the contra.[4]

In our interview, van Benthem shared this story: 'suppose that your wife wakes you up in the morning and says, "I have a question, what's the nature of our relationship?" ...your marriage is in serious trouble!'[5] Of course the question is worth asking when there is evidence that views differ. Some trouble can be avoided merely by understanding that views can differ. The differing views on the nature of logic do not need to lead to divorce, but they warrant a conversation.

I argue that there are multiple ways of conceiving the subject matter of logic. Supporting this point is evidence that many different strategies are used to explain the subject matter of logic in textbooks. However, teachers need to make pragmatic simplifications in their teaching, particularly at an introductory level. Each interview reveals this tension between what logicians think about logic and what they say to their students. Nevertheless, the diversity of their choices is indicative of a diversity of views on the subject matter of logic.

In this chapter, I will discuss what the interviews reveal about introductory teaching objectives and the pragmatic decisions that teachers need to make that may shape students' understanding of logic. I will also discuss what the interviewees think about logic in contrast to what they say to their students, and further explore what logic could be about.

7.2 WHAT DO WE WANT STUDENTS TO LEARN?

A common feature of all teaching, whether stated or unstated, is that it has a goal. Every teacher, whether they have thought about it or not has some intended goal in mind. Asking logicians about their teaching goals results in two sorts of insights: insights into practical pedagogy, and insights into what logicians think is essential in logic.

When I asked Russell about her goals, she said, 'I want a mixture of things. ...One of the things that I want is development of certain skills.' The examples of these skills she gave are:

1. Translating between English and some formal language;

2 Priest, 'Logical Pluralism', p. 196.

3 Harman, *Change in View*.

4 Harman wants to examine belief revision, but *not* using logic. Because his audience is philosophical, they expect that any study of belief revision not just will but *must* be done via logic. So Harman has to carve out this territory before he can conduct the investigation in the way he wants to.

5 See §7.20

2. Filling in a truth table accurately and reading off various logical properties, like being a tautology or being unsatisfiable, from truth tables;
3. Being able to give the truth conditions for the quantifiers or conjunction or other logical expressions;
4. Being able to complete a proof in some proof system. Russell introduces the natural deduction, but what is important is that they learn some form of proof system, not that they learn that particular proof system.

Then she says that sometimes it is valuable to just know that something can be done. So, one of the things she wants students to learn is what the possibilities are in logic. To some extent she also wants them to learn some respect for the expertise available in logic.[6]

Russell also wants students to learn some of the conceptual material. Here she talks about introducing what a logical consequence is, how it is related to provability, and that they are not quite the same. She also wants to communicate that not everything can be expressed in these formal languages and talk about what kind of things get left out. She says,

> I want them to have the sense of discovery, that they're seeing into things and understanding things. I want them to have those sort of 'aha' moments where it's not like I listed a bunch of facts and now they know the facts they have to learn, it's that they've come to understand something and see something through being in the class; to own it for themselves.[7]

Russell is clear about what she wants students to learn, and she is also clear about what she wants to avoid. She wants to ensure that her teaching is inclusive and that students do not see logic as some essentially arbitrary system with no relationship to the world except as a language/power game.[8] She wants students to see that validity and soundness are properties out there in the real world, and she wants students to recognise those properties when they occur.[9]

Ripley describes what he wants to achieve as showing students what he finds exciting or fascinating about logic. He says, 'I want to treat an introduction to logic as an introduction to what it means to do logic. ... I want them to see the thing I love doing."[10] Ripley does not expect that every student will be excited about logic in the way he is excited. However, Ripley designs his course to give himself opportunities to present material that he sees as genuinely interesting throughout the course. Ripley's efforts focus on making what he finds exciting available to his students.

6 See §4.107
7 See §4.116
8 See 4.112-15
9 See §4.112
10 See §5.41

Ripley's approach is different from Russell's in that he does not talk about what he wants students to be able to do after taking a course in logic. His way of thinking about teaching focuses more on the notion of sharing. It is an offer he makes to students designed to show students the best of what Ripley does in a way that is maximally accessible. It is an approach to teaching which is more focused on the teacher than the student.

I did not ask Ripley about what he does to get to know his students, but I assume that he, like Russell, makes that effort. Similarly, he is just as focused on making it clear to students that understanding logic does not come from innate ability. He has spent time building exercises and homework for students to do in his course. He says this is not the point of logic, but 'you have to do it to be able to see the point.'[11] Then built into those exercises are exercises which demonstrate things about logic which Ripley finds genuinely interesting. This structure makes students engage with doing the work and challenges their understanding, hopefully enabling them to see the point of what they are being shown.

Van Benthem's focus is more cognitive than behavioural. He does talk about what he would like students to be able to do, which he sees in the context of real-world problem-solving. In that context, he would like students to be able to formalise a problem to some extent – to be able to write down a few essential things about a problem in formulas. However, he is more inclined to describe what he wants students to know, rather than do. He wants students to:

1. Become sensitive to some abstract notions like the difference between syntax and semantics;
2. Develop formal skills; without giving the impression that this is the essence of problem solving or mathematics;
3. Understand that it is possible to represent the essential features of a real problem in a logical language;
4. Appreciate the notion of a logical system; without giving the impression that logic is the science of formal systems;
5. Be able to see the potential for many broader connections and analogies.

Van Benthem wants to give students some training in using and thinking about formal systems. However, he wants to make sure that students never lose sight of the application. This focus on, and connection to, application makes the title of his course *Logic in Action* very apt.

Like Russell, van Benthem believes that students should walk away from logic having learned something useful from their studies, and he feels this very strongly. He is not prescriptivist about what this value must be, only that thinking about the aims of teaching is a critically im-

11 See §5.54

portant for teachers. It is a moral obligation on teachers to think about the subject's value for students even if students do not enjoy the subject or would prefer to study something else. No matter how students arrived in a classroom, but especially if they are required to be there, it is an integral part of a teacher's job to deliver something valuable to students.

As I said initially, the insights we can gain through this comparison of objectives are mixed between practical pedagogy and what logicians think is essential in logic. Chapter 8 is devoted to practical pedagogic questions. In that chapter, I return to the topic of intended learning outcomes – the outcomes teachers hope to achieve in teaching. Here, it is enough to observe that these are always present in teaching in some form or another. They may be explicitly articulated and linked to a teaching programme, as I discuss in chapter 8, or they may only exist informally in the teacher's thoughts and feelings. Learning outcomes also express something about the value of learning.

Today it is common for people to have strong opinions on the value of learning. A driver for these strong feelings about the value of learning is its associated costs. Today it is common for education to cost money, so when the cost is discussed that is the most natural focal point. However, even if education did not cost money, there would be a cost. Our time alive is limited, so there is always the cost associated with the choice to spend time learning this, rather than doing something else. Learning outcomes articulate what teachers hope students will get from the time they spend learning.

The moral question in teaching is whether what is being taught has any value. Learning outcomes articulate some part of that value. So, when I ask what these teachers hope to achieve in teaching, I am, in part, asking what they think about the value of learning logic. It would be astounding to find that a logic teacher thinks that learning logic is valueless. Nevertheless, the value of learning anything is context dependant. There is not much in modern formal education that is so valuable that it would be valuable for all people at all times and places. In the right context, any learning can be valuable. When teachers talk about the value of learning logic, much is assumed about the context.

What teachers say about logic is linked with what they think about the value of learning logic. The statements that assert what logic is about could easily be prefixed with 'learning logic is valuable because...'. It is the sort of statement that would sit naturally just before an expression of learning outcomes.

The common factor between all three interviewees in what they are aiming for is for students to learn a few key things about how formal systems work. All three also want students to be able to appreciate and enjoy formal systems for themselves. Formal systems are not all there is to logic. The discipline is much richer than that, but anyone who wants to explore the discipline today must understand how the formal systems work. As Ripley points out, teachers must

choose what to include and what to focus on,[12] but an essential part of a modern introduction to logic is this introduction to formal systems.

7.3 WHAT CAN WE SAY TO EXPLAIN LOGIC?

At the beginning I pointed out that pedagogic necessities may influence the way the subject matter of logic is expressed in teaching. The interviews presented an opportunity to explore how those necessities influence teachers' explanations of the subject matter of logic.

All of the interviewees talk about the need to make pragmatic simplifications in their teaching. Russell talks about what she calls fibbing to beginners, explaining that she feels she does it all the time. This happens because she is motivated by practical considerations like avoiding distracting hedging. She wants to create situations where students can see patterns emerging without cluttering the space with more technical, sophisticated, or detailed points.[13] She says that perhaps some necessary assumptions could be set up using conditionals and that students probably would not notice that feature of the course, but it might be a better way to go.[14]

She talks about how it seems only reasonable when teaching adults to say something at the beginning about what the subject matter that the course will cover. Moreover, she talks about how she deals with that in practice by opening the first class with example arguments and leading the class through a discussion focused on the question 'which are the good ones?' That culminates with an explanation that logic is about validity and that 'you might have come to suspect by looking at some of the arguments that any argument of that form is going to be valid. ...formal logic tries to capture that form.'[15] Russell thinks that 'logic is about the world; and to some extent the linguistic world'[16] and she wants students to see that.

Russell explains that you must think very carefully about what information you are going to present students with, and what you want them to get out of it. There are crucial learning experiences, and the setup and design must be right to make those experiences available to students. Russell notes that if you do not understand who your students are, you can make mistakes like pitching 'your class too high, or too formal, or assume a background your students don't have.'[17] These mistakes create an environment that excludes students. Students will not be able to see what they are missing. They might not be able to make sense of the difference between the knowledge and skills and the knowledge or skills that the teacher assumes they

12 See §5.77 and §5.84
13 See §4.38
14 See §4.41
15 See §4.46
16 See 4.112
17 See §4.118

have. They are vulnerable to assuming that the gap is magic that they do not have and cannot have, because they are not not a wizard – only wizards have that magic.

The complexities that surround the subject matter of logic could easily overwhelm a beginner, and presenting this complexity too early may distract from more important lessons. Chapter 3 demonstrates just how common simple explanations of the subject matter are. However, this is not a sign of poor teaching. The explanation given to a beginner needs to be simple. The explanations for beginners are not improved by making them more complex. They are improved by being careful about how they are phrased and pitched.

The simplicity of the explanation of the subject matter is one pedagogic necessity. Another pedagogic necessity is the existence of that explanation.

Ripley points out that he doesn't really think that that logic is about anything.[18] His discussion of the explanation in terms of proof reveals the kind of pressure that Russell alludes to when she talks about teaching logic to adults. She says that college students come into the course with the reasonable expectation that the teacher will start with some introduction that says what this is supposed to be.[19] The practical need to provide a sensible introduction to what the student will learn during the course, forces teachers to say something about what logic is about, even if they would prefer not to. Ripley explains that he solves the problem by looking first at the formal system he will introduce and asking himself, 'what is this good for?' This type of thinking allows him to select an explanation that is well matched to the formal system he introduces.[20]

Ripley explains that he begins with proof because his experiences have taught him that it is a more straightforward concept to grasp.[21] Ripley's approach to logic instruction is atypical. His explanation of logic is grounded in proof-theory. In contrast, it is more common to ground introductory logic instruction in model-theory. Another unusual feature of Ripley's approach is that he does not introduce validity until the middle of the course.[22] The more typical approach is to introduce validity at the beginning. Grounded in proof-theory as Ripley's approach is, he starts by introducing the language and its syntax. Ripley's approach is similar to Woller's approach. Woller describes his approach, saying:

> In the first teaching session, one introduces the symbolic language (the symbols and the formation rules), the notion of a proof *as a game* (the entire point of which is to reach a desired string of symbols from some given strings using definite rules), and one of those rules. One then demonstrates by examples how this rule

18 See §5.36
19 See §4.44
20 See §5.37
21 See §5.79-81
22 See §5.77

> is used in playing the game. A homework assignment is then given using a set of game exercises.[23]

Woller describes a similar experience to Ripley's. By introducing proof systems before semantic systems, it becomes easier for students to grasp specific metatheoretical points. He says:

> A fourth advantage also lies with the syntactical nature of this approach, for the notion of validity is a syntactical one. By having learned to play the games as they have, the students have thus actually experienced what it is to proceed syntactically. They are thus in a somewhat better position to understand those end of the course comments on what it is that logicians do in the study and the construction of formal systems.[24]

This explanation also demonstrates why this approach also works so well for Ripley, who wants to show students what logicians do.[25]

Like Russell, Ripley also confronts the problem of teaching things that you do not think are true: fibbing to beginners. Ripley cites teaching classical logic as a case of teaching something which he disagrees with.[26] Ripley describes the process that he uses to deal with this challenge: first think carefully about what the logic he is going to teach is useful for, then to think about how to best expose students to what he finds fascinating in logic. He talks about the challenge of having to start somewhere. Wherever you start that point will be intimately connected to other topics. For Ripley, the challenge of teaching is to chart a course where students will both see what is going on and see what it is about this material that he finds so captivating. Ripley presents the model-theoretic view to students, despite it not being his preferred view. However, he presents it in a way that places it alongside the proof-theoretic view so that students can understood the two in light of each other.

Like Russell and Ripley, van Benthem faces the problem of selecting what to say to students, and what to omit. His interest is in logics which are much more complicated than classical logic. Yet, there are practical reasons for starting with that logic. He says that he starts with propositional logic, predicate logic, and the syllogistic, to show students how we work. The rest of the course broadens the agenda, but this does not mean that we give up the established style of working; it is a historical achievement.[27]

Van Benthem intentionally shares his pluralistic view of logic with students though he is aware that students will not necessarily react to this the way he does. For him, the potential to

23 Raymond Woller, 'Beginning with Proofs in Introductory Logic', *Teaching Philosophy* 3, no. 2 (1979): p. 169.
24 Woller, p. 172.
25 See §5.41
26 See §5.40
27 See §6.75

connect to many different topics is what makes logic exciting. However, he knows that instead of finding this exciting, some students will find it threatening or overwhelming. He has various aims for his course, but he is aware that no matter what set of aims he chooses, they will not work equally well for all students.[28]

Van Benthem also mentions a point close to my heart. He says, 'the first logic course that people get is so important because it sets their mind and expectations. If you want to change something, it's going to be very hard after the first logic course because their mind has been set, both explicitly and implicitly in the methodology.'[29] An interesting illustration of this point is in the way van Benthem characterises logic in terms of information. Even though he now might not have an explicit memory of what he was told about logic in the first courses he took, the textbook used for his first formal course – Rescher's *Introduction to Logic*[30] – uses the idea of information to characterise logic. This choice is unusual enough to offer some anecdotal support for the powerful way in which an introductory logic course might shape a student's approach and attitude to logic.

The question of the subject matter of logic is a rich and complex topic and the difficulty confronting teachers is that they must draw on this challenging material to design a simple explanation of the subject matter which is only there to set the scene for what is otherwise a technical course. The introduction must walk a fine line. It must be accurate enough that students who will not study logic further do not form mistaken beliefs, and it must prime students who will progress and prepare them for what they will learn later.

7.4 WHAT DO WE THINK ABOUT LOGIC?

Logic courses are aimed at adults, so they naturally start with an introduction that says what this is supposed to be. Sometimes, that explanation also covers the value of learning it. This sort of introduction makes sense because adults have the right to decide whether the potential value of learning something will outweigh the cost. The teacher must say *something* about the subject matter of logic. However, they are constrained because students will not have the context necessary for a more elaborate discussion. As Ripley says, 'You've got to start somewhere, and wherever you start is intimately related to eighty other things that you can't introduce because you have to start somewhere.'[31]

In the end, textbooks represent what teachers think is the best way to explain the subject matter of logic to beginners, not how teachers think about logic. Each interview reveals some

28 See §6.39-40 and §6.86
29 See §6.42
30 Nicholas Rescher, *Introduction to Logic* (New York: St. Martin's Press, 1964).
31 See §5.84

tension between what logicians think about logic and what they say to their students. Nevertheless, the diversity of their choices is indicative of a diversity of views on the subject matter of logic.

Russell's approach to logic is model-theoretic. This is also the approach of all 38 of the introductory texts I examined. Hodges introduces model theory this way:

> Model theory began with the study of formal languages and their interpretations, and of the kinds of classification that a particular formal language can make. Mainstream model theory is now a sophisticated branch of mathematics (see the entry on first-order model theory). But in a broader sense, model theory is the study of the interpretation of any language, formal or natural, by means of set-theoretic structures, with Alfred Tarski's truth definition as a paradigm. In this broader sense, model theory meets philosophy at several points, for example in the theory of logical consequence and in the semantics of natural languages.[32]

Russell would not deny that logic could be taken in alternate ways. However, her approach is distinctly grounded in model-theory. She says that logic is about capturing validity and what she means is capturing patterns of truth preservation among sentences. Her responses show this interest in interpreting languages, formal or natural, using set-theoretic structures. Using the modelling description, her view is that logic models patterns of truth preservation which occur in natural language. However, as Hodges explains, 'model' has two different meanings: 'To model a phenomenon is to construct a formal theory that describes and explains it. In a closely related sense, you model a system or structure that you plan to build, by writing a description of it. These are very different senses of "model" from that in model theory.[33]' What makes Russell's view model-theoretic is her approach to understanding meaning. In Russell's view, the data for logic is related to the semantic theories for natural languages. In this way, despite the differences, there is a strong connection between Russell's view and Ripley's.

Russell accepts that there are multiple projects which might use formal systems.[34] I suspect that she accept their right to be called logic. The point to observe here is that different projects might result in quite different formal systems. Ones that have nothing to do with looking at patterns of truth preservation among sentences.

Russell calls attention to Etchemendy's argument in *The Concept of Logical Consequence* and how it points out that there are two ways of thinking about what has happened when a model is changed. One can think about different possible worlds or think in terms of different

32 Wilfrid Hodges, 'Model Theory', in *The Stanford Encyclopedia of Philosophy*, Fall 2018, ed. Edward N. Zalta (17 July 2013).
33 Hodges.
34 See §4.55

possible meanings. Russell has a view, and supporting arguments, on which of these options is better. Still, her response is also pluralistic because she allows for the possibility of different reasons and different conclusions.[35]

At the end of the conversation, Russell asked me, 'do you think just because two logics disagree on the extension of the relation of validity that they'll have different definitions of validity?' My response is to point out that there is a sense in which a definition of validity cannot be separated from the philosophy which supports it. Russell talks about how the way she defines validity changes through her introductory course saying, 'What changes is what I'm quantifying over: It starts with possible situations, and then it moves the rows of the truth table, and then it moves to models.'[36] However, her definitions all come from the same model-theoretic framework. When discussing pluralism, she says that she thinks that different logical systems are different theories of logical consequence. What they are competing over is the extension of the logical consequence relation. Because I see the definition of validity as intimately connected to its philosophy, I see the possibility for formal systems that have the same language and agree on the extension of the relation of validity and yet have fundamentally different ways of explicating validity.

When I asked Ripley what logic was about his first points out there is an important sense in which logic is not really about anything.[37] I think this is a crucial point. Russell echoes this point when she says that the subject matter has a kind of independence with some formal topics. There is a way of looking at logic just as formal systems divorced from their application. I think that seeing this point is essential for forming a framework for understanding logic flexible enough to cover the wide range of topics and activities that occupy logicians time and effort. Ripley clarifies that he would prefer not to claim that logic is about anything, but that if pressed into it he would explain logic as being about proof.

Ripley explains that he works in a broadly proof-theoretic tradition. In the *Stanford Encyclopedia of Philosophy* article on proof-theoretic semantics Schroeder-Heister explains:

> in general proof theory we are not solely interested in whether B follows from A, but in the way by means of which we arrive at B starting from A. In this sense general proof theory is intensional and epistemological in character, whereas model theory, which is interested in the consequence relation and not in the way of establishing it, is extensional and metaphysical ... Proof-theoretic semantics is an alternative to truth-condition semantics. It is based on the fundamental assumption that the central notion in terms of which meanings are assigned to certain ex-

35 See §4.83-9
36 See §4.103
37 See §5.36

> pressions of our language, in particular to logical constants, is that of *proof* rather than *truth*.[38]

Ripley explains that his work uses the sequent calculus's proof-theoretic tools to examine coherence and the conditions under which collections of speech acts (as assertions and denials) do or do not fit together.[39] Despite the difference in background theory, Russell and Ripley's interests coincide. They are both interested in the relationship between logic and natural language semantics. They have different semantic theories and they examine language in different ways, but at least some of what is interesting about logic to both of them is what we can learn about natural language semantics by studying logic. Regardless of their preferred semantic theory, both see logic as modelling natural language. They also both agree that evidence in logic is linked to semantic theories of natural languages.

There are several other commonalities between Ripley and Russell. Neither of them connects logic with reasoning, and neither of them thinks that logic is normative. Finally, we can see that the tradition of thought they are each grounded in plays a part in the way they construct and present logic to their students.

In contrast, van Benthem has a much more pluralistic view of logic. He says that 'a subject is, in some sense, the sum total of all topics that its practitioners have found interesting.'[40] and introduces the Zwarts syllogism as an amusing way to make this point.[41] He does not think there needs to be a unifying feature or subject matter to make the discipline valuable. Van Benthem adopts a mode of thinking which I think is very useful. He uses categories to sort and make sense of the landscape of topics in logic. He relates the theory of deduction to proof and inference, which is connected to proof-theory. He connects definition to the use of language and model-theory. These are topics that I have introduced and discussed. However, van Benthem goes on to add another topic: algorithm or computation. He connects the topic of computation to recursion theory. He points out that in introductory logic teaching even though the material may cover all these topics the explanation may emphasise only one topic and suppress the other two.

Van Benthem makes consistent use of a strategy of thinking in twos and threes: breaking a topic down into two or three components or looking at a group as being composed of two types. He is not fixed in how he views these categories; they are a useful way of grouping things to build a perspective, but it is only that: a perspective. It is a way to gain insight into a complex world, not an assertion of permanent or fixed truth. I think this is a useful approach for

38 Peter Schroeder-Heister, 'Proof-Theoretic Semantics', in *The Stanford Encyclopedia of Philosophy*, Spring 2018, ed. Edward N. Zalta (1 February 2018).

39 See §5.66

40 See §6.23

41 See §6.21-2

dealing with complex topics. The categories themselves do not have to be entirely accurate, just a good enough approximation to give insight into a complicated landscape. For instance, van Benthem understands some proof-theorists think of it as an alternative to semantics; thinking that meaning comes from the proof rules. So, when he relates definition to model-theory, he is also aware that at least some, if not all, proof theorists take a different approach to *definition*. Nevertheless, that does not undermine the utility of the rough and approximate division that he offered. He talks at least twice of people dividing into roughly two groups. However, he is careful to point out that this is not a fixed and immutable division. It is just a heuristic, a way of remembering that people's understandings, experiences, and responses will vary.

Van Benthem's approach to logic reflects this very adaptable way of thinking. He sees the world as full of structures, and logic as a useful tool for investigating those structures. He says, 'There are a lot of interesting structures in reality, which could be good for logic. I don't see any problem with that. Just imagine a wonderful new theory of formal systems which consist of rules plus memory. And think of new Gödel theorems about what you can and cannot achieve with such an architecture.'[42] Like Ripley, van Benthem resists the idea that logic, as a discipline, needs to be about something. There are the formal methods used in logic, but that is not all there is to logic. There is also the topic that you are studying. However, van Benthem resists the view that there must be a single unifying topic of interest for the whole discipline.

What gets van Benthem excited is the study of information in the broad sense. He is interested in how information is structured, but also in how it is assembled and modified. He talks about the role of inference, observation, and communication in solving problems in an information space. He says, 'Logic is about all rational information seeking activities.'[43]

Van Benthem also recognises three different approaches to validity: the semantic approach where validity is explicated in terms of truth which he connects to Tarski and Bolanzo pointing out that this does not represent a monolithic view; the proof-theoretic approach where validity is explicated in terms of proof which he connects to Gentzen; and a pragmatic or game-theoretic approach where validity is explicated in terms of having a winning strategy which he connects with Lorenzen.[44]

Van Benthem explains that he sees logic as lying at a crossroads. A point of connection between different things. From logic, you could travel to computer science, psychology, mathematics, philosophy, and more. This pluralistic view creates a challenge, as not all students enjoy exploring these connections.[45]

42 See §6.67
43 See §6.48
44 See §6.53-4
45 See §6.39

Van Benthem's view is unusual in logic in two ways. First, much of what he is interested in researching brings human activity back into the picture of what logic is about.[46] Secondly, his view is more explicitly pluralistic. It sets an agenda for logic that is much broader than could be captured by any simple slogan.

7.5 WHAT COULD LOGIC BE ABOUT?

Through the interviews, there are diverse views on the subject matter of logic. In contrast to G. Russell and Ripley, van Benthem's view on logic is more pluralistic. However, even when logicians have preferred views on logic's subject matter (or lack thereof), they may still be open to different ways of understanding it.

There are multiple possible ways to understand the overall project of logic. This is evident in the different strategies that teachers use to explain the project of logic. It is also evident in the different ways that logicians themselves conceive the project of logic. We can talk about logic in a unified way as an investigation into validity. However, in this context, validity is ambiguous.

Clarification about what is truly meant by validity is not necessary for the business of logic to begin, and much fruitful logical work can be conducted without this clarification. However, to answer questions about correctness that extend beyond the technical soundness and completeness proofs, this clarification is required. When asking whether double negation is genuinely valid, we need to do more to specify what constitutes genuine validity.

In seeking this clarification logicians sometimes appeal to a 'pre-theoretic' definition of validity. I offer a different approach. Instead of seeking some pre-theoretic definition of validity, we need a framework for constructing explanations of validity. I propose developing different ways of understanding validity by defining different possible logical projects. These logical projects establish boundaries for the definition of validity. They are the background against which particular definitions must be tested.

A logical theory is a theory of how the domain in which a logic is interpreted behaves. The logical project is that domain. The logical theory is tied to the logic's definition of validity. The logical project supplies an inter-theoretic validity – the kind of validity against which the technical definitions generated inside the logical project may be tested. Logical theories belonging to the same logical project can be tested against each other and the project's boundaries. Logical theories which do not belong to the same project cannot meaningfully be compared. Within each logical project, pluralism is an open question. Each project must fact the question of whether it could contain multiple logical theories each equally correct.

46 See §6.31

Below is a discussion of how different logical projects might be conceived, their objectives and their boundaries. I think there are more possibilities than what I outline below, but these are enough to make my point that there is more than one way of thinking about logic. Each of these projects covers a broad scope, with perhaps some overlap of interest with other projects. I use a topical breakdown, but what distinguishes domains from one another has less to do with the research topic, and more to do with what counts and does not count as data.

One possible project goes back to the traditional subject matter of logic. This view could be characterised by the slogan 'logic is about reasoning'. The data in this project is psychological. It could be characterised as a project to model human reasoning or inferring.

A considerable risk for this project is pre-judging reasoning processes. Where relevant systems exist like classical logic, mathematics, and statistics, those systems might be applied to judge human conclusion drawing processes as right or wrong. These judgements are a distraction from questions which are more central to the project. This project focuses on what processes there are, right or wrong according to some system or other does not come into it.

The project is part psychological and part logical. Psychological to gather data on reasoning, logical in generating models of conclusion drawing systems. The systems are always models because they are built to capture some features and not others, but could further idealise, by assuming some level of performance. For instance, a project in this research programme may disregard non-neurotypical or abnormal performance. The project may constrain itself to only the most normal cases occurring in ideal conditions.[47] Individual projects in this research programme also do not have to model all possible types of reasoning. It is acceptable for a project to focus only on modelling some subset of reasoning. In this way, classical logic may belong to this logical project if it can lay claim to accurately modelling some human reasoning process. However, being only one small fragment of the reasoning processes available, classical logic cannot lay claim to being all there is to this project.

This project may overlap with, or perhaps exist as a subset of the project which van Benthem sketches under the tagline 'logic is about information.'[48] Yet these projects are easily distinguished by their data. Projects which are about reasoning maintain a strong connection to psychological data and psychological data alone. Projects about information may consider information more abstractly and include more than human reasoning systems. Including information transfer in physical systems, or information updating in computer systems.

Also, in considering 'logic as information,' we may take the idea further as Allo and Mares do when they argue for treating information semantics as 'a genuine alternative explication of

47 So the reasoning considered is idealised, but not idealised according to the standards of some pre-determined system. Idealised in the sense that only optimal performance is considered.

48 See §6.47-9

the notion of logical consequence alongside the traditional model-theoretical and the proof-theoretical accounts.'[49]

Psychological and informational projects have no metaphysical concerns. Questions of whether the conclusions reached are true are simply outside the project's scope. Similarly, how information is conveyed in a language is outside the project's scope. Those questions are no less logical, but they belong to other projects.

Another possible project could be characterised by the slogan 'logic is about good reasoning'. This project aims to descriptively model norms which govern activities like inferring, asserting or believing. The project is part sociological and part logical. Sociological, to gather data on the norms which exist. Logical, in the attempt to generate consistent rule systems which conform to that data.

This project has something in common with other sociological projects investigating how societies are structured while understanding that they could be structured differently. The commonality is the potential for an implied critique of human activity. For instance, what if we observe that there is no consistent model for some group of norms? Or, what if the rules that we would endorse for believing would lead us to accept inconsistent beliefs in some situations? How should we respond to these discoveries individually and as a society? It is easy to conduct the sociological study that uncovers that shaved legs are typical among women in modern western societies. It is also possible to point out that social processes encourage people to conform to this norm, but that the norm itself is not inevitable and could be changed. Which raises the question 'should women shave their legs?'

The focus of this project is on the rules which govern the relationships between situations. What may, or must, be believed given certain other things are believed. This project explores how we might model these rule systems, the advantages and disadvantages of the rule systems, and whether they can capture the rules which govern the target activities.

Other norms may govern assertion or belief like etiquette, political correctness, and prohibitions against hate speech which are not within this project's scope. Inference marks this project's concerns, but it is still distinctly normative. The project is less concerned with the inferences themselves and more concerned with how we judge them.

It is not clear to what extent advocates for a normative understanding of logic like Field might endorse this project. Field says, 'Whatever other merits proof-theoretic and model-theoretic accounts of validity may have, they are not remotely plausible as accounts of the meaning of valid.'[50] He also says, 'validity attributions regulate our beliefs,'[51] and, 'a disagreement about

49 Patrick Allo and Edwin Mares, 'Informational Semantics As a Third Alternative?', *Erkenntnis* 77, no. 2 (December 2011): p. 167.

50 Field, 'What Is Logical Validity?', p. 33.

51 Field, p. 41.

validity ...is a disagreement about what constraints to impose on one's belief system.'[52] Which seems to indicate affiliation with the project I describe. However, Field gives no indication of what data he would consider relevant. He might consider the sociological data of what constraints people are inclined to impose on their belief systems irrelevant and reject the project I sketch entirely. His question may be more akin to asking 'should women shave their legs?' than to whether it is common for people in a society at a given time to endorse leg shaving as an activity for women. More like, 'what is the logic we should use to constrain beliefs?' than studying the constraints on beliefs which we endorse. I do not see a straight path to answering that sort of meta-sociological question. Still, some theorists do pursue them.

A third (or fourth, depending on how you are counting) possible project could be characterised by the slogan 'logic is the science of truth' This project is distinctly metaphysical. It has to do with the properties of truth and the modelling of space-time. The data for this project is the material world. Since it is a project about the material world, it inherits epistemic and measurement problems common to all projects that have to do with the material world. Problems with observation, object constitution and identity, particularly identity over time.

This project's risk is that we confuse how we speak of the physical world for how the physical world is. In this project language, or how we speak of the world is not data; it is irrelevant. This project is not interested in the possible truth values of sentences. This project is concerned with assigning truth values to propositions and the effects on the models of space-time of those decisions. It explores questions relating to the ontologies generated by different logical systems and whether they must be accepted as existing in space-time.

This is the sort of project which Williamson[53] might endorse. As Russell notes,[54] he too may accept that there is more than one way to conceive logic. However, the logical project he contributes to most is this investigation of the world's structure.

In this project, logic is related to physics and mathematics. It is the approach to logic required to understand Putnam's argument for choosing quantum logic over classical logic in light of findings in quantum mechanics[55] and the downstream arguments. For instance, Williamson argues that where there is a case external to mathematics for revising logic, for instance, Putnam's, the implications on mathematics that may arise because of that revision should be

52 Field, 'What Is Logical Validity?', p. 42.

53 id: https://www.worldcat.org/identities/lccn-n90665594 or, https://orcid.org/0000-0002-4659-8672

54 See §4.131

55 Hilary Putnam, 'Is Logic Empirical?', in *Boston Studies in the Philosophy of Science: Proceedings of the Boston Colloquium for the Philosophy of Science 1966/1968*, ed. Robert S. Cohen and Marx W. Wartofsky (Dordrecht: Springer Netherlands, 1969), 216–241; Hilary Putnam, 'How to Think Quantum-logically', *Synthese* 29, nos. 1/4 (1974): 55–61.

taken seriously.[56]

Another way of thinking about logic is to consider language, linguistic expression, and meaning in languages, both natural and formal. It is this territory that G. Russell points to when she says:

> Take first order Tarski models of a language; you've assigned extensions to the predicates and referents to the names, but what are those models supposed to represent? And when you changed a model – so you've changed the extension of some predicate or the referent of some name – what does that difference between the two models represent? And Etchemendy says, there are two ways of thinking about it. One is the models represent different ways the world could be. So, you could think of the different first-order models as different possible worlds. And that's what you were sketching there. And then another way, which is what Tarski had in mind originally, is that you're thinking about different possible meanings the predicates could have; and you're thinking about reinterpreting the non-logical parts of the language.[57]

This view is not so amenable to slogans, but G. Russell's 'patterns of truth preservation among sentences.'[58] and Ripley's 'mathematical models of language.'[59] are both in this space. We also see some of this linguistic focus in Cook's expression of philosophical logical pluralism: 'The correct logic (relative to the goals of philosophical logic) is relative to the linguistic phenomenon being represented.'[60]

In this project, meaning and speech acts are central, and natural language forms the background data. G. Russell and Ripley approach this project in different ways with G. Russell interested in theories of meaning and interpretations for the language and Ripley interested in the coherence of collections of speech acts. G. Russell works in the model-theoretic tradition while Ripley belongs to the proof-theoretic tradition. Further, Allo and Mares[61] could be seen as setting up a third to explore this project with an informational account of meaning. As with all the other projects, this project is rich with possibilities: different ways to interpret and explore the topic while still belonging to the same broad project.

The last possible view of logic that I will discuss could be characterised by the slogan 'logic is about argument'. Van Benthem discussed this possibility saying:

56 Timothy Williamson, 'Alternative Logics and Applied Mathematics', *Philosophical Issues* 28, no. 1 (2018): 399–424.
57 See §4.86
58 See §4.61
59 See §5.63
60 Cook, 'Let a Thousand Flowers Bloom: A Tour of Logical Pluralism', p. 494.
61 Allo and Mares, 'Informational Semantics As a Third Alternative?'

> there's a third view of validity which you often don't get to see in textbooks,[62] which was brought out by Lorenzen namely the pragmatic or game-theoretic intuition. Lorenzen in his work on dialogue games in the nineteen fifties said that if you look at the history of logic, it's likely that the patterns that were considered valid had something to do with what was successful in argumentation. Lorenzen's made this view precise. He said he here's another intuition of validity: a conclusion C follows from premises P, if in argument or dialogue a person who claims the conclusion against an opponent who is willing to grant the premises has a winning strategy. So, you might say here that the intuition is that validity is something that's compelling.[63]

Hintikka presents a view like this in 'The Role of Logic in Argumentation.'[64] In this project, logic becomes interactive, with interactions understood as moves in a game. The data is the process of dialogue or information seeking. The logicians objective is to formalise that interactive framework. This project – the study of argumentation – is currently an active research area attracting much interdisciplinary attention.

The point of discussing these many ways in which logic can be conceived is not to insist that all activities that belong to logic do belong to some well-defined project or other. It is also not to suggest that they should. I think that would be stiffing. Equally, I do not expect that a well-defined project or consistent views are necessary pre-requisites for productive logical research.

The point of explaining logic in as possible projects is to create a framework for understanding activities in logic. So that when it becomes necessary to make sense of or understand, some logical system or approach, we have a place to turn. Ripley describes going through this process intuitively. He says:

> I think, well, what is this language immediately good for? What is what's its real use?... We got it out of the Hilbertian and pre-Hilbertian moment where people were trying to formalise the notion of proof. We had the practice of mathematical proof starting to reflect on itself. And there is a reason they ended up at something like classical logic. It's not a coincidence; they didn't do a bad job of it. It's a good answer to a question in that area.[65]

When we are called upon to make sense of logical systems, to give an account of the point of

62 The reader may wonder at this comment given that earlier I pointed out that argument is the most frequently used word in the textbooks I examined, but recall, in those textbooks authors treat 'argument' and 'reason' as synonymous. So the use in textbooks is quite different to the dialogical sense of argument which van Benthem is using here.

63 See §6.53-4

64 Jaakko Hintikka, 'The Role of Logic in Argumentation', *The Monist* 72, no. 1 (1989): 3–24.

65 See §5.37

some activity or other, I think the best process to follow is one that constructs that meaning by reflecting on what that system would be suitable for. With formal logics, we might ask, 'what is this system a good model of?' These kinds of questions help us supply satisfactory explanations when we are called upon to make them.

TEACHING INTRODUCTORY LOGIC

> Observe the bee or the fisherman,
> you will not find any trace of
> reasoning or geometry; you will find
> only a stupid attachment to custom.
>
> Alain, Propos d'un Normand,
> September 1, 1908

8.1 EDUCATION THEORY IN THE 21ST CENTURY

Alain's poetic description obscures the role of theorising in the development of boat design. As with boat design, theorising may not be necessary for developments to occur in education. Nevertheless, theorising compliments the demands of the environment in driving development. Late in the 20th century, educationalists began to discuss the changes to the education system that would be needed for the 21st century.

With our increasingly interconnected world, ideas like this are seldom explored within one jurisdiction alone. Yet each jurisdiction varies, so common global ideas are often adapted to a local context. The 2012 report 'Supporting future-oriented learning and teaching: A New Zealand perspective' discusses the demands of the social environment, setting the context for theorising. It explains:

> During the latter half of the 20th century, international thinking about education began to shift to a new paradigm. This shift was driven by an awareness of massive and ongoing social, economic and technological changes, and the exponentially increasing amount of human knowledge being generated as a result. International thinking began to seriously examine questions about the role and purposes of education in a world with an unprecedented degree of complexity, fluidity and uncertainty.
>
> Alongside economic, social, political and technological changes, many serious challenges characterise the 21st century world. Some authors describe these as 'wicked problems.' They are 'highly complex, uncertain, and value-laden,' spanning multiple domains: social, economic, political, environmental, legal and moral. It is argued that learners – *and* teachers, school leaders and families/communities

> – need support to actively develop the capabilities they need to productively engage in 21st century wicked problem solving.[1]

The need to change the Industrial Age-based traditions of learning and teaching is an explicit theme embedded throughout the report. Driven by globalisation[2] and massification,[3] developments in the past half-century in higher education are at least as dramatic as the developments in the 19th century, which shaped the modern research university.[4]

The 2012 New Zealand report reflects what education has been in the 20th century, and where it might go in the 21st century as global social and economic change continues. It discusses a model of education which 'foregrounds the development of learners' dispositions, capacities or competencies to deal with new situations and environments, including those with high degrees of complexity, fluidity and uncertainty.'[5]

One example of theorising, which supports this model of education, is the outcomes-based teaching and learning approach of Biggs and Tang. They propose a process of constructive alignment in which 'we state what we intend the general outcomes a graduate of a university should achieve, and following from that, we derive the content-based programme and specific course level outcomes.'[6] Tertiary institutions commonly develop and publish a graduate profile which expresses the attributes the institution aims to develop in the students. An often-cited definition of 'graduate attributes' is:

> Graduate attributes are the qualities, skills and understandings a university community agrees its students should develop during their time with the institution. These attributes include but go beyond the disciplinary expertise or technical knowledge that has traditionally formed the core of most university courses. They are qualities that also prepare graduates as agents of social good in an unknown future.[7]

Biggs and Tang explain that 'intended learning outcomes (ILOs) apply at the institutional level as graduate attributes, or as we prefer, graduate outcomes, and at the programme and

1 Rachel Bolstad et al., *Supporting Future-oriented Learning & Teaching – a New Zealand Perspective* (Prepared for the New Zealand Ministry of Education, 2012), p. 2.

2 The increasingly economically interconnected world with a corresponding rise in communication and cultural exchange.

3 The growing rate of enrolment in tertiary education (indicatively depicted in figure 2.16).

4 Philip G. Altbach, Liz Reisberg and Laura E. Rumbley, 'Trends in Global Higher Education: Tracking an Academic Revolution', in *2009 World Conference on Higher Education* (Paris: United Nations Educational, Scientific / Cultural Organization, 2009).

5 Bolstad et al., *Supporting Future-oriented Learning & Teaching – a New Zealand Perspective*, p. 13.

6 John B. Biggs and Catherine Tang, *Teaching for Quality Learning at University* (Maidenhead, UK: McGraw-Hill Education, 2011), p. 10.

7 Bowden *et al.* 2000 cited in Simon C. Barrie, 'Understanding What We Mean by the Generic Attributes of Graduates', *Higher Education* 51, no. 2 (2006): p. 217

course levels. Graduate outcomes provide useful guidelines for designing programme outcomes, which are in turn addressed by the outcomes of specific courses.'[8]

While there are significant practical challenges involved in constructing graduate and programme outcomes, educators have a duty to engage in conversations about the utility of what they are teaching. The perennial question for educators must be, as van Benthem so aptly put it, 'what are we teaching, and why?'[9] Teachers need to think carefully about what will be most useful for their students in future contexts. Engaging in dialogues about the intended outcomes of learning is one way to fulfil that duty. Debating which graduate and programme outcomes best fit the context is one way in which experienced educators can engage in these valuable conversations.

The reason that considering the intended learning outcomes of a philosophy programme is so essential when designing a logic course is that it helps with the decision whether to teach a logic course *at all*. The discussion of the intended outcomes of a university education informs the discussion of the intended outcomes of the philosophy programme. These discussions must always be relative to the local context – taking into account the educational backgrounds, gender and ethnic diversity, and socio-economic circumstances of the students.

The needs of the philosophy programme, inside of its institutional context, should dictate whether the programme contains logic courses. Critical thinking and argument analysis are skills which are often central in a philosophy programme. Sometimes teachers assume that in learning logic, students will learn to think critically and analyse arguments – however this assumption does not hold.

Critical thinking and argument analysis are likely to be high on the list of desired outcomes for any philosophy programme, but this does not imply the need for logic instruction. Those outcomes might be equally, and more reliably, achieved in other courses specifically designed to achieve them.[10,11] Logic instruction is not required for learning critical thinking; and both teachers' experience and relevant research seem to disprove the claim that studying logic will develop students' abilities in identifying, reconstructing and analysing natural language arguments.[12] Critical thinking, argumentation theory and informal logic are areas of active research closely related to logic. These subjects may lay equal claim to belonging in a philosophy programme and may prove to be better foundations for introductory teaching depending on the needs of the programme.

8 Biggs and Tang, *Teaching for Quality Learning at University*, p. 113.

9 See §6.79

10 Nada J. Alsaleh, 'Teaching Critical Thinking Skills: Literature Review', *TOJET: The Turkish Online Journal of Educational Technology* 19, no. 1 (2020): 21–39.

11 Raymond S. Nickerson, 'Chapter 1: On Improving Thinking through Instruction', *Review of Research in Education* 15, no. 1 (January 1988): 3–57.

12 Alsaleh, 'Teaching Critical Thinking Skills: Literature Review', p. 170.

In 1987 Nisbett and colleagues articulated the doctrine of formal discipline: that 'the study of abstract rule systems trains the mind for reasoning about concrete problems'[13] They go on to say: 'One of the first endeavors of the new discipline of psychology was to provide experimental research that cast doubt on the formal discipline concept. The most effective antagonist was Thorndike, who undertook a program of empirical research on transfer of training effects that remains impressive by today's standards. Thorndike rarely found strong transfer effects.[14]' This failure to transfer in the manner assumed is referred to as the problem of transfer. Nisbett goes on to argue that it is possible to use instruction in formal methods to improve student's reasoning. Notably, Nisbett finds that this result does not hold for training in propositional logic.

This lack of improvement was demonstrated with the Wason Selection Task. In this task participants are presented with a conditional: *If P then Q*, and four cards; as shown in figure 8.1. Participants are told that the cards below have information about four situations. Each card represents one situation. One side of a card tells whether *P* happened, and the other side of the card tells whether *Q* happened.

Figure 8.1: Wason Selection Task

Participants are asked to indicate only those card(s) which definitely need to be turned over to see if any of these situations violate the conditional. The conditional is violated by any situation in which *P* happens, and *Q* does not. The correct answer is to choose only the *P* card (to see if there is a '*not Q*' on the back) and the '*not Q*' card (to see if there is a *P* on the back).

There is a consistently high, widely replicated error rate on this task which logic training does not ameliorate. Nisbett's study reflects these findings. The error rate on the abstract[15] Wason Selection Task is surprising, but this information might not be as relevant as it seems since it's not clear that improvement on this specific task is, or should be, the principal goal of logic instruction.

13 R. Nisbett et al., 'Teaching Reasoning', *Science* 238, no. 4827 (October 1987): p. 625.

14 Nisbett et al., p. 625.

15 As opposed to a task based on a social rule like the legal drinking age where *P* and *Q* are replaced by 'drinking beer' and a person's age. Tasks bases on social rules produce more reliably correct card selection.

The problem of transfer is perhaps best understood the way van Benthem framed it when he said that when we teach logic, we should be teaching something useful.[16] In the exchange between teacher and learner, it is the teacher's duty to ensure that what students learn will be useful. In this sense, the problem of transfer is a general problem for many disciplines, not an issue which is specific to logic instruction.

The problem of transfer seems especially acute for logic because of the inflated claims about the power of logic instruction in the popular conscience. These cure-all claims plagued even Whately, who wrote:

> On the utility of Logic many writers have said much in which I cannot coincide, and which has tended to bring the study into unmerited disrepute. By representing Logic as furnishing the sole instrument for the *discovery of truth* in all subjects, and as teaching the use of the *intellectual faculties* in general, they raised expectations which could not be realised, and which naturally led to a re-action. The whole system, whose unfounded pretensions had to be blazoned forth, came to be regarded as utterly futile and empty: like several of our most valuable medicines, which when first introduced, were proclaimed, each, as a panacea, infallible in the most opposite disorders; and which consequently, in many instances, fell for a time into total disuse; though, after a long interval, they were established in their just estimation, and employed conformably to their real properties.[17]

Learning logic, just like learning any other subject, offers no guarantee of learning skills which will be useful for other applications. Educationalists have devoted much effort to producing solutions to this problem, and supplying the tools teachers need to ensure that logic teaching will be useful.

Logic, just like any other discipline, can be used to enable the development of transferable skills like critical thinking. But the way this is achieved is not by assuming that these skills will be automatically developed, or learning transferred. Formal logic should be thought of as part of the disciplinary expertise or technical knowledge which may or may not be required by the programme. The needs of the philosophy programme dictate whether this disciplinary expertise is needed. They also provide the framework for determining what sort of skill development should be embedded in logic instruction, if it takes place. If the curriculum delivers a clear case for logic teaching, there are two other critical challenges for 21st century logic teaching: inclusive instruction, and the two-hump curve.

The need for inclusive teaching is driven by changing social attitudes, massification, and

16 See §6.79

17 Whately, *Elements of Logic*, p. viii.

increasing diversity in the classroom. The two-hump curve describes the situation in which the class divides into two groups: one group understands the course content and finds the course too easy, while the other finds the course impenetrable.

8.2 INCLUSIVE INSTRUCTION

The most comprehensive discussion of the first challenge – inclusive teaching – is found in feminist discourse. Frederique Janssen-Lauret opens the abstract of her paper on strategies to redress the gender imbalance in logic classes with the statement that 'Logic is one of the most male-dominated areas within the already hugely male-dominated subject of philosophy.'[18] She goes on to say, 'Men dominate not only the literature, but also the classroom, both as teachers and students. As lecturers and tutors on courses in formal and philosophical logic, we know that these courses generally contain more male students than female students.'[19] Research into the potential sources of gender imbalance indicates that gender schemas/stereotypes, beliefs about innate ability, and implicit biases are the most likely sources.[20]

We live in a world with a legacy of sexist and racist beliefs. As social attitudes turn away from these beliefs, and the diversity in classrooms increases, classrooms become a situation in which sexist and racist beliefs can be challenged. Teachers who want to challenge these beliefs design classroom activities which stress that success in logic is not a matter of innate ability, and include women and non-western authors in the syllabus.[21,22]

Janssen-Lauret mentions the difficulty of including authors who are women in formal logic courses which rely on textbooks, noting that few textbooks are authored by women.[23] This claim is borne out in my collection of 38 textbooks, where only three of those have women's names presented as the sole author; one where a woman appears as the first author in a group of three authors, and two where a woman is an editor of a recent edition of an established textbook.

Another problem with logic textbooks, besides their authorship, is assumptions about the generic student, and using terms which package assumptions about the normal or default case. A writer may present the generic case in the masculine form, or otherwise package assumptions about common household items, actions, experiences, and social relations in a way which marks a definite 'other'.

18 Frederique Janssen-Lauret, 'Making Room for Women in Our Tools for Teaching Logic: A Proposal for Promoting Gender-inclusiveness', in *4th International Conference on Tools for Teaching Logic*, ed. M. Antonia Huertas et al. (Rennes; France, June 2015), p. 65.

19 Janssen-Lauret, p. 65.

20 Tom Dougherty, Samuel Baron and Kristie Miller, 'Female Under-representation among Philosophy Majors: A Map of the Hypotheses and a Survey of the Evidence', *Feminist Philosophy Quarterly* 1, no. 1 (2015): 1–30.

21 Janssen-Lauret, 'Making Room for Women in Our Tools for Teaching Logic'.

22 Anand Jayprakash Vaidya, 'Does Critical Thinking and Logic Education Have a Western Bias? The Case of the Nyaya School of Classical Indian Philosophy', *Journal of Philosophy of Education,* 51, no. 1 (2017): 132–160.

23 Janssen-Lauret, 'Making Room for Women in Our Tools for Teaching Logic', p. 71.

It is almost inevitable that textbooks will reflect something of the cultural context of the time and place in which they are written. For example, Whately's 1826 textbook contains:

> If penal laws against Papists were enforced, they would be aggrieved: but penal laws against them are not enforced: therefore the Papists are not aggrieved.[24]

While Kalish and Montague's textbook from 1964 includes:

> All Communists are Marxists. Some Communists are American.
> ∴ Some Marxists are American.[25]

Reflecting the contemporary social and cultural context is not inherently wrong, but sometimes textbooks can include pernicious notions better left out. For instance, Lewis Carroll's *Symbolic Logic*[26] has striking examples, including:

- No one, who hangs up his hat on the gas-jet, can be a man that is kept in proper order by his wife
- No photograph of a lady ever fails to make her simper or scowl
- No Jews are honest
- [N-word]s have woolly hair

Othering can appear in many ways, from the obvious – such as in 'No Jews are honest' – to the less obvious. It can also be multi-dimensional. For instance, consider 'No one, who hangs up his hat on the gas-jet,[27] can be a man that is kept in proper order by his wife'. This example entangles the generic 'one' with a presumption of maleness, expresses social expectations of husbands and wives, while also implicitly suggesting that every man wears a hat and has a gas-jet.

The textbook's exercises are the most obvious source for examples of pernicious notions, but they can be embedded throughout. Where sexist and racist beliefs are present in society, it is almost inevitable that they will find their way into the exercises of logic textbooks. For instance, in Whately, we find:

2. None but Whites are civilized: the ancient Germans were Whites: therefore they were civilized.
3. None but Whites are civilized: the Hindoos are not Whites: therefore they are not civilized.[28]

While Kalish and Montague present:

24 Whately, *Elements of Logic*, pp. 240-241.
25 Kalish and Montague, *Logic*, p. 117.
26 Lewis Carroll, *Symbolic Logic: Part I, Elementary* (London: Macmillan, 1896).
27 an archaic light fitting
28 Whately, *Elements of Logic*, p. 240.

31. If a father has only male children, then he does not have to provide a dowry for any one of them.[29]

This example exists alongside the examples from physics, algebra, and geometry such as:

33. If x is an integer greater than or equal to zero and every integer is divisible by x, then x is equal to 1.

All teaching is to some extent exclusionary – it divides people who know, from those who don't, and creates a hierarchy of achievement. There is always the chance that success or failure in learning will be mixed into a student's identity. However, logic teaching is a delicate matter because being logical is valued over being illogical; as opposed to being a musician, which is not better than being a non-musician.

Many logicians are quite familiar with the gulf between knowing logic and 'being logical'. Still the perception persists, and it is one which the conscientious teacher should bear in mind. 'Logic' is a culturally loaded concept. It is wrongly associated with 'rationality', western European cultural origin, and maleness. This too, can interact with students' identities both before and during teaching. Logic instruction can either passively reinforce these background associations or actively resist them.

Nye argues that logic is a tool developed by men to exclude women – to keep them from power.[30] But it is not *logic* which is used to exclude people; it is *language*. The communication which occurs in teaching is complex and multi-dimensional. It can, in many ways, contain information about in-groups and out-groups, who belongs and who does not.

The process of breaking down exclusionary structures requires the continuous critical examination of the messages contained in teaching. Some parts of the complex communication which occur in education are more tractable than others. Teachers and textbooks can emphasise that success does not come from innate ability. They can take steps to show the diversity of those who have made original contributions to logic.[31] They can check for the use of sexist and racist language. So long as in-group and out-group power dynamics are a feature of society, the job of critically examining teaching for exclusionary messaging will remain. Without continual work to break down exclusion and bias teaching will inevitably perpetuate them instead.

29 Kalish and Montague, *Logic*, p. 148.

30 Andrea Nye, *Words of Power: A Feminist Reading of the History of Logic* (New York: Routledge, 1990).

31 Where instruction contains a male and western bias, the availability heuristic would lead students to assume that this represents the actual distribution of contributors to logic. If instructors do not want to perpetuate a bias they must find ways to present a more likely distribution in the classroom, on which does not suggest to students that that maleness and western cultural origin are essential for being able to contribute to logic.

8.3 TRANSFORMING THE TWO-HUMP CURVE

The second challenge for the logic teacher is the two-hump curve, where students divide into those who intuitively understand the instruction and those who do not. This division works against any teacher who wants to build an inclusive environment. G. Russell associates classroom exclusion with assuming a background that students don't have. She elaborates:

> If you're teaching it as if they're all computer science students when they're all philosophy students, or maybe you're teaching it as if they've all got strong backgrounds in set theory and they haven't. So, they don't understand what's going on; they don't have certain bits of vocabulary, or certain proof techniques that you're assuming. They're going to end up feeling that it is some sort of magic, and they don't have it – they're not wizards. And so, they're excluded by it.[32]

There is also the danger that what is being asked of students appears arbitrary, turning tests of learning into reinforcements of elitism. Russell explores this point in our interview saying:

> there's this possible view you could have of logic that it's somehow elitist as if it's 'you better learn these special rules; we're going to use them to see who gets into the right club.' And you could find yourself sitting in a logic class thinking 'I don't get these rules, I find them hard to remember, I'm never going to be in the club.'[33]

One of the main techniques for breaking down exclusion in the classroom is to counteract beliefs about innate ability by emphasising effort.[34] Though, if there are students for whom no amount of effort will lead to success elitism is inescapable. This is a real risk.

Schnee explains that logic course often end up partitioned into those who find the course intuitive and those who find it impenetrable. He says:

> A problem that arises often in introductory formal logic classes is not that the class is too hard or that the class is too easy. It's that the class is too hard and too easy—too hard for one half but too easy for the other. A symptom of this problem is the bimodal or two-hump distribution curve: many students in the A or B range and many students in the D or F range, with few students in between.[35]

While Russell's solutions are about knowing your students and communicating well with them, Schnee suggests paying attention to the cognitive levels of exercises and assessment items.

Schnee explains, 'Students learn best when they face the right level of challenge—difficult

32 See §4.118
33 See §4.115
34 Dougherty, Baron and Miller, 'Female Under-representation among Philosophy Majors'.
35 Ian Schnee, 'Bactrians and Dromedaries', *Teaching Philosophy* 40, no. 4 (2017): p. 463.

but surmountable'[36] Courses should be designed to provide this degree of challenge for most students. Achieving a normal grade distribution is not the objective, it's an indication of whether the course is providing an optimal level of challenge. Schnee discusses using Bloom's taxonomy to analyse the cognitive levels of assessment items.

Bloom's taxonomy[37] is a model used by educationalists to classify educational learning objectives into levels of complexity and specificity. It was initially published in 1956 and revised in 2001. Krathwohl discusses the two-dimensional structure of the Revised Bloom Taxonomy and explains how it can be used to evaluate the intended learning outcomes of a course.[38]

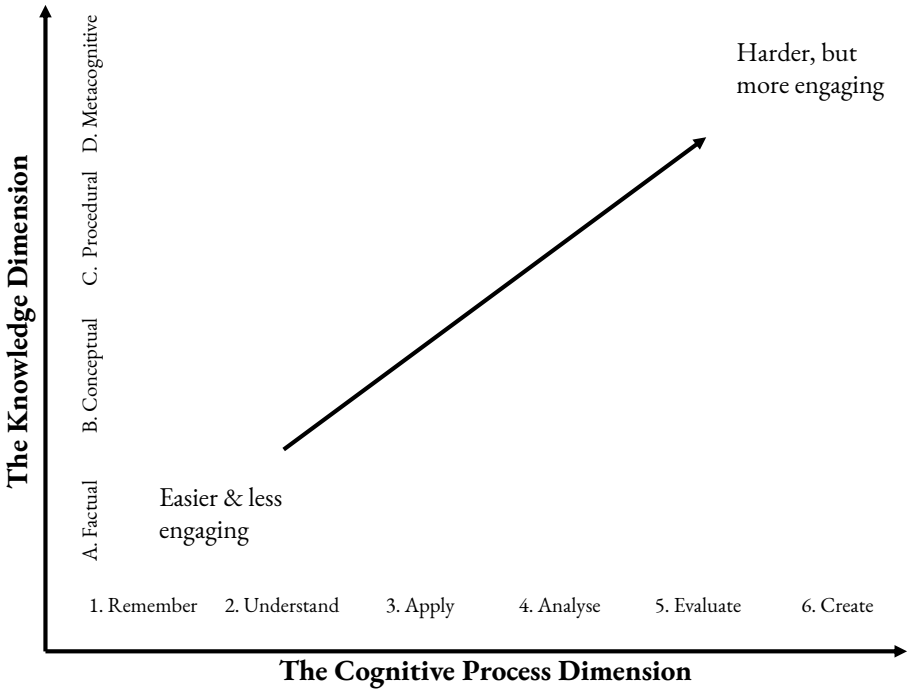
While Krathwohl's discussion focuses on applying the taxonomy to the course learning objectives, Schnee discusses the application at the level of specific exercises and assessment items. But as Krathwohl mentions, the principals in both cases are the same.[39]

Krathwohl points out that intended learning outcomes are usually framed 'in terms of (a) some subject matter content and (b) a description of what is to be done with or to that content.'[40] So (b) is a cognitive process verb, which expresses what is to be done with the knowledge-based noun (a). Thus, the two dimensions of the Revised Bloom Taxonomy are: the cognitive process dimension (expressed as a verb) and the knowledge dimension (expressed as a noun or noun phrase). Krathwohl presents a Taxonomy Table (see figure 8.4) and explains that, 'any objective could be classified in the Taxonomy Table in one or more cells that correspond with the intersection of the column(s) appropriate for categorizing the verb(s) and the row(s) appropriate for categorizing the noun(s) or noun phrase(s).'[41]

Krathwohl provides details on the structure of each of these two dimensions, shown in table 8.1,[42] and explains that the cognitive process dimension is hierarchical 'in the sense that the six major categories ...are believed to differ in their complexity, with *remember* being less complex than *understand*, which is less complex than *apply*, and so on.'[43] However, the hierarchy is not strict – there is some overlap between categories. This is most clearly illustrated with *Understand* because 'some cognitive processes associated with *Understand* (e.g., *Explaining*) are more cognitively complex than at least one of the cognitive processes associated with *Apply* (e.g., *Executing*).'[44]

36 Schnee, 'Bactrians and Dromedaries', p. 463.
37 B. S. Bloom et al., *Taxonomy of Educational Objectives: The Classification of Educational Goals*, Handbook 1: Cognitive Domain, ed. B. S. Bloom (New York: David McKay, 1956).
38 David R. Krathwohl, 'A Revision of Bloom's Taxonomy: An Overview', *Theory Into Practice* 41, no. 4 (2002): 212–219.
39 Krathwohl, p. 217.
40 Krathwohl, p. 213.
41 Krathwohl, p. 215.
42 Krathwohl, pp. 214-215.
43 Krathwohl, p. 215.
44 Krathwohl, p. 215.

Figure 8.2: The Revised Bloom Taxonomy - Two Dimensional



Motivated by the desire to create the right level of challenge for students, Schnee used the Revised Bloom Taxonomy to develop a tool for assessing the level of difficulty in exercises and assessment items – shown in table 8.2.[45]

45 Schnee, 'Bactrians and Dromedaries', pp.472-473.

Table 8.1: Structure of the Revised Bloom Taxonomy

	The Cognitive Process Dimension		**The Knowledge Dimension**
1.0	**Remember** – Retrieving relevant knowledge from long-term memory. **1.1 Recognising** **1.2 Recalling**	A.	**Factual Knowledge** – The basic elements that students must know to be acquainted with a discipline or solve problems in it. **Aa. Knowledge of terminology** **Ab. Knowledge of specific details and elements**
2.0	**Understand** – Determining the meaning of instructional messages, including oral, written, and graphic communication. **2.1 Interpreting** **2.2 Exemplifying** **2.3 Classifying** **2.4 Summarising** **2.5 Inferring** **2.6 Comparing** **2.7 Explaining**	B.	**Conceptual Knowledge** – The interrelationships among the basic elements within a larger structure that enable them to function together. **Ba. Knowledge of classifications and categories** **Bb. Knowledge of principles and generalisations** **Bc. Knowledge of theories, models, and structures**
3.0	**Apply** – Carrying out or using a procedure in a given situation. **3.1 Executing** **3.2 Implementing**	C.	**Procedural Knowledge** – How to do something; methods of inquiry, and criteria for using skills, algorithms, techniques, and methods. **Ca. Knowledge of subject-specific skills and algorithms** **Cb. Knowledge of subject-specific techniques and methods** **Cc. Knowledge of criteria for determining when to use appropriate procedures**
4.0	**Analyse** – Breaking material into its constituent parts and detecting how the parts relate to one another and to an overall structure or purpose. **4.1 Differentiating** **4.2 Organising** **4.3 Attributing**	D.	**Metacognitive Knowledge** – Knowledge of cognition in general as well as awareness and knowledge of one's own cognition. **Da. Strategic knowledge** **Db. Knowledge about cognitive tasks, including appropriate contextual and conditional knowledge** **Dc. Self-knowledge**
5.0	**Evaluate** – Making judgements based on criteria and standards. **5.1 Checking** **5.2 Critiquing**		
6.0	**Create** – Putting elements together to form a novel, coherent whole or make an original product. **6.1 Generating** **6.2 Planning** **6.3 Producing**		

Table 8.2: Schnee's Bloom Tool

Topic	**Remember**	**Understand & Apply**	**Analyze**	**Evaluate**
Logical truth	Define tautology	Explain the relation between tautologies and first-order validities	Prove via an example that not all first-order validities are tautologies	Assess a sentence (new, difficult) for types of necessary truth
Equivalence	State DeMorgan's Law	Compute simple transformations with equivalences	Prove a formula can be put into NNF, justifying each step	Evaluate two sentences for types of equivalence
Translation	Simple translations	Moderately difficult but not unfamiliar translations	Difficult or unfamiliar translations, or devising one's own predicates to capture the logical form of an argument	Translating an ambiguous sentence in multiple ways; assessing which reading is stronger
Truth tables	State the truth function for Sheffer stroke	Compute truth tables	Use a joint truth table to show an argument is valid or invalid	Assess a propositional argument for validity
Syntax	Identify which formulas are literals; determine how many free variables are in a formula	Provide an alternate but equivalent definition of a literal	Prove with induction that every formula has an even number of parentheses	Assess whether two definitions of formulas or sentences are equivalent
Informal proofs	Identify the inferences made in a given proof	Summarize in your own words a complicated informal proof	Produce an informal proof	Evaluate an argument (not formalized)
Formal proofs	Label the rules and lines in a given formal proof	Provide your own examples of each proof rule	Produce a formal proof	Evaluate a formal argument
Truth functional completeness	Identify which sets of connectives are truth functionally complete	Use the Boolean connectives to express a given truth function	Show that a newly given connective is truth functionally complete	Assess whether $\{\neg, \leftrightarrow\}$ is truth functionally complete

Schnee argues that careful attention to the cognitive levels demanded of students in exercises and assessments will result in a closer to normal grade distribution. Schnee identifies some familiar topics in introductory logic and gives example tasks associated with each item, which corresponds with the cognitive process dimension of the Revised Bloom Taxonomy. Because of how the tasks at the level of *Understand* and *Apply* overlap in logic, Schnee collapses those two categories. He also omits *Create* tasks because 'it is rare to include such tasks in an introductory logic class.'[46]

The application of these sorts of tools in course design allows teachers to prepare a course which can be tuned to not only cover essential topics in logic but also deliver an appropriate level of challenge to students. Education theory supplies the tools for dealing with the practical problems for teaching in the 21st century, where our rapidly changing conditions make tradition a less reliable guide.

8.4 THE SUBJECT MATTER OF LOGIC

In addition to the many practical challenges for logic teaching, I have presented a theoretical problem. I have argued that in many cases, the explanation used to introduce the subject matter of logic in introductory teaching conflicts with the core content of the course – an introduction to classical logic. I believe that this is flawed pedagogy, but let us not forget why the problem exists.

The challenge of writing an introduction to a logic textbook comes from the fact that the explanation must walk a fine line. It must make sense of the subject matter of logic for a mixed audience, of those who may take only the one course and those who will later specialise in the discipline. For students who will take only one course the teacher has to ensure that they walk away with an understanding of the subject which is as accurate and as useful as possible. The concern for students who will specialise in logic is that they develop an understanding of the foundations which they do not have to deconstruct later; one which will allow them to build a full understanding of the subject as they proceed, rather than one limited in some way by their initial teaching.

Crafting an introduction for a logic textbook is also difficult because it draws on contentious philosophic material, which must be made appropriate for introducing the subject matter to a beginner. Meanwhile the introduction merely sets the scene for what is an otherwise technical, almost mathematical, course.

In my introduction, I speculated that the difficulty of writing an introduction encourages teachers to fall back on traditional ways of introducing the subject matter. In chapter 2 I dis-

46 Schnee, 'Bactrians and Dromedaries', p. 473.

cussed traditional ways of introducing logic and the history that produced them.

In chapter 3, I argued that many ways of introducing the subject matter of logic conflict with classical logic. I claimed that there are two explanations better aligned with classical logic: semantic theorising, and metaphysical analysis. Other accounts are still legitimate explanations of the subject matter of logic; they belong in the discipline. However, a misalignment between the explanation of the subject matter, and the logical system introduced, will undermine learning.

Logic as a discipline is the study of several inter-related subjects using formal methods; there is no single, distinct, unambiguous subject matter of logic. However, it is possible to construct coherent explanations which make sense of certain sorts of logical projects. In logic, there is space for explorations of language, psychology, argumentation, metaphysics and more. The subject matter of the project supplies a notion of inter-theoretic validity. The logical system is an attempt to model that validity.

The logic-as-modelling view that I advance in this thesis accepts the possibility of multiple, equally legitimate, inter-theoretic validities. Normativity is one, but there are others. This is a trivial form of pluralism which relativises the correctness of a logical system to the logical project. But within the bounds of any given logical project, the question of whether there can be multiple equally correct logical theories or only one correct theory, is far from trivial.

Logic as a discipline becomes a way in which different topics can be explored and elucidated. No topic can legitimately lay claim to being all there is to the subject. Nor is logic the only means by which the chosen topic could be explored. Logic can be focused on many different topics; what brings it together as a discipline is a family resemblance.

It is common for introductory textbooks to appeal to some hegemonic conceptual role for logic. This is unnecessary and unwise. Naturally, to give some explanation of the point of the formalism, introductions must be given relative to a topic of interest. Nevertheless, a topic of interest does not need to be presented as if it is the only subject with which logicians are concerned. Explicitly presenting this relativism is a departure from tradition, but not a departure from sense.

There is also no need for a single, distinct, unambiguous subject matter of logic. Logical projects and their accompanying formal systems can be characterised without needing to appeal to some hegemonic conception of logic. Logic as a discipline is the study of several inter-related subjects using formal methods. Formal methods are subject matter independent – there is a general process of formalisation which is not tied to any given subject matter. Introductory logic teaching presents an opportunity to introduce and explain that process, and draw attention to how those methods could be applied to model many phenomena.

Logic is a diverse discipline. One way that teachers might handle that diversity is to focus on

explicating the process of formalisation and to some extent showing what is necessary to create a formal system. The process is then made concrete by showing how the formal system serves as a model of the topic of interest.

The solution that I propose is to frame logic as a triad composed of the general process of formalisation, a formal system, and a topic of interest. There are many equally legitimate options for the topic of interest. However, some choices for the topic of interest will work better than others depending on the formal system introduced, as some combinations will lead to the conclusion that the logic presented is flawed or inadequate. This is especially the case for pairing the topic of argument or reasoning with classical logic.

Another way to think about alignment between the formal system and the topic of interest can be found in Mates' questions:

- Is logic about the way people think, the way they ought to think, or neither of these?
- Is it principally concerned with language or with the extralinguistic world?
- Are the logician's artificial languages to be regarded as simplified but essentially faithful *models* of natural languages, or are they to be thought of as proposed *replacements* for natural languages, or is their utility to be explained in some other way?[47]

Concerning classical logic, the way I answer these questions is neither; the extralinguistic world; and as a replacement respectively.

The important methodological point here is that philosophy of logic should not be used to restrict the class of formal systems which are considered legitimate. Instead, the philosophy of logic must be constructed to align with the formal system. Formal systems can be designed to achieve a wide variety of aims, and when the aim varies so does both the formal system and its philosophy. For example, when a logic aims to model natural language, classical logic is not particularly good, but that doesn't mean that classical logic is not good for some other aim. The fact that classical logic is perfectly good for some other aim does not invalidate the aim of using logic to model natural language. The key question is whether the logic and the philosophy of that logic are well aligned.

In introductory logic teaching, there is an opportunity to highlight the method of formalisation and draw attention to how the method could be applied to model many phenomena. Pragmatically, a teacher may only have time to introduce one or two simple formal systems. Classical logic is a simple system which is commonly taught and is a reasonable choice for an introductory logic course. The challenge then is selecting a topic of interest which aligns with classical logic.

I suggest approaching this problem by reflecting on the parts of the formal system. Very

47 Mates, *Elementary Logic*, p. 1.

roughly a formal system can be broken into three parts: the symbols and their meaning (often referred to as the language), the consequence relation or definition of validity, and the interpretation. For classical logic, this means the traditional symbols and their meanings (both proof-theoretic and model-theoretic), and appropriate definitions of validity. The topic of interest can be thought of as the interpretation – what the formal system models. This is, in effect, the translation between '$\wedge$' and 'and'. This presents the opportunity to talk about the differences in meaning between '$\wedge$' and 'and'; how and when '$\wedge$' serves as a model of the meaning of 'and'. We can get students to explore both the power and utility of modelling with a formal system as well as showing its limitations. When looking at classical logic the cluster of concepts to draw into instruction includes validity, proof, truth, consistency, modelling, and meaning. Identifying the concepts used to introduce leads to the next question: how, and in what order should they be introduced?

Another way of structuring key concepts in logic which may help with organising their introduction is to group the concepts into three main areas: language, proof systems (truth-trees, axiomatic systems, natural deduction, and so on), and meta-theory (soundness and completeness, the value and limits of formalisation, and so on). Thinking this way suggests a progression. First would be introducing the symbols and some explanation of their meaning (usually model-theoretic). Next comes an introduction to proofs and a definition of validity. Finally, finishing off with meta-theory where proof-theoretic and model-theoretic approaches to meaning can be examined. The advantage of this conceptual progression is its natural scaffolding: to understand proof systems, students must first be introduced to the language. To make sense of the meta-theory they have to grasp both the language and at least one proof system.

Sometimes meta-theory is left out of introductory logic courses. However, meta-theory is crucial for explaining what the point, use, or usefulness of classical logic is. Without meta-theory, it is hard to explain and justify teaching students the language and the proof system. A discussion of soundness and completeness rounds out a discussion of the meaning of the symbols and their interpretation. Without this discussion, the language and the proof system might be fun and interesting to students (though the mileage here varies quite a lot), but it will lack an explanation of what the point was. Students might be able to figure out the point, but why leave this to chance?

What does this mean for introducing logic? The introduction should be a foreshadowing of the essential point that the whole course will build to. It is something which should be tailored to suit each course just as each course should be customised to suit its context. The framework I have outlined for developing an introduction to logic belongs in the context of course design. Course design, ultimately, should produce the introduction.

8.5 DESIGNING INTRODUCTORY LOGIC COURSES

There are two models of course design which could be used: constructive alignment[48] or backwards design.[49] Constructive alignment connects course design to the programme and institution, and is a useful model from that perspective. However, the backwards design model, which focuses more at the level of an individual course, supplies a more straightforward framework for discussing the possibilities of course design in introductory logic. Wiggins and McTighe identify a three-stage process of backward design,[50] illustrated in figure 8.3.

Figure 8.3: Backwards design process of Wiggins and McTighe

Wiggins and McTighe give little advice on the practical business of how to carry out the three stages of backwards design because the practical choices will be driven by the goals, evidence, and context of learning and teaching. They explain that sometimes the desired result entails the direct presentation of information and at other times it entails experiential learning – it all depends on the context. They say:

> If you are lost while driving and you stop to ask someone for directions, you want direct instruction. You don't want Joe Socrates endlessly asking, 'And why are you trying to get there as opposed to some other place? What does it mean that you are driving? How do you think you became lost? Have you considered that maybe you are not lost and have found something important?' No, you want Joe to inform you on how to get to Main Street. On the other hand, if your goal is to learn how to cook, you would be profoundly disappointed to be given 30 lectures about every angle on cooking without ever setting foot in a kitchen and 'doing' some cooking.[51]

Using backwards design as a model means that the practical planning of any given introductory logic course is centred around the desired results of the course. In other words, the intended learning outcomes. The two-dimensional nature of the revised taxonomy can be used

48 Biggs and Tang, *Teaching for Quality Learning at University*.
49 Grant P. Wiggins and Jay McTighe, *Understanding by Design*, 2nd ed. (Alexandria, VA: Association for Supervision and Curriculum Development, 2005).
50 Wiggins and McTighe, p. 18.
51 Wiggins and McTighe, p. 240.

to begin planning by focusing on the knowledge dimension. What topics will the course cover? The first step in the backwards design process is writing a draft course prescription: a brief statement that describes the purpose and content of the course in about 50 (and no more than 100) words. For example:

> Logic is a diverse field with relationships to language, psychology, argumentation, computation, metaphysics, and more. What brings this diverse field together is similar formal methods. In this course, students will be introduced to two formal systems: propositional logic and quantificational logic. They will learn how validity is defined in those logics, how those logics might be used as a tool to analyse English arguments, and the limits to their application. Students are introduced to the use of techniques such as truth tables, truth trees, and natural deduction.

The course prescription expresses the disciplinary expertise or technical knowledge that the course aims to develop. It is a summary of the course content, which also signals the approach to presenting logic that will be taken in the course. Ideally, it is informed by the philosophy programme that the course belongs to so that the technical knowledge and the skills developed will be appropriate for that programme. The programme should be sensitive to student demographics and the broader context of education, and the course should be in line with the programme's goals in that context.

The next step of the backwards design process is expanding on the course prescription by drafting some intended learning outcomes (ILOs). These are constructed by identifying between four and six topics to include in the course. For instance, the topics might be introduction to formal methods (introducing the theory of symbols and their meaning), truth tables, validity and other relationships, proof systems, and meta-theory. The aims might then be for students to be able to:

ILO1. Recognise the symbols of propositional and predicate logic and remember the different theories of how those symbols get their meaning.

ILO2. Identify truth-functional English sentences and translate them into sentences in propositional or predicate logic.

ILO3. Evaluate symbolised statements or arguments for consistency, contingency, contradiction, tautology, and validity using truth tables and truth trees.

ILO4. Explain what makes symbolised statements or arguments consistent, contingent, contradictory, tautologous, or valid.

ILO5. Construct natural deduction proofs of the validity of symbolised arguments.

ILO6. Explain how truth tables relate to truth trees and natural deduction proofs.

Figure 8.4: Testing ILOs using the Revised Bloom Taxonomy

	The Cognitive Process Dimension					
The Knowledge Dimension	1. Remember	2. Understand	3. Apply	4. Analyse	5. Evaluate	6. Create
A. Factual Knowledge	ILO1.					
B. Conceptual Knowledge		ILO2. , ILO4. & ILO6.				
C. Procedural Knowledge			ILO5.		ILO3.	
D. Metacognitive Knowledge						

The learning outcomes can then be tested using the Revised Bloom Taxonomy, checking for good coverage across knowledge and cognitive process dimensions, as illustrated in figure 8.4. Once the intended learning outcomes have been constructed, the next step in the backwards design process is determining what evidence would demonstrate that a student has achieved the intended learning outcomes. Table 8.3 contains a description of how each of the six learning outcomes listed above could be demonstrated.

Table 8.3: Determine evidence that ILOs have been achieved

	Evidence of Learning Achievement
ILO1.	Reproducing appropriate symbols in translations; Identifying the truth tables and proof rules associated with each symbol; Identifying whether a description of how symbols get their meaning is proof-theoretic or model-theoretic.
ILO2.	Selecting only statements from a list of English sentences which includes commands, questions, and exclamations; Correctly translating English statements into propositional or predicate logic.
ILO3.	Correctly constructing truth tables & interpretating the results; Correctly applying truth tree rules to construct accurate trees & interpreting the results of a truth tree correctly.
ILO4.	Produce explanations which refer to the truth-functional conditions for consistency, contingency, contradiction, tautology, and validity.
ILO5.	Correctly applying natural deduction rules to construct proofs with appropriate use of assumptions and *reductio ad absurdum*.
ILO6.	Producing explanations which make a syntax/semantics distinction and discuss the role of soundness and completeness proofs.

The third and final stage of backwards design is planning the learning experiences and instruction. Here worksheets like Sumisson and Goodfellow's[52] curriculum mapping template and course outlines can be helpful. Templates of this nature supply a framework for reviewing the broad objectives and structure of the course. Most tertiary institutions will have their versions of these designed to suit them. Figure 8.5 is a sample of a planning worksheet based on Victoria University of Wellington Course Description template, incorporating an adaptation of Sumisson and Goodfellow's template. With the broad overview of the course in mind, the last step is detailed course planning. Figure 8.6 is a sample lesson planning worksheet.

Good teaching is intentional. It is something in which teachers invest time and effort. It is evident in the effort which goes into producing textbooks, as well as their number and variety. It is present in my interviews with Gillian Russell, Johan van Benthem, and Dave Ripley, where all three have spent time thinking about how to best introduce students to logic.

The best way to develop an introduction to logic through a broad understanding of the context in which education is taking place, and to use the tools of education theory to craft an explanation of the meaning and value of logic suitable to the setting. There is no single thing which makes logic meaningful or valuable, and logic instruction will not always be relevant. Sensitivity to the way the context changes the value of education is crucial. As van Benthem warns, 'a logic course is also an excellent way of making enemies ...sometimes powerful enemies who can do us harm.'[53] There is no single best way to introduce a beginner to logic because introductions are so intimately connected to context. But in developing an introduction, as with so many other aspects of education design, alignment is a virtue. If the explanation of the subject matter of logic and the formal systems are misaligned the meaning and value of the logic course will be undermined.

In my analysis, I did not set out to condemn the stock of existing logic textbooks, and yet I have uncovered a wide range of philosophic, social, and pedagogic problems which plague them. First, the philosophy of logic presented can be misaligned with the formal system introduced. Most commonly this takes the form of a normative philosophy of logic presented to explain classical logic. Next, they may make unproven claims about the utility of learning logic. This error seems to persist despite the damage it occasionally does to the reputation of logic as a discipline and the warnings and reminders logicians give one-another about not over-stating logic's utility. Finally, textbooks may have embedded sexism, racism, classicism, and may be a vehicle for the perpetuation of bias.

There is another limitation to textbooks which is worth considering: their static form. The

52 Jennifer Sumsion and Joy Goodfellow, 'Identifying Generic Skills through Curriculum Mapping: A Critical Evaluation', *Higher Education Research & Development* 23, no. 3 (2004): 329–346.

53 See §6.88

for-print design locks textbooks into a single shape; all the limitations frozen into print. Russell says, 'obviously whenever you have a textbook you probably don't like every single aspect of it.'[54] This may just be that it contains some unfortunate stereotypes and some sexist language. It might have to do with the structure and pacing of the content, the formal systems it introduces, or the philosophy of logic which it uses. But these are perhaps the lesser deficits; the great deficit of the static textbook is that it cannot be adapted to suit the teaching context.

Textbooks are supposed to support teaching – to help teachers design and deliver courses – but they limited by their nature. The intended learning outcomes, exercises, and content are fixed. If those do not suit the context, the textbook cannot be used. With luck, the textbook will be roughly aligned with the institutional context, but there will likely always be some degree of misalignment. Hopefully, the textbook will include a wide enough range of exercises and content that there's an opportunity to raise or lower the cognitive level demanded to meet students with an appropriate level of challenge, but more exercises and content will inevitable raise the price of the textbook.

One solution which addresses all these drawbacks is a collaborative project to build a collection of teaching resources. The project would have a collection of logic problems which teachers could use in exercises and assessments. Each problem would be accompanied by notes such as a peer-reviewed assessment of its cognitive level, according to the Revised Bloom Taxonomy. It would include links to seminal articles in logic again with notes to help teachers decide whether those articles would be appropriate as assigned reading for their students. It would also provide teachers with an opportunity to share and discuss course outlines and notes on course content.

This project would supply substantial support to teachers worldwide enabling them to adapt their courses to respond to both their interests as subject matter experts and the demands of their environments.

54 See §4.96

Figure 8.5: Course Planning

Course Prescription

Logic is a diverse field with relationships to language, psychology, argumentation, computation, metaphysics, and more. What brings this diverse field together is the family resemblance created by the use of similar formal methods. In this course, students will be introduced to two formal systems: propositional logic and quantificational logic. They will learn how validity is defined in those logics, as well as how those logics might be applied to analyse English arguments. Students are introduced to the use of techniques such as truth tables, truth trees, and natural deduction.

Teaching/learning summary

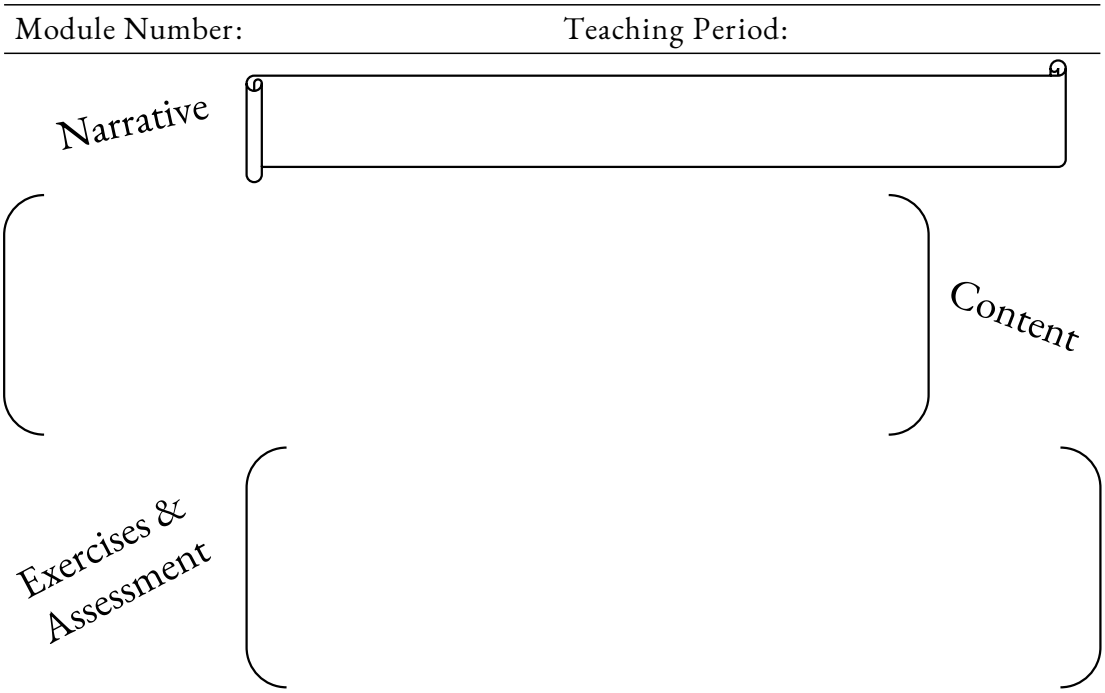
Students will spend time in lectures learning the how's and why's of logic, attend tutorials where they will practice completing exercises, and complete online quizzes and peer review exercises to consolidate learning.

Course intended learning outcomes (ILOs)		Taught	Modelled	Practised	Assessed
1	Recognise the symbols of propositional and predicate logic and remember the different theories of how those symbols get their meaning	×		×	×
2	Identify truth functional English sentences and translate them into sentences in propositional or predicate logic	×	×	×	×
3	Evaluate symbolised statements or arguments for consistency, contingency, contradiction, tautology, or validity using truth tables and truth trees	×	×	×	×
4	Explain what makes symbolised statements or arguments consistent, contingent, contradictory, tautologous, or valid	×	×	×	×
5	Construct natural deduction proofs of the validity of symoblised arguments	×	×	×	×
6	Explain how truth tables relate to truth trees and natural deduction proofs	×		×	×

Assessment items and workload per item		Assessment weighting	ILO(s)
1	Online quizzes & homework (50 Hours)	10%	1, 2, 3, 4, 5, & 6
2	In-class Test 1 (10 hours preparation time)	15%	1 &2
3	In-class Test 2 (10 hours preparation time)	15%	2 & 3
4	Peer review exercise (6 hours)	5%	4
5	Natural Deduction Assignment (8 hours)	15%	5
6	Final 3-hour Examination (20 hours preparation time)	40%	1, 2, 3, 4, 5, & 6

Programme intended learning outcomes		Assumed	Taught	Modelled	Practised
1	Analytic Skills	×	×	×	×
2	Problem-solving			×	×
3	Decision-making				
4	Communication	×		×	×
5	Influencing skills				
6	Research skills				

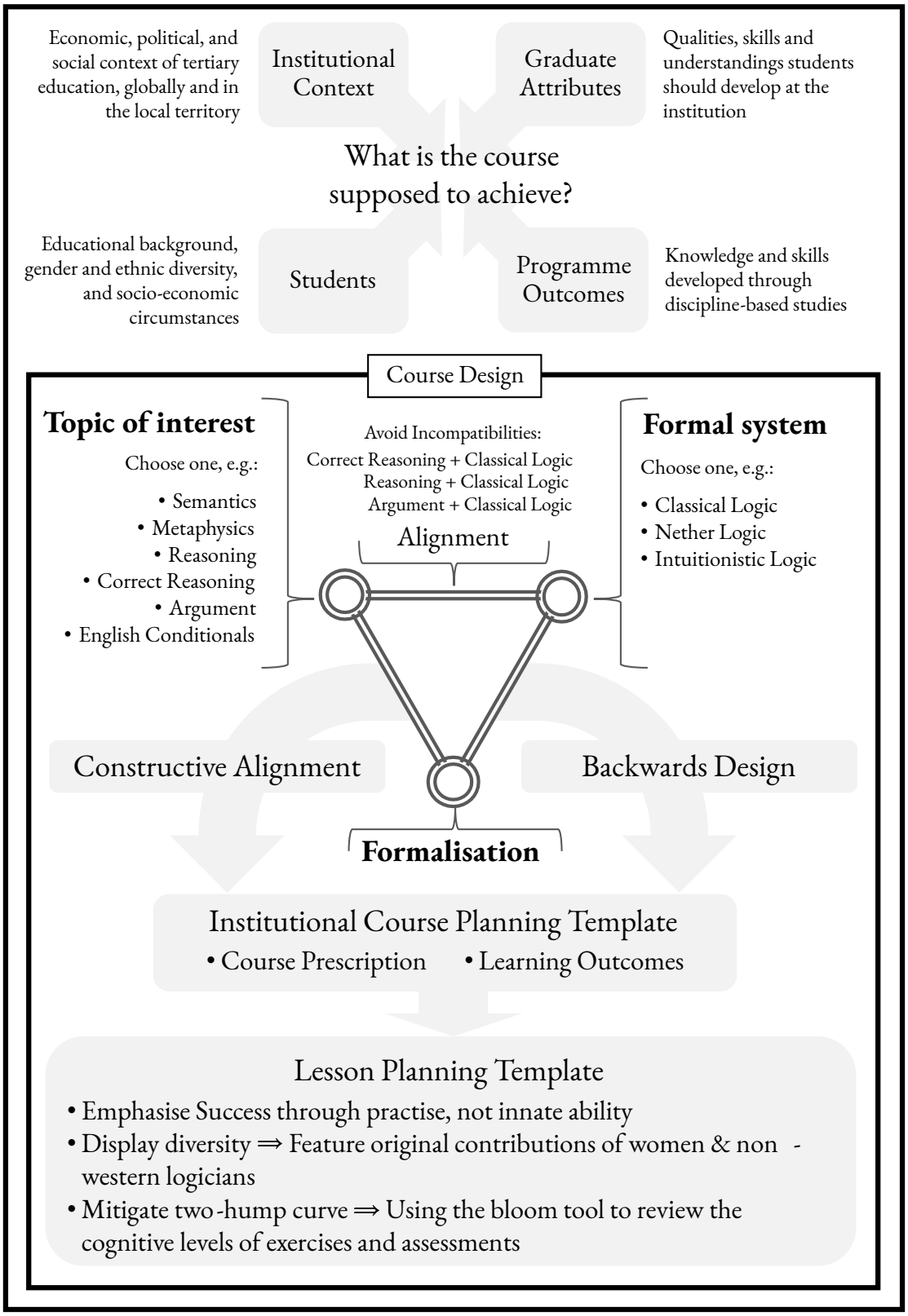
Figure 8.6: Lesson Planning



Course intended learning outcomes (ILOs)		Taught	Modelled	Practised	Assessed
1	Recognise the symbols of propositional and predicate logic and remember the different theories of how those symbols get their meaning				
2	Identify truth functional English sentences and translate them into sentences in propositional or predicate logic				
3	Evaluate symbolised statements or arguments for consistency, contingency, contradiction, tautology, or validity using truth tables and truth trees				
4	Explain what makes symbolised statements or arguments consistent, contingent, contradictory, tautologous, or valid				
5	Construct natural deduction proofs of the validity of symoblised arguments				
6	Explain how truth tables relate to truth trees and natural deduction proofs				

Graduate Outcomes		Assumed	Taught	Modelled	Practised	Assessed
1	Specialised understanding of the chosen field					
2	Critical and creative thinking					
3	Intellectual autonomy & integrity					
4	Able to work both independently and collaboratively					
⋮						

Figure 8.7: Course Building Guide



CONCLUSION: RE-FRAMING VALIDITY

RE-FRAMING VALIDITY

> We are like sailors who on the open sea must reconstruct their ship but are never able to start afresh from the bottom. ...the ship can be shaped entirely anew, but only by gradual reconstruction.
>
> Otto Neurath[1]

It is a truism that in the introduction you tell readers what you are going to tell them, then you tell them, then in the conclusion tell them what you told them. While telling readers what you told them is certainly something you should do in conclusions, it misses their essential role: telling them *why it mattered*.

This thesis has focused on logic pedagogy, but while logic pedagogy is valuable, it is not the fundamental motivation. What drove this exploration of logic pedagogy is a fundamentally philosophical question: what is logic about? This question has pedagogic applications but is not itself pedagogical.

Central to the thesis is an investigation into the subject matter of logic which becomes an argument about validity. It is relevant to two contemporary philosophical debates: pluralism and Normativity. To pluralism, my observation is that much of this debate rests on defining some singular conceptual role for logic. I argue there is not one, but many. To Normativity, my contribution is the simple point that while Normativity is a legitimate notion of inter-theoretic validity, it is not the only one.

However, this thesis is more than just a contribution to contemporary debates. It is an argument for a way of thinking about validity before a formal definition. This way of thinking about validity is an approach which allows us to more clearly understand the intricacies of meaning contained within that notion. I show that there are multiple notions of inter-theoretic validity. Understanding that there are multiple notions of inter-theoretic validity helps make sense of the diversity of the activities and investigations in the discipline and can help strengthen our pedagogy. With a notion of inter-theoretic validity in hand we are able to examine the relative merits of rival logical theories. Without such a notion rivalry is meaningless.

1 Otto Neurath, 'Anti-Spengler', in *Empiricism and Sociology*, ed. Marie Neurath and Robert S. Cohen (Dordrecht: Springer, 1973), p. 199

9.1 PLURALISM AND NORMATIVITY

A substantial literature about pluralism arose following a series of papers by Beall and Restall written between 2000 and 2002 which culminated in their book *Logical Pluralism*. In this book, they argue that within the 'settled core' of logic there are multiple equally legitimate logics.[2] In response, Priest argued that while there are many senses in which pluralism is uncontroversially correct, 'with respect to the correct logic for canonical application... one can quite coherently view this situation from a monist perspective,'[3] taking the position that there is only one *correct* logic. This monism *vs.* pluralism debate has also inspired discussion of different forms of pluralism. Some of these forms of pluralism are trivial in the sense that they seem so obviously right that they are unworthy of debate, while others result in substantial disagreement.

To frame their account of pluralism Beall and Restall rely on there being a 'settled core' in logic. One of the themes in this debate is how to judge whether a logic is correct. Cook describes the structure of an idea which appears in the literature:

> In judging the correctness of a logic ...we evaluate that logic in terms of how well it 'matches up' with logical consequence. Given a logic $\langle L, \Rightarrow \rangle$, such an evaluation is typically carried out along something like the following lines:
>
> 1. Identify a subset LV of the primitive symbols of L – this is the logical vocabulary of L.
> 2. Construct a (partial) translation function T from LV to approximate bits of natural language – this projects the logical/nonlogical distinction in $\langle L, \Rightarrow \rangle$ onto our natural language.
> 3. Determine whether or not the following *Correctness Principle* (CP) holds:
>
> CP: Given any recursive mapping I from L to statements in our natural language (i.e. an interpretation) which agrees with T on LV, and given any statement Φ and set of statements Δ from L: $I(\Phi)$ is a logical consequence of $I(\Delta)$ if and only if $\Delta \Rightarrow \Phi$.
>
> Put a bit more loosely, a logic (plus an identification and interpretation of logical vocabulary) is correct if and only if, for any way of interpreting the nonlogical vocabulary, the logic validates a particular argument if and only if the natural language statement corresponding to the conclusion of that argument is a logical con-

2 Beall and Restall, *Logical Pluralism*.
3 Priest, 'Logical Pluralism', p. 208.

> sequence of the natural language statements corresponding to the premises of that argument.[4]

This is similar to Priest's statement that 'A vernacular inference is valid iff its translation into the formal language is valid in the pure logic.'[5] However, as G. Russell explains, a worry is that there may be no notion of logical consequence for formal validity to 'match up' with. She says: 'The worry in the background here is that ...there is nothing for the theories to be right about. ...no such thing as "valid simpliciter" but only "valid$_I$" and "valid$_C$" etc. But now what is to stop us making up any logic we like? Logic threatens to be purely conventional.[6]' Ripley's response to this question in our interview is illuminating. He says: 'I strongly suspect that there is nothing there. Officially, I don't care whether there is anything there or not. If there is something there I never talk about it. So, it's fine. But there's a reason I'm never talking about it; that's because I really think there's nothing there.'[7] Not everyone is concerned by the idea that logic may be purely conventional – that there may be no such thing as genuine validity – but it is certainly a concern to some. At the very least it would be good to have an interpretation which renders dialogue between logicians who support rival logical theories intelligible. This is the point that Field highlights when he says:

> Whatever other merits proof-theoretic and model-theoretic accounts of validity may have, they are not remotely plausible as accounts of the meaning of 'valid'. And not just because they involve technical notions like 'model' and 'proof' that needn't be possessed by a speaker who understands the concept of valid inference. The more important reason is that competent speakers may agree on the model-theoretic and proof-theoretic facts, and yet disagree about what's valid ...advocates of different logics presumably disagree about something – and something more than just how to use the term 'valid', if their disagreement is more than verbal. It would be nice to know what it is they disagree about. An they don't disagree about what's classically valid (as defined either model-theoretically or proof-theoretically); nor about what's intuitionistically valid, or LP-valid, or whatever. So what do they disagree about?[8]

Field proposes Normativity as a solution to the background worry brought up in the discussion of pluralism. Normativity – the idea that logic is about how we ought to reason – makes sense of what logicians disagree about when they argue about rival definitions of validity. This

4 Cook, 'Let a Thousand Flowers Bloom: A Tour of Logical Pluralism', pp. 495-496.
5 Priest, 'Logical Pluralism', p. 196.
6 Russell, 'Logic Isn't Normative', p. 378.
7 See §5.75
8 Field, 'What Is Logical Validity?', pp. 33-34.

solution comes from MacFarlane's 2004 proposal that the Normativity might be a way to go 'beyond intuitions and get a clearer understanding of what the concept of logical validity is *for*.'[9] However, Normativity is not universally accepted. Moreover, in 2019 Steinberger argued that Normativity and logical pluralism are incompatible. He argues that if logic is normative 'pluralism collapses into monism with respect to either the strongest or the weakest admissible logic,'[10] and concludes, 'if logic is normative, competition between logics may be inevitable.'[11]

Normativity can add tremendous value. It renders inter-theoretic dispute intelligible and allows us to move beyond potentially contaminated intuitions. It is a notion which clarifies the phenomena under investigation and points to the data required to support rival theories. If it were uncontentious, it might well stand as an account of genuine validity, but the lack of universal agreement disqualifies it. I expect that the same would go for any candidate notion presented as 'genuine validity'. I argue that there is no genuine account of the meaning of validity. Instead, validity is a complex cluster of inter-related meanings. It is unreasonable to expect that we will be able to summon a definition that captures all the things that we might mean when we claim that an argument is valid or that some premise implies a conclusion. There is no notion of validity which is so uncontested that it could stand as a reasonable candidate for *genuine* validity.

The lack of some genuine validity threatens to make the exercises of logic pointless. For logic to have a point, it has to *mean* something. Or, to put this another way, to make the exercise of developing formal systems valuable they must match up with something. The trouble is figuring out what that something is. The solution is a general theory of the subject matter of logic. Normativity is one way of developing validity – explicating the subject matter – but it is one way among many. G. Russell provides evidence of this pluralism with the claim that 'logic is the study of patterns of truth-preservation on truth-bearers.'[12]

There are many candidate concepts which could be given as the subject matter: reasoning, argument, truth, proof, information, and so on. All are equally legitimate, but they do not all apply equally at the same time. Rival logical theories can be dealing with the same essential subject matter, in which case, we have a genuinely meaningful dispute. However, if they do not have a shared notion of inter-theoretic validity, any dispute is meaningless. Sadly, it will not always be obvious whether there is a shared notion of inter-theoretic validity, because as other pluralist discussions show, the boundaries which individuate logical calculi, consequence rela-

9 John MacFarlane, 'In What Sense (if Any) Is Logic Normative for Thought?' (2004), p. 2.
10 Florian Steinberger, 'Logical Pluralism and Normativity', *Philosophers' Imprint* 19, no. 12 (2019): p. 1.
11 Steinberger, p. 17.
12 Russell, 'Logic Isn't Normative', p. 382.

tions, and logical theories are unclear.[13,14] The same calculus can express distinct consequence relations,[15] and the same consequence relation can be expressed in different calculi. What this means for logical theories is unclear.

We can give different accounts of validity. Informally and couched in natural language, or more formally, using mathematical language. Logic is good for supplying accounts of validity, and in logic we explore validity. In doing so, we develop formal systems, consequence relations, and logical theories. Sometimes the project which is pursued creates a need to choose between these, and interrogating the subject matter of that investigation can help with the decision.

The subject matter is also useful for creating a framework for understanding what features are important in a formal system. For instance, while there is overlap between reasoning and truth, the data which is relevant to each is quite different; one is psychological, the other metaphysical. Formal systems designed to handle one data type might look quite different from those designed to handle another. Being clear about the target subject matter is beneficial when there is a need to assess formal systems.

9.2 DEMARCATION AND THE DISCIPLINE

Johan van Benthem made the very insightful comment that 'a subject is, in some sense, the sum total of all topics that its practitioners have found interesting' and provided the Zwarts Syllogism to illustrate the point.[16] Moreover, there is something both indelicate and unjust to an argument about the subject matter of logic which would rule out the investigations of some logicians. Out of courtesy to our colleagues, we need an account which does not embroil us in some undignified identity politics.

At the same time, there are many ways in which being able to make a principled demarcation advances research in logic. For instance, demarcation plays a role in the choice of logical constants. Rather than giving up on demarcation entirely, the ideal situation would be to have a general theory which is flexible enough to account for the many varied forms of logical research; one that allows the specific kinds of demarcation which they require, and helps make sense of different research projects.

Logicians can have quite different aims when they develop logics. They could be exploring syntactic validity in natural language, the provability of mathematical statements, truth preservation (in natural language or a formal structure), constraints on belief, or assertability and

13 Ole Thomassen Hjortland, 'Logical Pluralism, Meaning-variance, and Verbal Disputes', *Australasian Journal of Philosophy* 91, no. 2 (June 2013): 355–373.

14 Bogdan Dicher, 'Variations on Intra-theoretical Logical Pluralism: Internal Versus External Consequence', *Philosophical Studies* 177, no. 3 (2020): 667–686.

15 Dicher, p. 671.

16 See §6.21

deniability conditions for speech acts. The goal has an impact on how we consider the suitability of a logic.

Still, research need not be constrained by whether it has an aim. Sobel need not have had any goal in mind in developing the falsity preserving *Nether Logic*. What matters in this situation is that we can develop explanations after the fact. We can pursue formal research for its own sake; working backwards from the resulting systems to the kind of phenomena they could be used to model. An exploration of the subject matter of logic can move both ways – from an explanation of the subject matter to specific characteristics in a formal system, and from a formal system to an explanation of its subject matter.

A flexible logic-as-modelling view allows all this. It enables acknowledging the work of other logicians while at the same time being able to give the demarcations necessary to further individual research programmes. We can contrast the goals and commitments of various researchers and research programmes and use that to account for the diverse array of logics which we have today.

9.3 TEACHING AND TRADITION

Sometimes an early lesson can have a profound effect. One of my early childhood memories is of my mother teaching me a lesson about tradition. This lesson was delivered (as so many of the lessons she gave me were) in the form of a story. Long ago, when she was young, a friend came to visit. Her mother told her not to put her friend's coat on her bed. Seeing no clear logic to the instruction, she asked her mother for an explanation. Her mother replied, 'that's what *my* mother always told me to do'. Feeling that this was no explanation at all, my mother took the issue up with her grandmother. Her grandmother explained that it was because the neighbour children had lice.

Like many of the lessons my mother gave me, the moral is complex. The first message is not to accept, 'it is the way it has always been done' as a legitimate reason. When I read Alain's words, 'all their science stops there: to copy what is, to do what has always done' I feel my mother's disdain. The second message is that understanding the reasons which support actions is crucial. In knowing what those reasons are, we are better equipped to judge for ourselves which actions to take. The original course of action was reasonable. It might even be a sensible precaution to take in all future cases. The point is that without keeping an understanding of the reason, we lose the ability to judge whether the old action is still useful in new circumstances.

I have argued that the difficulty which surrounds the development of an introductory chapter for an introductory logic text leads to a reliance on tradition. Furthermore, tradition supplies an approach which is not well suited to the modern context. But there is something which has

gone unsaid, that is the potential long-lasting effects and repercussions of the way that logic is introduced. Van Benthem talks about his desire to change the agenda of modern logic in his course *Logic in Action*, and says:

> You might ask, 'why do it for students at that level?' and it's because the first logic course that people get is so important because it sets their mind and expectations. If you want to change something, it's going to be very hard after the first logic course because their mind has been set, both explicitly and implicitly in the methodology.[17]

He shares my intuition that the way that logic is introduced will have a long-lasting effect on the way that students conceive logic. Teachers should bear this possibility in mind, both for students that will take only one or two courses, and for those who will eventually specialise in logic.

Teachers invest a great deal of care and effort in designing courses for their students. They consider how to ensure that students will get something valuable from learning the course material. They think about how to structure and present content, and how their students might experience learning the material. Teachers care deeply about their students learning. In a tertiary setting, teachers in research institutions are also subject matter experts. Philosophy of logic is a less common specialisation in the broader field of logic. Nevertheless, just as every teacher has a teaching philosophy, every logician has a philosophy of logic. Just as a teaching philosophy affects what teachers say and do in the classroom, so too will the logician's philosophy of logic.

In the context of pedagogy, philosophy of logic is not a trivial consideration. An example is Ripley, who disavows interest in the philosophy of logic. However, he holds a strong philosophy of logic which is then evident in his teaching. His position – that there is no such thing as validity in itself – directs his choice of material and its order and pacing. The same holds for G. Russell and van Benthem. Their views on the subject matter of logic impact what and how they teach.

A fully considered view is not needed for good logic teaching, as is clear from van Benthem's course which, he explains, is not based on a fully considered view.[18] However, what teachers must do is ensure that the philosophy of logic they use aligns with the formal system which they introduce. The simplest way to ensure alignment is similar to the backwards design process in education design: first selecting the formal systems to introduce, and then designing an expression of the subject matter which aligns those systems.

17 See §6.42
18 See §6.47

9.4 FINAL REMARKS

The logic-as-modelling view that I advance in this thesis accepts the possibility of multiple, equally legitimate, inter-theoretic validities – logical projects. Normativity is one, but there are others. The inter-theoretic validity which a logic models is its subject matter.

There is no single, distinct, unambiguous subject matter of logic. Nor does there need to be. We can characterise logical projects and their accompanying formal systems without needing to appeal to some hegemonic conception of logic.

It is common for introductory textbooks to appeal to some hegemonic conceptual role for logic. This is unnecessary and unwise. Naturally, to give some explanation of the point of the formalism, introductions must be given relative to a subject of interest. Nevertheless, a subject of interest does not need to be presented as if it is the only subject with which logicians are concerned.

In this thesis, I explored the subject matter of logic and the way it is taught I used two focus questions. The first is 'What concepts are embedded in introductory explanations of logic?' The second, 'How should introductions to logic be structured?'

To explore the first question, I examined a small part of the history of modern logic instruction as well as a corpus of 38 contemporary introductory logic textbooks. Chapters 2 and 3 were devoted to these topics. I argued that the history of modern logic instruction offers bad examples of how to explain the subject matter of logic. The primary deficit here is in explaining logic in terms of either reasoning or good reasoning. On that explanation of the subject matter of logic, the classical propositional calculus is not fit for purpose. The misalignment between explanation and content combined with the dogmatic presentation of the subject matter produces a poor pedagogic model.

To explore the second question, I interviewed three logicians. These interviews are presented in chapters four, five, and six. Each interview demonstrates concern for students and reflective teaching. We see the challenges inherent in developing introductory material: you must start somewhere, and you cannot cover everything. Chapter seven synthesises the interviews and the lessons drawn from examining introductory textbooks. It discusses other concerns in modern logic pedagogy and presents a framework for solving the pedagogic problem of introducing the subject matter of logic.

The pedagogic focus supplied data for a fundamentally philosophical investigation. By examining modern introductory textbooks, I was able to show the variety of subjects that logic could be about. By conducting interviews with teaching logicians, I explored how a logician's philosophy of logic manifests in their instruction. The final test of the philosophy of logic that I advance is that it should produce an accurate and clear explanation of the subject suitable for

an introductory text on logic. While there is no single best instance of this, the theory which I advance does generate guidance on how to develop a suitable explanation; one which is aligned with the logic which will be introduced. I suggest a collaborative project to build a collection of teaching resources to overcome the limitation of the fixed-format textbook.

TEXTBOOKS

Table A.1: Included textbooks

Book	Type	Language	Editions	First Published	Last Printed	Uses
Barker-Plummer, Barwise and Etchemendy, *Language, Proof, and Logic*	Normative	English	2	1999	2011	11
Hurley and Watson, *A Concise Introduction to Logic*	Normative	English	13	1982	2018	7
Copi, Cohen and McMahon, *Introduction to Logic*	Normative	Chinese, English, German, Hebrew, Spanish	15	1953	2019	4
Magnus, *Forall* ***x***	Normative	English	1	2005	2017	4
Smith, *An Introduction to Formal Logic*	Validity	English	1	2003	2013	4
Klenk, *Understanding Symbolic Logic*	Normative	English, Spanish	5	1983	2009	3
Enderton, *A Mathematical Introduction to Logic*	Psychological	English, Spanish	3	1972	2013	3
Gensler, *Introduction to Logic*	Normative	English	3	2002	2017	3
Halbach, *The Logic Manual*	Language	English, Italian	1	2010	2015	3
Tomassi, *Logic*	Argument	English	1	1999	2013	3
Bergmann, Moor and Nelson, *The Logic Book*	Validity	English	6	1990	2014	2
Kalish, Montague and Mar, *Logic*	Normative	English, French	2	1964	2003	2
Priest, *An Introduction to Non-Classical Logic*	Validity	English, German	2	2000	2017	2
Tymoczko and Henle, *Sweet Reason*	Normative	English	2	1995	2012	2
Forbes, *Modern Logic*	Argument	English	1	1994	2006	2
Goldfarb, *Deductive Logic*	Normative	English	1	2003	2004	2
Lemmon, *Beginning Logic*	Argument	Dutch, English, Italian, Romanian	1	1965	2012	2
Restall, *Logic*	Normative	English	1	1999	2010	2
Teller, *A Modern Formal Logic Primer*	Normative	English	1	1989	1989	2
Sinnott-Armstrong and Fogelin, *Understanding Arguments*	Argument	English	9	1978	2015	1
Munson and Black, *The Elements of Reasoning*	Argument	English	7	1990	2017	1
Mendelson, *Introduction to Mathematical Logic*	Psychological	English, German, Italian, Russian	6	1963	2015	1
Salmon, *Introduction to Logic and Critical Thinking*	Normative	English	6	1968	2014	1
Dalen, *Logic and Structure*	Language	English, German	5	1979	2013	1

Book	Type	Language	Editions	First Published	Last Printed	Uses
Jeffrey, *Formal Logic*	Validity	English, Finnish, Spanish	4	1967	2006	1
Ben-Ari, *Mathematical Logic for Computer Science*	Psychological	English, Polish	3	1990	2012	1
Layman, *The Power of Logic*	Normative	English	3	1998	2005	1
Arthur, *An Introduction to Logic*	Argument	English	2	2011	2017	1
Hodges, *Logic*	Consistency	English, German, Italian	2	1977	2001	1
Nolt, Rohatyn and Varzi, *Logic*	Argument	English	2	1998	2011	1
Bell, Solomon and De Vidi, *Logical Options*	Normative	English	1	2001	2001	1
Heil, *First-Order Logic*	Psychological	English	1	1994	1994	1
Howson, *Logic with Trees*	Validity	English	1	1996	2005	1
Jennings and Friedrich, *Proof and Consequence*	Normative	English	1	2006	2006	1
Lee, *Logic*	Normative	English	1	2017	2017	1
Smith, *Logic*	Truth	English	1	2012	2012	1
Smullyan, *Logical Labyrinths*	Truth	English, Japanese	1	2009	2009	1
Suppes, *Introduction to Logic*	Normative	English, French, Italian	1	1956	2013	1

A.1 PUBLICATION DETAILS

Arthur, Richard T. W. *An Introduction to Logic: Using Natural Deduction, Real Arguments, a Little History, and Some Humour*. 2nd ed. Revision of: Natural Deduction: An Introduction to Logic with Real Arguments, a Little History, and Some Humour. Peterborough, Ontario: Broadview Press, 2017. ISBN: 9781554813322.

Barker-Plummer, Dave, Jon Barwise and John Etchemendy. *Language, Proof, and Logic*. 2nd ed. Stanford, Calif.: CSLI Publications, 2011. ISBN: 9781575866321.

Bell, John L., Graham Solomon and David De Vidi. *Logical Options: An Introduction to Classical and Alternative Logics*. Peterborough: Broadview Press, 2001. ISBN: 1551112973.

Ben-Ari, Mordechai. *Mathematical Logic for Computer Science*. 3rd ed. London: Springer, 2012. https://doi.org/10.1007/978-1-4471-4129-7.

Bergmann, Merrie, James Moor and Jack Nelson. *The Logic Book*. 6th ed. Boston: McGraw-Hill, 2014. ISBN: 9780078038419.

Copi, Irving M., Carl Cohen and Kenneth McMahon. *Introduction to Logic*. 14th ed. ProQuest Ebook Central, New York: Routledge, Taylor & Francis Group, 2016. ISBN: 9781315510873.

Dalen, Dirk van. *Logic and Structure*. 5th ed. London: Springer, 2013. https://doi.org/10.1007/978-1-4471-4558-5.

Enderton, Herbert B. *A Mathematical Introduction to Logic*. 2nd ed. San Diego: Harcourt, 2007. ISBN: 0122384520.

Forbes, Graeme. *Modern Logic: A Text in Elementary Symbolic Logic*. New York: Oxford University Press, 1994. ISBN: 0195080297.

Gensler, Harry J. *Introduction to Logic*. 3rd ed. ProQuest Ebook Central, New York: Taylor & Francis, 2017. ISBN: 9781317436119.

Goldfarb, Warren D. *Deductive Logic*. ProQuest Ebook Central, Indianapolis: Hackett, 2003. ISBN: 9781603845854.

Halbach, Volker. *The Logic Manual*. Oxford: Oxford University Press, 2010. ISBN: 9780199587841.

Heil, John. *First-Order Logic: A Concise Introduction*. Boston: Jones / Bartlett Publishers, 1994. ISBN: 9780867209570.

Hodges, Wilfrid. *Logic*. 2nd ed. London: Penguin Books, 2001. ISBN: 9780141003146.

Howson, Colin. *Logic with Trees: An Introduction to Symbolic Logic*. 1st ed. ProQuest Ebook Central, London: Routledge, Taylor & Francis Group, 1997. ISBN: 9780203976739.

Hurley, Patrick J., and Lori Watson. *A Concise Introduction to Logic*. 13th ed. Boston, MA: Cengage Learning, 2017. ISBN: 9781305958098.

Jeffrey, Richard. *Formal Logic: Its Scope and Limits*. 4th ed. Edited by John P. Burgess. ProQuest Ebook Central, Indianapolis: Hackett Publishing Company, Inc, 2006. ISBN: 9781624666063.

Jennings, R. E., and N. A. Friedrich. *Proof and Consequence: An Introduction to Classical Logic*. Peterborough, Ont.: Broadview Press, 2006. ISBN: 9781551115474.

Kalish, Donald, Richard Montague and Gary Mar. *Logic: Techniques of Formal Reasoning*. 2nd ed. Edited by Robert J. Fogelin. New York: Harcourt Brace Jovanovich Inc., 1980. ISBN: 0155511815.

Klenk, Virginia. *Understanding Symbolic Logic*. 3rd ed. Upper Saddle River, N.J: Prentice Hall, 1994. ISBN: 9780536632524.

Layman, C. Stephen. *The Power of Logic*. 3rd ed. Boston: McGraw-Hill, 2005. ISBN: 0072875879.

Lee, Siu-Fan. *Logic: A Complete Introduction*. Teach Yourself, 2017. ISBN: 9781473608436.

Lemmon, E. J. *Beginning Logic*. 2nd ed. Boca Raton: Chapman & Hall/CRC, 1998. ISBN: 0412380900.

Magnus, P. D. *Forall **x**: An Introduction to Formal Logic*. Version 1.4. 29 December 2017. http://www.fecundity.com/logic.

Mendelson, Elliott. *Introduction to Mathematical Logic*. 4th ed. 1st reprint, New York: Chapman & Hall/CRC, 2001. ISBN: 0412808307.

Munson, Ronald, and Andrew G. Black. *The Elements of Reasoning*. Boston, MA: Cengage Learning, 2017. ISBN: 9781305585935.

Nolt, John Eric, Dennis A. Rohatyn and Achille C. Varzi. *Logic*. Abridged. Edited by Alex M. McAllister. Schaum's easy outlines. Based on Schaum's outline of theories and problems of logic. New York: McGraw-Hill, 2006. ISBN: 0071455353.

Priest, Graham. *An Introduction to Non-Classical Logic: From If to Is*. 2nd ed. Cambridge Introductions to Philosophy. ProQuest Ebook Central, Cambridge: Cambridge University Press, 2008. ISBN: 9780511392306.

Restall, Greg. *Logic: An Introduction*. Fundamentals of Philosophy. ProQuest Ebook Central, London: Routledge, Taylor & Francis Group, 2006. ISBN: 9780415400671.

Salmon, Merrilee H. *Introduction to Logic and Critical Thinking*. 6th ed. Australia: Wadsworth Cengage Learning, 2013. ISBN: 9781111841522.

Sinnott-Armstrong, Walter, and Robert J. Fogelin. *Understanding Arguments: An Introduction to Informal Logic*. Ninth Edition, Concise. Stamford, CT: Cengage Learning, 2015. ISBN: 9781285197395.

Smith, Nicholas J. J. *Logic: The Laws of Truth*. ProQuest Ebook Central, Princeton, NJ: Princeton University Press, 2012. ISBN: 9781400842315.

Smith, Peter. *An Introduction to Formal Logic*. 7th printing with corrections, Cambridge, UK: Cambridge University Press, 2013. ISBN: 9780521008044.

Smullyan, Raymond M. *Logical Labyrinths*. Wellesley, MA: A. K. Peters, 2009. ISBN: 9781568814438.

Suppes, Patrick. *Introduction to Logic*. Reprint, Mineola, N.Y: Dover Publications, 1999. ISBN: 0486406873.

Teller, Paul. *A Modern Formal Logic Primer: Sentence Logic*. Vol. 1. Englewood Cliffs, NJ: Prentice-Hall, 1989. ISBN: 0139031707.

Tomassi, Paul. *Logic*. 1st ed. ProQuest Ebook Central, Ann Arbor, MI: Routledge, Taylor & Francis Group, 2002. ISBN: 9780203197035.

Tymoczko, Thomas, and James Henle. *Sweet Reason: A Field Guide to Modern Logic*. Reprint, New York: Springer, 2000. ISBN: 0387989307.

TEXT EXCERPTS: LOGIC IS...

B.1 ARGUMENT

Arthur, *An Introduction to Logic*

[Spends a lot of time introducing argument, makes no statement about logic.]

Forbes, *Modern Logic*

> Symbolic logic is usually described as the study of the difference between *valid* and *invalid* arguments [p. 3]

Lemmon, *Beginning Logic*

> It is not easy, and perhaps not even useful, to explain briefly what logic is. Like most subjects, it comprises many different kinds of problem and has no exact boundaries; at one end, it shades off into mathematics, at another, into philosophy. The best way to find out what logic is is to do some. None the less, a few very general remarks about the subject may help to set the stage for the rest of this book.
>
> Logic's main concern is with the soundness and unsoundness of arguments[1] [p. 1]
>
> To sum up the contents of this section, we may define logic as the study, by symbolic means, of the exact conditions under which patterns of argument are valid or invalid [p. 5]

Munson and Black, *The Elements of Reasoning*

[Spends a lot of time introducing argument, makes no statement about logic.]

Nolt, Rohatyn and Varzi, *Logic*

> Logic is the study of arguments. [p. 1]

Sinnott-Armstrong and Fogelin, *Understanding Arguments*

[Spends a lot of time introducing argument, makes no statement about logic.]

Tomassi, *Logic*

> To study logic is to study argument. Argument is the stuff of logic. Above all, a logician is someone who worries about arguments. [p. 2]
>
> The central problem which worries the logician is just this: how, in general, can we tell good arguments from bad arguments? [p. 2]

1 Lemmon equates soundeness with validity

B.2 CONSISTENCY

Hodges, *Logic*

> Logic can be defined as the study of consistent sets of beliefs; this will be our starting-point. Some people prefer to define logic as the study of valid arguments. Between them and us there is no real disagreement, as section 11 will show. But consistency makes an easier beginning. [p. 1]
>
> Logic is about consistency – but not all types of consistency. [p. 1]
>
> The type of consistency which concerns logicians is not loyalty or justice or sincerity; it is *compatibility of beliefs*. [p. 1]

B.3 NORMATIVE

Barker-Plummer, Barwise and Etchemendy, *Language, Proof, and Logic*

> ...*all* rational inquiry depends on logic, on the ability of people to reason correctly most of the time, and, when they fail to reason correctly, on the ability of others to point out the gaps in their reasoning.[p. 1]
>
> And what are the techniques by which we can distinguish correct or 'valid' reasoning from incorrect or 'invalid' reasoning? More basically, what is it that *makes* one claim 'follow logically' from some given information, while some other claim does not?[p. 1]
>
> To study logic is to use the methods of rational inquiry on rationality itself. [p. 2]

Bell, Solomon and De Vidi, *Logical Options*

> While philosophers claim logic as one of the traditional sub-disciplines of their field (along with metaphysics, epistemology, and ethics), nowadays logic is also studied by mathematicians, computer scientists, cognitive scientists and linguists, among others. But to understand the basics of modern logic, it is still worthwhile to begin by considering the subject's roots as the study of good reasoning. [p. 1]
>
> ...logic is concerned with investigating this relation of 'following from'.
>
> *Logic*, one could say without too much violence to the traditional meaning of the word, *is the science of valid inference*. [p. 4]
>
> *Logic* is the science of good argument, but only one component of being a good argument is investigated, namely the relationship between premises and conclusions. But not all cases in which the premises support the conclusion are of interest, only those where the truth of the premises would guarantee the truth of the conclusion. And not even all of these arguments are of interest. Rather, the cases of interest are those in which this guarantee is due (somehow) to the form of the argument. [p. 4]

Copi, Cohen and McMahon, *Introduction to Logic*

> Logic is the study of the methods and principles used to distinguish correct from incorrect reasoning. [p. 2]
>
> Reasoning is an art, as well as a science. It is something we do, as well as something we understand. Giving reasons for our beliefs comes naturally, but skill in the art of building arguments, and testing them, requires practice. One who has practiced and strengthened these skills is more likely to reason correctly than one who has never thought about the principles involved. Therefore we provide in this book very many opportunities for practice in the analysis of arguments. [pp. 8-9]

Gensler, *Introduction to Logic*

> **Logic** is *the analysis and appraisal of arguments*. [p. 1]
>
> Logic is a useful tool to clarify and evaluate reasoning [p. 1]

Goldfarb, *Deductive Logic*

> Logic is the study of principles of reasoning. It is concerned not with how people actually reason, but rather with how people ought to reason if they wish to ensure the truth of their results. That is, by 'principles of logic' we mean those that yield correct reasoning. [p. xiii]

Tymoczko and Henle, *Sweet Reason*

> Logic is the study of reasoning and arguments. [p. 1]
>
> Logic is concerned with what makes reasoning good and what makes arguments valid. [p. 1]

Hurley and Watson, *A Concise Introduction to Logic*

> **Logic** may be defined as the organized body of knowledge, or science, that evaluates arguments. [p. 1]
>
> The purpose of logic, as the science that evaluates arguments, is thus to develop methods and techniques that allow us to distinguish good arguments from bad. [p. 2]

Jennings and Friedrich, *Proof and Consequence*

> Throughout almost the whole of its history, the primary interest of logicians has lain in the articulation of what could be called Canons of Correct Reasoning: the formulation of rules by which we could assess the arguments of others, and be guided in our own inferences. [p. 4]
>
> The central historical interest in arguments was a practical one: how to tell a good argu-

ment from a bad one. [p. 8]

Logicians have been interested in arguments whose goodness or badness turned upon our understanding of the words having occurrences in them, rather than upon the material facts [p. 9]

Kalish, Montague and Mar, *Logic*

Logic is concerned with arguments, good and bad. [p. 1]

Virtue among arguments is known as validity. [p. 1]

Klenk, *Understanding Symbolic Logic*

The only thing logic is concerned with is whether arguments are good or bad, correct or incorrect. Logic is a *normative* enterprise; its job is to *evaluate* arguments [p. 5]

Layman, *The Power of Logic*

Roughly speaking, **logic** is the study of methods for evaluating arguments. More precisely, logic is the study of methods for evaluating whether the premises of an argument adequately support (or provide good evidence for) its conclusion. [p. 1]

Logic is the study of methods for evaluating whether the premises of on argument adequately support (or provide good evidence for) its conclusion. [p. 3]

Lee, *Logic*

Logic is the study of the methods and principles used to distinguish between good and bad reasoning. It is a **normative** discipline, in the sense that it does not survey and describe *how* we actually reason (which is the job of the psychologist) but what we *should* do in reasoning. [p. 2]

Logic is the study of good and bad reasoning. [p.21]

Magnus, *Forall* ***x***

Logic is the business of evaluating arguments, sorting good ones from bad ones. [p. 5]

Restall, *Logic*

Logic is the study of good reasoning, and in particular, what makes good reasoning *good*. [p. 6]

Salmon, *Introduction to Logic and Critical Thinking*

Logic is the field of study concerned with analyzing arguments and appraising their correctness or incorrectness. ... Logic, however, is broader than critical thinking because it

does not confine itself to examining particular arguments but is a formal systematic study of the principles of valid inference and correct reasoning. [p. 12]

Suppes, *Introduction to Logic*

In modern times logic has become a deep and broad subject. We shall initially concentrate on that portion of it which is concerned with the theory of correct reasoning, which is also called the theory of logical inference, the theory of proof or the theory of deduction. [p. xi]

Finally, it should be remarked that no precise definition of *logic* is attempted in these pages. In the narrow sense, logic is the theory of valid arguments or the theory of deductive inference. A slightly broader sense includes the theory of definition. A still broader sense includes the general theory of sets. Moreover, the theory of definition together with the theory of sets provides an exact foundation for the axiomatic method, the study of which is informally considered part of logic by most mathematicians. [p. xiv]

Teller, *A Modern Formal Logic Primer*

A good way of thinking about logic, when you are beginning to learn, is to say that logic is the study of this reason-giving connection. I like to say, more generally, that logic is the science of arguments. Logic sets out the important properties of arguments, especially the ways in which arguments can be good or bad. [p. 2]

B.4 PSYCHOLOGICAL

Ben-Ari, *Mathematical Logic for Computer Science*

Logic formalizes valid methods of reasoning. [p. 1]

Mathematical logic formalizes reasoning. [p. 5]

Enderton, *A Mathematical Introduction to Logic*

Symbolic logic is a mathematical model of deductive thought. ... Symbolic logic is a model in much the same way that modern probability theory is a model for situations involving chance and uncertainty. How are models constructed? You begin with a real-life object, for example an airplane. Then you select some features of this original object to be represented in the model, for example its shape, and others to be ignored, for example its size. And then you build an object that is like the original in some ways (which you call essential) and unlike it in others (which you call irrelevant). Whether or not the resulting model meets its intended purpose will depend largely on the selection of the properties of the original object to be represented in the model. [p. xi]

Heil, *First-Order Logic*

> Arguments exhibit repeatable *patterns*. Some of these patterns represent valid reasoning – their premises imply their conclusions – and some do not. [p. 2]
>
> Logic provides a way of studying and classifying repeatable forms or patterns of reasoning. [p. 2]
>
> formal logic provides a powerful technique for assessing the validity and invalidity of arguments. [p. 3]

Mendelson, *Introduction to Mathematical Logic*

> One of the popular definitions of logic is that it is the analysis of methods of reasoning. In studying these methods, logic is interested in the form rather than the content of the argument. [p. 1]
>
> The systematic formalization and cataloguing of valid methods of reasoning are a main task of logicians. If the work uses mathematical techniques or if it is primarily devoted to the study of mathematical reasoning, then it may be called *mathematical logic*. We can narrow the domain of mathematical logic if we define its principal aim to be a precise and adequate understanding of the notion of *mathematical proof*. [p. 1]

B.5 TRUTH

Smith, *Logic*

> this does not mean that logic is itself the science of reasoning. Rather, logic is the science of truth. [§1.1]
>
> Logic, then, is primarily concerned with truth, not with reasoning. Yet logic is very usefully applied to reasoning – for we want to avoid reasoning in ways that could lead us from true starting points to false conclusions. [§1.1]

Smullyan, *Logical Labyrinths*

> [Introduces Knight and Knaves puzzles about lying and truth-telling.]

B.6 LANGUAGE

Dalen, *Logic and Structure*

> [Makes no direct statement about logic. Talks about mathematical language.]

Halbach, *The Logic Manual*

> Philosophers think and reason. If they do so properly, they reason correctly and produce valid arguments. ...develop theories about sound reasoning and the validity of arguments, that is, they began to do logic. [p. 1]

B.7 VALIDITY

Bergmann, Moor and Nelson, *The Logic Book*

> The hallmark of deductive logic is **truth-preservation**. Reasoning that is acceptable by the standards of deductive logic is always truth-preserving; that is, it never takes one from truths to a falsehood. [p. 1]

Howson, *Logic with Trees*

> There is much more to logic than the question of what makes *inferences* deductively valid or invalid, but to most people that is what logic is all about, so that is where we shall begin. [p. 3]

Jeffrey, *Formal Logic*

> Formal logic is the science of deduction. It aims to provide systematic means for telling whether or not given conclusions follow from given premises, i.e., whether arguments are valid or invalid. [p. 1]

Priest, *An Introduction to Non-Classical Logic*

> The point of logic is to give an account of the notion of validity: what follows from what. [p. 3]

Smith, *An Introduction to Formal Logic*

> The business of logic is the *systematic evaluation of arguments for internal cogency*. And the kind of internal cogency that will especially concern us is *deductive validity*. [p. 1]
> The business of logic, then, is the evaluation of stretches of reasoning. [p. 1]

BIBLIOGRAPHY

Alain. 'XXXVIII- 1er Septembre 1908'. In *Propos d'un Normand 1906-1914*. Paris: Gallimard, 1956.

Allo, Patrick, and Edwin Mares. 'Informational Semantics As a Third Alternative?' *Erkenntnis* 77, no. 2 (December 2011): 167–185. https://doi.org/10.1007/s10670-011-9356-1.

Alsaleh, Nada J. 'Teaching Critical Thinking Skills: Literature Review'. *TOJET: The Turkish Online Journal of Educational Technology* 19, no. 1 (2020): 21–39.

Altbach, Philip G., Liz Reisberg and Laura E. Rumbley. 'Trends in Global Higher Education: Tracking an Academic Revolution'. In *2009 World Conference on Higher Education*. Paris: United Nations Educational, Scientific / Cultural Organization, 2009. https://unesdoc.unesco.org/ark:/48223/pf0000183219.

Altonn, Helen. 'Educator Earned Worldwide Fame for Work in Logic'. *Honolulu Star-Bulletin*, 1 September 2002. http://archives.starbulletin.com/2002/09/01/news/story11.html.

Barrie, Simon C. 'Understanding What We Mean by the Generic Attributes of Graduates'. *Higher Education* 51, no. 2 (2006): 215–241. https://www.jstor.org/stable/29734975.

Beall, J. C., and Greg Restall. *Logical Pluralism*. Repr. Oxford: Clarendon Press, 2008. https://doi.org/10.1093/acprof:oso/9780199288403.001.0001.

Beaney, Michael. 'Series Editors Forward'. In *Susan Stebbing and the Language of Common Sense*, Kindle Edition, by Siobhan Chapman. London: Palgrave Macmillan UK, 2013. https://doi.org/10.1057/9781137313102.

———. 'Stebbing, Lizzie Susan (1885–1943)'. In *The Dictionary of Twentieth-Century British Philosophers*, edited by Stuart Brown, 995–1002. Bristol: Thoemmes, 2005. https://doi.org/10.5040/9781350052437-0409.

———. 'Susan Stebbing on Cambridge and Vienna Analysis'. In *The Vienna Circle and Logical Empiricism: Re-evaluation and Future Perspectives*, edited by Friedrich Stadler, 10:339–350. Kluwer Academic Publishers, 2003.

Beaney, Michael, and Siobhan Chapman. 'Susan Stebbing'. In *The Stanford Encyclopedia of Philosophy*, Summer 2017, edited by Edward N. Zalta. 8 May 2017. https://plato.stanford.edu/archives/sum2017/entries/stebbing.

Beth, Evert Willem. *Aspects of Modern Logic*. Edited by E. M. Barth and J. J. A. Mooij. Translated by D. H. J. de Jongh and S. De Jongh-Kearl. Springer Netherlands, 1970. https://doi.org/10.1007/978-94-010-3332-9.

Biggs, John B., and Catherine Tang. *Teaching for Quality Learning at University*. Maidenhead, UK: McGraw-Hill Education, 2011.

Blake-Turner, Christopher, and Gillian Russell. 'Logical Pluralism without the Normativity'. *Synthese*, September 2018. https://doi.org/10.1007/s11229-018-01939-3.

Blanchette, Patricia. 'The Frege-hilbert Controversy'. In *The Stanford Encyclopedia of Philosophy*, Fall 2018, edited by Edward N. Zalta. 9 August 2018. https://plato.stanford.edu/archives/fall2018/entries/frege-hilbert/.

Bloom, B. S., M. D. Engelhart, E. J. Furst, W. H. Hill and D. R. Krathwohl. *Taxonomy of Educational Objectives: The Classification of Educational Goals*. Handbook 1: Cognitive Domain. Edited by B. S. Bloom. New York: David McKay, 1956.

Bolstad, Rachel, Jane Gilbert, Sue McDowall, Ally Bull, Sally Boyd and Rosemary Hipkins. *Supporting Future-oriented Learning & Teaching – a New Zealand Perspective*. Prepared for the New Zealand Ministry of Education, 2012. https://www.educationcounts.govt.nz/publications/schooling/supporting-future-oriented-learning-and-teaching-a-new-zealand-perspective.

Brent, Richard. 'Whately, Richard (1787–1863), Church of Ireland Archbishop of Dublin and Philosopher." Oxford Dictionary of National Biography'. In *The Oxford Dictionary of National Biography*. Oxford: Oxford University Press, 2004. https://doi.org/10.1093/ref:odnb/29176.

Carnap, Rudolf. *Introduction to Symbolic Logic and Its Applications*. Translated by William H. Meyer and John Wilkinson. New York: Dover Publications, 1958. https://catalog.hathitrust.org/Record/001387508.

Carroll, Lewis. *Symbolic Logic: Part I, Elementary*. London: Macmillan, 1896. https://catalog.hathitrust.org/Record/001831800.

Chapman, Siobhan. *Susan Stebbing and the Language of Common Sense*. Kindle Edition. London: Palgrave Macmillan UK, 2013. https://doi.org/10.1057/9781137313102.

Cook, Roy T. 'Let a Thousand Flowers Bloom: A Tour of Logical Pluralism'. *Philosophy Compass* 5, no. 6 (2010): 492–504. https://doi.org/10.1111/j.1747-9991.2010.00286.x.

Copi, Irving M. *Introduction to Logic*. 2nd ed. New York: The Macmillan Company, 1961. https://catalog.hathitrust.org/Record/001387418.

Dicher, Bogdan. 'Variations on Intra-theoretical Logical Pluralism: Internal Versus External Consequence'. *Philosophical Studies* 177, no. 3 (2020): 667–686. https://doi.org/10.1007/s11098-018-1199-z.

Dougherty, Tom, Samuel Baron and Kristie Miller. 'Female Under-representation among Philosophy Majors: A Map of the Hypotheses and a Survey of the Evidence'. *Feminist Philosophy Quarterly* 1, no. 1 (2015): 1–30. https://doi.org/10.5206/fpq/2015.1.4.

Duluth Public Library. 'Abraham Kaplan'. Vintage Duluth, 4 June 2007. Accessed 25 May 2020. https://dplreference.wordpress.com/2007/06/04/abraham-kaplan/.

———. 'Abraham Kaplan and Irving Copi'. Vintage Duluth, 29 June 2007. Accessed 25 May 2020. https://dplreference.wordpress.com/2007/06/29/abraham-kaplan-and-irving-copi/.

———. 'Irving Copi'. Vintage Duluth, 21 June 2007. Accessed 25 May 2020. https://dplreference.wordpress.com/2007/06/21/irving-copi/.

Eemeren, Frans H. van, and Arnolda Francisca Snoeck Henkemans. *Argumentation: Analysis and Evaluation*. New York; London: Routledge, 2017. https://www.taylorfrancis.com/books/e/9781315401140.

Etchemendy, John. *The Concept of Logical Consequence*. Stanford, CA: Center for the Study of Language & Information, 1999.

Feferman, Anita Burdman, and Solomon Feferman. *Alfred Tarski: Life and Logic*. Cambridge, UK: Cambridge University Press, 2004.

Field, Hartry. 'Pluralism in Logic'. *The Review of Symbolic Logic* 2, no. 02 (9 June 2009): 342–359. https://doi.org/10.1017/s1755020309090182.

———. 'What Is Logical Validity?' In *Foundations of Logical Consequence*, edited by Colin R. Caret and Ole Thomassen Hjortland, 33–70. New York: Oxford University Press, 2015. https://doi.org/10.1093/acprof:oso/9780198715696.003.0002.

Francez, Nissim, and Gilad Ben-Avi. 'Proof-theoretic Reconstruction of Generalized Quantifiers'. *Journal of Semantics* 32, no. 3 (August 2015): 313–371. https://doi.org/10.1093/jos/ffu001.

Franklin, James. 'Ludicrous, but Interesting'. *New Criterion* 36, no. 4 (2017): 79–82. https://newcriterion.com/issues/2017/12/ludicrous-but-interesting.

Frost-Arnold, G. *Carnap, Tarski, and Quine at Harvard: Conversations on Logic, Mathematics, and Science*. Vol. 5. Full Circle: Publications of the Archive of Scientific Philosophy. Chicago: Open Court, 2013.

Gabbay, Dov M., Ralph H. Johnson, Hans Jürgen Ohlbach and John Woods, eds. *Handbook of the Logic of Argument and Inference*. Amsterdam: Elsevier, 2002.

Godden, David. 'Psychologism in the Logic of John Stuart Mill: Mill on the Subject Matter and Foundations of Ratiocinative Logic'. *History and Philosophy of Logic* 26, no. 2 (2005): 115–143. https://doi.org/10.1080/01445340412331332809.

Gómez-Torrente, Mario. 'Alfred Tarski'. In *The Stanford Encyclopedia of Philosophy*, Spring 2019, edited by Edward N. Zalta. 29 January 2019. https://plato.stanford.edu/archives/spr2019/entries/tarski/.

Gopen, George D., and Judith A. Swan. 'The Science of Scientific Writing'. *American Scientist* 78, no. 6 (1990): 550–558. https://www.jstor.org/stable/29774235.

Güzeldere, Güven. 'An Interview with Maria Reichenbach and David Kaplan'. In *Turkish Studies in the History and Philosophy if Science*, edited by Gürol Irzik and Güven Güzeldere, 7–24. Dordrecht: Springer, 2005. https://doi.org/10.1007/1-4020-3333-8_2.

Haaparanta, Leila, ed. *The Development of Modern Logic*. Oxford University Press, June 2009. https://doi.org/10.1093/acprof:oso/9780195137316.001.0001.

Hanson, William H. 'The Concept of Logical Consequence'. *The Philosophical Review* 106, no. 3 (July 1997): 365–409. https://doi.org/10.2307/2998398.

Harman, Gilbert. *Change in View*. Chap. 2. London; Cambridge, Mass.: The MIT Press, 1986.

———. 'Internal Critique'. In *Handbook of the logic of argument and inference*, edited by Dov M. Gabbay, Ralph H. Johnson, Hans Jürgen Ohlbach and John Woods, 171–186. Amsterdam: Elsevier, 2002.

Heathcote, Adrian. 'Hume's Master Argument'. In *Hume on Is and Ought*, edited by Charles R. Pigden, 92–117. New York: Palgrave Macmillan, 2010.

Heim, Irene, and Angelika Kratzer. *Semantics in Generative Grammar*. Malden, Mass: Blackwell, 1998.

Hilbert, David, and Wilhelm Ackermann. *Principles of Mathematical Logic*. Edited by Robert E. Luce. Translated by Lewis M. Hammond, George G. Leckie and F. Steinhardt. New York: Chelsea Publishing Co., 1950. https://catalog.hathitrust.org/Record/001387534.

Hintikka, Jaakko. 'The Role of Logic in Argumentation'. *The Monist* 72, no. 1 (1989): 3–24. http://www.jstor.org/stable/27903120.

Hjortland, Ole Thomassen. 'Logical Pluralism, Meaning-variance, and Verbal Disputes'. *Australasian Journal of Philosophy* 91, no. 2 (June 2013): 355–373. https://doi.org/10.1080/00048402.2011.648945.

———. 'What Counts as Evidence for a Logical Theory?' *The Australasian Journal of Logic* 16, no. 7 (November 2019): 250. https://doi.org/10.26686/ajl.v16i7.5912.

Hodges, Wilfrid. 'Model Theory'. In *The Stanford Encyclopedia of Philosophy*, Fall 2018, edited by Edward N. Zalta. 17 July 2013. https://plato.stanford.edu/archives/fall2018/entries/model-theory.

Hume, David. *A Treatise of Human Nature*. A New Edition. Vol. 2. London: Printed for Thomas & Joseph Allman, 1817. https://catalog.hathitrust.org/Record/100751199.

Janssen-Lauret, Frederique. 'Making Room for Women in Our Tools for Teaching Logic: A Proposal for Promoting Gender-inclusiveness'. In *4th International Conference on Tools for Teaching Logic*, edited by M. Antonia Huertas, João Marcos, María Manzano, Sophie Pinchinat and François Schwarzentruber, 65–74. Rennes; France, June 2015. arXiv: 1507.04697. http://arxiv.org/abs/1507.04697.

Kalish, Donald, and Richard Montague. *Logic: Techniques of Formal Reasoning*. New York: Harcourt, Brace & World, 1964. https://catalog.hathitrust.org/Record/004449809.

Kelley, Earl C. *The Workshop Way of Learning*. New York: Harper & Brothers, 1951. https://books.google.co.nz/books?id=5vY7AAAAIAAJ.

Krathwohl, David R. 'A Revision of Bloom's Taxonomy: An Overview'. *Theory Into Practice* 41, no. 4 (2002): 212–219.

Kusch, Martin. 'Psychologism'. In *The Stanford Encyclopedia of Philosophy*, Winter 2015, edited by Edward N. Zalta. 1 December 2015. https://plato.stanford.edu/archives/win2015/entries/psychologism/.

———. *Psychologism: A Case Study in the Sociology of Philosophical Knowledge*. Philosophical Issues in Science. London; New York: Routledge, 1995.

MacFarlane, John. 'In What Sense (if Any) Is Logic Normative for Thought?' 2004. http://johnmacfarlane.net/normativity_of_logic.pdf.

Mancosu, Paolo, Richard Zach and Calixto Badesa. 'The Development of Mathematical Logic from Russell to Tarski, 1900–1935'. In *The Development of Modern Logic*, edited by Leila Haaparanta, 318–470. Oxford: Oxford University Press, 2009. https://doi.org/10.1093/acprof:oso/9780195137316.003.0029.

Mates, Benson. *Elementary Logic*. New York: Oxford University Press, 1965.

Mill, John Stuart. *A System of Logic, Ratiocinative and Inductive: Being a connected view of the principles of evidence, and methods of scientific investigation.* Vol. 1. London: John W. Parker, West Strand, 1843. https://catalog.hathitrust.org/Record/009730733.

Neurath, Otto. 'Anti-Spengler'. In *Empiricism and Sociology*, edited by Marie Neurath and Robert S. Cohen, 158–213. Dordrecht: Springer, 1973. https://doi.org/10.1007/978-94-010-2525-6_6.

Newton-Smith, W. H. *Logic: An Introductory Course*. Taylor & Francis, 2003.

Nickerson, Raymond S. 'Chapter 1: On Improving Thinking through Instruction'. *Review of Research in Education* 15, no. 1 (January 1988): 3–57. https://doi.org/10.3102/0091732X015001003.

Nisbett, R., G. Fong, D. Lehman and P. Cheng. 'Teaching Reasoning'. *Science* 238, no. 4827 (October 1987): 625–631. https://doi.org/10.1126/science.3672116.

Nye, Andrea. *Words of Power: A Feminist Reading of the History of Logic*. New York: Routledge, 1990.

Peckhaus, Volker. 'The Mathematical Origins of Nineteenth-century Algebra of Logic'. In *The Development of Modern Logic*, edited by Leila Haaparanta, 159–195. Oxford: Oxford University Press, 2009. https://doi.org/10.1093/acprof:oso/9780195137316.003.0018.

Pigden, Charles R. 'Comments on 'Hume's Master Argument''. In *Hume on Is and Ought*, edited by Charles R. Pigden, 128–142. New York: Palgrave Macmillan, 2010.

———. 'Introduction'. In *Hume on Is and Ought*, edited by Charles R. Pigden. New York: Palgrave Macmillan, 2010.

———. 'Snare's Puzzle/Hume's Purpose: Non-cognitivism and What Hume Was Really up to with No-ought-from-is'. In *Hume on Is and Ought*, edited by Charles R. Pigden. New York: Palgrave Macmillan, 2010.

Priest, Graham. 'Logical Pluralism'. Chap. 12 in *Doubt Truth to be a Liar*, Reprint, 194–211. Oxford: Clarendon Press, 2009. https://doi.org/10.1093/0199263280.003.0013.

———. 'Two Dogmas of Quineanism'. *The Philosophical Quarterly* 29, no. 117 (1979): 289–301. https://doi.org/10.2307/2219445.

———. 'Validity'. In *The Nature of Logic*, edited by Achille C. Varzi, 183–206. European review of philosophy. Stanford, Calif.: CSLI Publications, 1999.

Prior, Arthur. 'The Runabout Inference-ticket'. *Analysis* 21, no. 2 (1960): 38–39. https://doi.org/10.1093/analys/21.2.38.

Putnam, Hilary. 'Is Logic Empirical?' In *Boston Studies in the Philosophy of Science: Proceedings of the Boston Colloquium for the Philosophy of Science 1966/1968*, edited by Robert S. Cohen and Marx W. Wartofsky, 216–241. Dordrecht: Springer Netherlands, 1969. https://doi.org/10.1007/978-94-010-3381-7_5.

Quine, W. V. 'Homage to Rudolf Carnap'. In *Rudolf Carnap, Logical Empiricist: Materials and Perspectives*, edited by Jaakko Hintikka, 73:xxiv–xxvii. Synthese Library. Springer Netherlands, 1975. https://doi.org/10.1007/978-94-010-1807-4.

Quine, Willard Van Orman. *Elementary Logic*. Boston: Ginn / Company, 1941. https://catalog.hathitrust.org/Record/001958442.

———. *The Time of My Life*. Cambridge, Mass.: MIT Press, 1985.

Reichenbach, Hans. *Elements of Symbolic Logic*. New York: The Free Press, 1966. https://catalog.hathitrust.org/Record/006196618.

———. *Selected Writings 1909-1953: With a Selection of Biographical and Autobiographical Sketches*. Edited by Maria Reichenbach and Robert S. Cohen. Translated by Elizabeth H. Schneewind. Vol. 1. Vienna circle collection. Dordrecht: D. Reidel, 1978.

Rescher, Nicholas. *Introduction to Logic*. New York: St. Martin's Press, 1964. https://catalog.hathitrust.org/Record/001387398.

Resnik, Michael David. 'The Frege-hilbert Controversy'. *Philosophy and Phenomenological Research* 34, no. 3 (March 1974): 386–403. https://doi.org/10.2307/2107085.

Rigg, James McMullen. 'Whately, Richard'. In *Dictionary of National Biography: Watson - Whewell*, edited by Sidney Lee, 60:423–429. London: Elder Smith & Co., 1899. https://en.wikisource.org/wiki/Whately,_Richard_(DNB00).

Russell, Bertrand. *The Autobiography of Bertrand Russell 1914-1944*. Boston: Little, Brown, 1969. http://www.worldcat.org/oclc/496844317.

Russell, Gillian. 'Logic Isn't Normative'. *Inquiry* 63, nos. 3-4 (April 2020): 371–388. https://doi.org/10.1080/0020174X.2017.1372305.

———. 'One True Logic?' *Journal of Philosophical Logic* 37, no. 6 (2008): 593–611. https://doi.org/10.1007/s10992-008-9082-6.

Sainsbury, Richard Mark. *Paradoxes*. 2nd ed. Cambridge: Cambridge University Press, 1999.

Schnee, Ian. 'Bactrians and Dromedaries'. *Teaching Philosophy* 40, no. 4 (2017): 463–481. https://doi.org/10.5840/teachphil201813080.

Schroeder-Heister, Peter. 'Proof-Theoretic Semantics'. In *The Stanford Encyclopedia of Philosophy*, Spring 2018, edited by Edward N. Zalta. 1 February 2018. https://plato.stanford.edu/archives/spr2018/entries/proof-theoretic-semantics.

'The Fifth International Congress for the Unity of Science at Harvard University'. *Science* 88, no. 2292 (December 1938): 519. https://doi.org/10.1126/science.88.2292.519.

Sigmund, Karl. *Exact Thinking in Demented Times: The Vienna Circle and the Epic Quest for the Foundations of Science*. Basic Books, 2017.

Snyder, T. D. *120 Years of American Education: A Statistical Portrait*. Technical report. National Center for Education Statistics, January 1993. http://nces.ed.gov/pubs93/93442.pdf.

Sobel, Jordan Howard. 'Nether Logic'. *Teaching Philosophy* 17, no. 2 (1994): 161–171. https://doi.org/10.5840/teachphil199417217.

Spade, Paul Vincent, and Jaakko J. Hintikka. 'History of Logic'. Encyclopædia Britannica Online, September 2019. Accessed 12 November 2019. https://www.britannica.com/topic/history-of-logic.

Stadler, Friedrich. 'The Vienna Circle and Its Periphery: Biographies and Biobibliographies'. Chap. 12 in *The Vienna Circle: Studies in the Origins, Development, and Influence of Logical Empiricism*, 4:397–596. Vienna Circle Institute Library. Cham: Springer, 2015. https://doi.org/10.1007/978-3-319-16561-5_12.

Stebbing, L. Susan. *A Modern Introduction to Logic*. London: Methuen & co. ltd., 1930. https://catalog.hathitrust.org/Record/001387317.

Steinberger, Florian. 'Explosion and the Normativity of Logic'. *Mind* 125, no. 498 (April 2016): 385–419. https://doi.org/10.1093/mind/fzv203.

Steinberger, Florian. 'Logical Pluralism and Normativity'. *Philosophers' Imprint* 19, no. 12 (2019): 1–19. http://hdl.handle.net/2027/spo.3521354.0019.012.

Sumsion, Jennifer, and Joy Goodfellow. 'Identifying Generic Skills through Curriculum Mapping: A Critical Evaluation'. *Higher Education Research & Development* 23, no. 3 (2004): 329–346. https://doi.org/10.1080/0729436042000235436.

Tarski, Alfred. *Introduction to Logic: And to the methodology of deductive sciences*. Second Edition, Revised. Translated by Olaf Helmer-Hirschberg. New York: Oxford University Press, 1946. https://catalog.hathitrust.org/Record/000168956.

Vaidya, Anand Jayprakash. 'Does Critical Thinking and Logic Education Have a Western Bias? The Case of the Nyaya School of Classical Indian Philosophy'. *Journal of Philosophy of Education,* 51, no. 1 (2017): 132–160. https://doi.org/10.1111/1467-9752.12189.

Varzi, Achille C. 'On Logical Relativity'. *Philosophical Issues* 12 (2002): 197–219. http://www.jstor.com/stable/3050548.

Weber, Zach. 'A Guide to Logical Pluralism for Non-logicians'. *Think* 16, no. 47 (2017): 93–114. https://doi.org/10.1017/s1477175617000239.

Whately, Richard. *Elements of Logic*. Ninth ed. London: John W. Parker, 1853.

Wiggins, Grant P., and Jay McTighe. *Understanding by Design*. 2nd ed. Alexandria, VA: Association for Supervision and Curriculum Development, 2005.

Williamson, Timothy. 'Alternative Logics and Applied Mathematics'. *Philosophical Issues* 28, no. 1 (2018): 399–424. https://doi.org/10.1111/phis.12131.

Woller, Raymond. 'Beginning with Proofs in Introductory Logic'. *Teaching Philosophy* 3, no. 2 (1979): 169–172. https://doi.org/10.5840/teachphil19793225.

LIST OF FIGURES

IMAGE SOURCES

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Figure 1.1: Wikipe-tan wearing a version of 'The dress'. Figure design by Kasuga jawiki, vectorization by Editor at Large, 'The dress' modification by Jahobr. CC BY-SA 3.0. Retrieved July 2020 from: https://commons.wikimedia.org/wiki/File:Wikipe-tan_wearing_The_Dress_reduced.svg

Figure 2.1: Richard Whately, by Henry Meyer after Charles Grey, 1830s-1840s, Stipple engraving. Non-commercial/Student thesis or dissertation. Retrieved April 2020 from: https://www.npg.org.uk/collections/search/portrait/mw42280

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Figure 2.3a: Gottlob Frege. *Encyclopaedia Britannica*. Public Domain. Retrieved April 2020 from: https://www.britannica.com/biography/Gottlob-Frege#/media/1/218763/10839

Figure 2.3b: David Hilbert. *American Journal of Mathematics* 29, no. 1 (1907). Public Domain. Accessed July 30, 2020. www.jstor.org/stable/2369910.

Figure 2.5: Paris, Exposition Universelle 1900, Eugene Trutat, October 1900, photograph preserved by the muséum de Toulouse. Public Domain. Retrieved April 2020 from: https://commons.wikimedia.org/wiki/File:MHNT_PHa_912_P093.jpg

Figure 2.6: Meeting place of the Vienna Circle. Institute Vienna Circle. CC BY-SA 4.0. Retrieved April 2020 from: https://commons.wikimedia.org/wiki/File:Eingang_Mathematisches_Kolloquium.TIF

Figure 2.7: Lizzie Susan Stebbing, by Howard Coster photographer, 1939, half-plate film negative. Non-commercial/Student thesis or dissertation. Retrieved May 2020 from: https://www.npg.org.uk/collections/search/portrait/mw56414

Figure 2.8: Rudolf Carnap, professor of Philosophy at the University of Chicago, University of Chicago Photographic Archive, [apf1-01560], Special Collections Research Center, University of Chicago Library. Retrieved May 2020 from: http://photoarchive.lib.uchicago.edu/db.xqy?one=apf1-01560.xml

Figure 2.9: Hans Reichenbach. Hans Reichenbach Papers, University of Pittsburgh Archives & Special Collections. No known restrictions. https://digital.library.pitt.edu/islandora/object/pitt%3AUS-PPiU-asp197301/viewer

Figure 2.10: Alfred Tarski and Kurt Gödel in Vienna, photograph taken by Maria Kokoszyńska-Lutmanowa, 1935, Alfred Tarski papers, UC Berkeley, Bancroft Library. Public Domain. Retrieved May 2020 from: http://ark.cdlib.org/ark:/28722/bk0016t8g2w

Figure 2.11: W.V.O Quine in 1934, Junior Fellow at Harvard University. Courtesy of Douglas B. Quine, PhD; W. V. Quine Literary Estate

Figure 2.12: Chicago's Lake Front. Kaufmann & Fabry Co. ca. 1938. Photograph. No known restrictions. Library of Congress (USA). Retrieved May 2020 from: https://www.loc.gov/item/2007660861/

Figure 2.13: State Debates. Duluth Central High School's Zenith Yearbook. 1933. No known restrictions. Courtesy of Duluth Public Library.

Figure 2.14: Bertrand Russell lecturing at the University California, Los Angeles. c. Apr. 1940. No known restrictions. McMaster University Archives (Archive Box Number: RA3 Rec. Acq. 1200). https://library.mcmaster.ca/spaces/archives/russell

Figure 2.15: Photograph of Harvard Yard. *Encyclopedia Americana*. 1920. Public Domain. Retrieved April 2020 from: https://commons.wikimedia.org/wiki/File:Americana_1920_Harvard_University.jpg

Figure 2.17: Richard Montague c.1965–67, by the arrangement of the UCLA philosophy department as part of series of portraits of senior and prominent faculty members. No known restrictions. Photo courtesy of Dr Ivano Caponigro.

Figure 2.18: Embattled professors: Angela Davis with UCLA philosophy department Chairman Donald Kalish at news conference that followed her lecture. Los Angeles Times, Photographer: Whitney Fitzgerald. October 7, 1969. CC BY 4.0. Los Angeles Times Photographic Archive (Collection 1429). Library Special Collections, Charles E. Young Research Library, UCLA. Retrieved May 2020 from: http://www.oac.cdlib.org/ark:/13030/hb8xonb644/?order=1

LIST OF TABLES