Naturalized Modal Epistemology and Quasi-Realism

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Abstract

Given quasi-realism, the claim is that any attempt to naturalize modal epistemology would leave out absolute necessity. The reason, according to Simon Blackburn, is that we cannot offer an empirical psychological explanation for why we take any truth to be absolutely necessary, lest we lose any right to regard it as absolutely necessary. In this paper, I argue that not only can we offer such an explanation, but also that the explanation won't come with a forfeiture of the involved necessity. Using 'squaring the circle' as evidence, I show that, contrary to quasi-realism, absolute necessity won't be left out in attempts to naturalize modal epistemology.

Keywords: Naturalized Modal Epistemology; Quasi-realism; Absolute Necessity; Imagination; Cognitive Artifacts; Squaring the Circle

1. Introduction

For a considerable period, modal epistemology, the study of our knowledge of possibility and necessity, was done rationalistically, i.e., independently of the empirical sciences that study our experiences of modality. Following Quine's (1969) now famous *Epistemology Naturalized*,[1] however, we have been called to take findings from those sciences seriously when doing modal epistemology, i.e., we have been called to naturalize modal epistemology (Nolan 2017a). Elsewhere, (Omoge 2021), I tried to respond to this call. I argued that by scrutinizing the psychology of imagination, we can get a better handle on what it takes to use imagination to make metaphysical modal judgments. In essence, I offered a naturalistic epistemological account of metaphysical modality by way of psychologizing imagination.

However, since many philosophers take metaphysical modality to be *absolute*, i.e., modality in the widest sense or modality without qualification (Clarke-Doane 2019), and given Simon Blackburn's quasi-realism (1984 and 1993), my imagination-based psychologically[2] naturalized modal epistemological account seems to face one significant problem. According to Blackburn, any epistemological account of absolute necessity that operates by psychologizing imagination would explain away the involved necessity: "we cannot explain naturalistically our own failure to see what it would be for [an absolutely necessary falsehood] to be true. When we can see how, if [it] were true [...] we release it from impossibility. It does not deserve ruling out any more" (1993, 70). If so, and since some metaphysical necessities, e.g., logical (modus ponens) and arithmetical (1+1=2) ones, are archetypical examples of absolute necessity, then my imagination-based psychological analysis of metaphysical modality can be said to have inadvertently jeopardized the absoluteness of metaphysical modality.

In this paper, I argue against Blackburn's view that absolute necessity is antinaturalistic. In Section 2, I explain why he holds this view. In Section 3, I say more about some unclear aspects of his arguments, clarifying how I read him on those occasions. In Section 4, I consider Shaun Nichols' (2006) response to Blackburn, arguing that even though he addresses one important aspect of Blackburn's concerns, he is silent where it matters most—whether once we can naturalistically explain why we take absolute necessities to be absolutely necessary, they cease to be absolutely necessary. Consequently, in Section 5, I argue that 'squaring the circle' is evidence that they don't: we can naturalistically explain why we take it to be absolutely impossible, and we don't thereby lose our right to regard it as absolutely impossible. In Section 6, I address a possible quasi-realistic objection, and conclude in Section 7.

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[1] Post-Quine, naturalism has been divided into at least metaphysical and methodological. Naturalism is metaphysical if philosophical questions are taken to be scientific ones, but it is methodological if the scientific method (using 'scientific method' as an umbrella term to cover behavioural, natural, and human sciences) and/or scientific findings are taken as instrumental to philosophical analyses. See, e.g., Papineau (2009) for a discussion. See also n. 4.

[2] Naturalistic explanations needn't be psychological, however. See Section 3.

For clarity's sake, and also because I will bring other kinds of necessity as we progress, I will retain speaking in terms of 'absolute necessity', and not shorten it to 'necessity'.

2. Blackburn on the Antinaturalistic Nature of Absolute Necessity

Over the years, Blackburn has pushed for a project he calls *quasi-realism*, which he claims offers plausible treatments of a range of philosophical topics. One motivation for the project is that both realism and antirealism fail to correctly account for knowledge and truth. As he explains it, realism holds that knowledge and truth are grounded in an extra-natural realm of facts, whereas, antirealism denies such realm, encourages us to feel comfortable in claiming knowledge and truth as natural, but fails to tell us how to find them natural (Blackburn 1993, 32 & 35). This failure of antirealism, Blackburn argues, leaves the semantic machinery of realism enticing, such that rejecting the machinery alongside the extra-natural realm of facts realism postulates, is like throwing the baby away with the bath water.

The way out, in his view, is a midway between realism and antirealism: "This result accords with what I call 'quasi-realism', for it is another respect in which someone who approves of the anti-realist instinct over the priority of truth or virtue still ends up with the very thoughts that the realist took for his own" (Blackburn 1993, 48). In sum, quasi-realism is the midway between realism and anti-realism. Quasi-realism, therefore, asks "broadly anthropological questions: How are we to understand the roles and functions of the behaviour [or concept] in question, in the lives of the creatures concerned? What is its practical significance? Whence its genealogy?" (Macarthur and Price 2007, 95). By answering these questions, quasi-realism, Blackburn says, would yield empirical psychological explanations of our judgements in those situations.

Applying this quasi-realistic project to modality yields the stance that absolute necessity is antinaturalistic, in that (the blocks that issue) our judgments of absolute necessity becomes empirically psychologically unexplainable. According to him, this is because when we run modality through the anthropological questions quasi-realism asks, as above, we see that the whole point of modalizing is to *vary our beliefs with circumstance*, where 'variation of beliefs with circumstance' involves making sense of the constant changes in our knowledge as we interact with others and with the world. Hence, he says: "naturalized epistemology is largely a study of the variation of belief with circumstance" (1993, 66). In his view, modalizing helps us to do this and because doing so was/is beneficial to our social and cognitive life, we have evolved to effortlessly and non-collusively modalize. This, according to him, is why we modalize. To be clear then, his concern isn't to offer a naturalized epistemology of absolute necessity, but with one particular way—the psychological approach—of giving such an epistemology, like my (2021). For him, psychologizing *why* we absolutely modalize can't/mustn't succeed. Let me expatiate.

In his view, psychologically explaining modalizing involves '*making something of*', which he defines thus: "Here 'make something of' will include being able to explain how such a way of thought might arise, knowing how it might be rectified, understanding the practices of those whose thought it is, and so on" (1993, 66). Ultimately, 'make something of' explains why he thinks absolute necessity is antinaturalistic in that it clarifies which modal commitments can't be run through the anthropological questions quasi-realism asks. For instance, we can't answer whence comes the genealogy of 1+1=3: "Seeing it like that would require thinking the other side of the boundary: understanding how it might be, for instance, that although it is compulsory for us to use classical arithmetic, with a different cast of mind it might have been compulsory to use another arithmetic. And this we cannot do" (1993, 72).

Since he says naturalized epistemology consists largely in varying beliefs with circumstance, he follows up that, "[naturalized epistemology] can be done by us only when we can make something of the variation of belief involved" (1993, 66), i.e., 'make something of' captures the essence of naturalized modal epistemology. Since we can't 'make something of' absolutely necessary falsehoods, he concludes that any psychologically naturalistic explanation of why we absolutely modalize would/must fail. For the imaginative blocks we face when we endeavour to give such an explanation is not a contingent fact about us: "But it goes a little further, for in the light of what we have said, it will also be so that we cannot see the incapacity as *just* one we happen to be subject to; we cannot deem it a *mere* fact about ourselves, here, now" (1993, 71). Thus, in clearer terms, psychologizing why we absolutely modalize would/must fail because we can't psychologize why we face the imaginative blocks that issue our judgments of absolute necessity: "We now find that if *any* natural explanation of our imaginative block can be given, this attacks our right to make the commitment" (1993, 71).

To summarize: Our modal commitments enable us to vary our beliefs with circumstance via 'make something of'; we can't do this for some beliefs and this inability renders the beliefs absolutely necessary; consequently, the involved modal commitments must remain non-variable with circumstance; i.e., they must be impervious to naturalistic explanations. Thus, we can't naturalistically explain why absolute necessities are absolutely necessary because to do so would mean they weren't absolutely necessary in the first place, and if they ever were, they will therefrom cease to be absolutely necessary. Blackburn puts it forcefully: "In the case of the modal, the phenomenon is antinaturalistic at its core" (1993: 72).

Blackburn tells the story in a much more interesting manner and situate it within a much wider project, but this short rendition suffices for our purpose—namely, to show that naturalistically explaining why we absolutely modalize won't lead to the forfeiture of the involved necessity. To set us on our way, it would be best to clarify some aspects of Blackburn's argument, which he isn't so clear about.

3. Taking Stock of Blackburn's View of Absolute Necessity

It isn't clear to what degree we should read Blackburn on the antinaturalistic nature of absolute necessity. I, for one, tend to read him pessimistically—that he thinks we can't naturalistically explain why we regard absolute necessities as absolutely necessary, *come what may*. But there seems to be room for a less pessimistic reading. For instance, he says at the end:

> Or is this unduly pessimistic? Some relief might be got by teasing out more aspects of the core inability to 'make anything of' a way of thought that accepts a putative impossibility. Obviously, there are enterprises of thinking through what modifications in logic are possible or what would be missing in a way of thought that consistently tried to make 1+1=3 […] So, it ought to be possible to hold both that these laws are necessarily true and that we can 'make something of' ways of thought that lead people to deny them" (1993, 72).

Regardless, I think this less pessimistic reading ultimately reduces to my pessimistic reading. For no sooner had he cautioned his pessimism than he insisted that we hit a firmer imaginative block or forfeit the involved necessity:

> [The above quote] is not a serious obstacle to the direction of this essay. What we do is take a proposed deviation and follow it until either the way of thought seems possible—and we no longer modalize against it—or it breaks down. But 'breaks down' will mean: offends against something that we suppose essential to any scheme of thought (such as some distinction of truth and falsity […]). Eventually we voice an inability to make anything of transgression against these norms: this is the surd that remains (1993, 72).

In short, it seems he wants us to read him pessimistically even though he countenances that "it may do a little to moderate the antinaturalistic pessimism" (1993, 72).

This then provides one way of reading his view that our *inability* to make anything of absolutely necessary falsehoods isn't a contingent fact about us, which he isn't explicit about as well. Is the 'inability' absolutely or relatively necessary? I think he needs it to be absolutely necessary. After all, were it relatively necessary, i.e., were there a given body of statements to which the inability follows as a consequence (cf. Hale 1996), it would be logically possible that we can make something of absolutely necessary falsehoods. So, since he wants to insist that we can't make anything of them, come what may, then that inability must itself be absolutely necessary. But in what sense is it absolutely necessary? He doesn't say. I'll zero-in on this lacuna later in Section 6. For now, it suffices that I'll read him, in what follows, as saying that our 'inability' to make anything of absolutely necessary falsehoods is itself *absolutely* necessary.

From the foregoing, it is clear that Blackburn takes naturalistic explanations to be only psychological ones. But we now know that naturalistic explanations needn't be psychological; they can also be *linguistic* and *scientific* (Nolan 2017a). Linguistic ones consider the behaviour of modal terms in our natural languages; scientific ones, the role modality plays in scientific theorization. But a quasi-realist could insist that both explanations do not escape the challenge Blackburn raises. They would say a linguistic explanation presupposes making sense of the mental act involved in modalizing, and since that mental act is 'make something of', it reduces to a psychological one. Similarly, they would say explaining modality through understanding our best science entails regarding the world as such that we can't make anything of false scientific claims independently of what anyone may think, i.e., we can't give a psychological explanation of such an inability. So, in both instances, Blackburn's view stands.[3] Notice how this insistence supports reading Blackburn pessimistically. For were linguistic and/or scientific explanations of why we absolutely modalize feasible, then absolute necessity wouldn't be antinaturalistic at its core after all.

This psychological dimension then explains why Blackburn takes quasi-realism as buttressing Quine's (1960) view about absolute necessity. Quine says we should abandon modalizing in the absolute sense altogether, but Blackburn says we should at least know why we absolutely modalize before abandoning it.[4] Zangwill argues that Blackburn seems to have missed Quine's point in that where he says absolutely necessary beliefs can't be varied with circumstance, "Quine will say that all beliefs vary with circumstances, but some *more* than others" (1988, 146). Perhaps this is why Hale thinks Blackburn "confuses psychological facts about our sentiments with the 'real modal status of propositions'" (1986, 78). For Quine might just be saying that absolutely modalizing is naturalistically explainable albeit reductively. Even so, Blackburn seems to be saying that any such reductive naturalistic explanation would explain away the absoluteness of absolute modalizing. Again, my pessimistic reading of him is fair.

Spelling out Blackburn's qualms with absolute necessity in this way helps to show where available responses fall short. Notably, Nichols (2006) argues that, *pace* Blackburn, we can give a psychologically naturalistic explanation for the imaginative blocks that issue judgments of absolute necessity, but his view falls short in one significant respect. It doesn't say anything about whether the judgments it allows us to naturalistically explain lose their absoluteness. Let me quickly review his position then, to explain the gap he leaves open, which I want to fill here.

[3] Thanks to an anonymous reviewer for this journal for impressing this point upon me.

[4] Since Quine famously takes epistemological questions to be psychological ones, his naturalism about epistemology is metaphysical (see n. 1). If so, then Blackburn, despite ruling that absolute necessity is antinaturalistic, is also metaphysically naturalistic about modal epistemology. However, as Nolan (2017a) clarifies, modal epistemology is best naturalized methodologically, and so, a case can be made against Blackburn in modal epistemology in a manner similar to Nolan's (2017b) case against him in meta-ethics. While this avenue is promising, I won't it pursue here.

4. Nichols on Blackburn's View of Absolute Necessity

Nichols argues that Blackburn gives up too quickly. In his view, given what we now know about the neuropsychology of imagination, we can explain the imaginative blocks we face when we try to imagine how 1+1=3 could be true. He writes: "*Pace* Blackburn, a naturalist might be able to explain why we have an imaginative block against the proposition 1+1=3 without being able to make anything out of thought that 1+1=3 might be true" (2006, 247). What explanation might this be?

In an earlier work—Nichols and Stich (2003)—he argues that imagination isn't an unconstrained fantasy generator, because imaginings and beliefs are in the 'same code', i.e., they have the same contents, interact with the same cognitive mechanisms, and have the same logical form. Consequently, he dubs this view, the 'single code hypothesis' (Nichols 2004). The hypothesis is that "for any mechanism that takes input from both the [imagination] box and the belief box, the [imagination] representation *p* will be processed much the same way as the belief representation *p*" (Nichols 2004, 131).

In his (2006), he then uses the single code hypothesis to explain why we face imaginative blocks when we attempt to consider how it is that 1+1=3, for example. He gives this as the reason we face the blocks: "if the candidate belief that *p* would be immediately rejected by the inference mechanisms, then the candidate [imagination] representation that *p* will also be immediately rejected by the inference mechanisms" (2006, 250). Thus, according to him, we face imaginative blocks when we endeavour to see how 1+1 could be 3, because of "the fact that the [imagination] representation that $p\&\sim p$ would engage our normal inferential systems. And just as our inferential systems would expel the belief representation $p\&\sim p$, so too do they expel the [imagination] representation that $p\&\sim p$" (2006, 250). Simply, we face the blocks that issue judgments of absolute necessity because imagination is not an unconstrained fantasy generator in that it is hooked up to our inferential systems. Cognitive-architectural constraints on imagination, therefore, explain why we face the blocks that issue judgments of absolute necessity.

Since this sort of explanation doesn't rely on us to make anything of absolutely necessary falsehoods, Nichols argues that, contrary to Blackburn, 'make something of' doesn't capture the essence of naturalized modal epistemology, in that it isn't characterized by varying beliefs with circumstance. Put differently, Nichols' view is that we aren't obliged to understand the other side of the boundary, i.e., naturalistically explaining why 1+1≠3 is absolutely impossible doesn't necessarily involve understanding how it could be that 1+1=3. He puts it better: "in pursuing a naturalistic explanation of the imaginative blocks, we are not obligated to 'make sense' of propositions that are excluded by our imaginative blocks. We are merely obligated to explain how the blocks arise" (2006, 248; see also Björnsson 2004). In this way, he argues that naturalized modal epistemology can be done for absolute necessities. I agree.

However, a quasi-realist could take Nichols' strategy to explain 'make something of' away, as vindication that we can't after all, in the manner that Blackburn explains, make anything of absolutely necessary falsehoods. Put differently, the quasi-realist could argue that despite having a naturalistic explanation for the imaginative blocks that issue the judgment that $1+1\neq3$ is absolutely impossible, we still can't entertain a way of thought where $1+1=3$. Yes, Nichols has said we are not obliged to understand such a way of thought, that explaining how the blocks arise is what matters, but the quasi-realist could insist that an explanation of why the blocks arise is insufficient. They might say that we must at least, be able to explain why our inferential systems rebel at contradictions, and that this would, in one way or other, involve understanding the other side of the boundary, which they would then say, Blackburn has stressed that we can't do. For if we can, then we would understand whatever our inferential systems are blocking out, i.e., we would understand how a world where those blocked-out propositions are true looks like.

I agree that we can't do this. Although Nichols doesn't say, I suspect that he agrees as well—that's partly why he focuses on explaining how the blocks arise. At any rate, the gap he leaves open, which allows the quasi-realist to demand understanding what is being blocked out, is what I want to bridge here. The gap is that given what he says, it is unclear why we needn't understand it.[5] After all, it is one thing to respond to Blackburn's view that we *can't* understand what is being blocked out by saying we *needn't* understand it, and quite another to give the reason(s) why we needn't understand it. In short, Nichols' view that we needn't understand the other side of the boundary doesn't tell us whether our right to regard the involved judgments as absolutely necessary is secured or lost in not needing to understand the other side of the boundary.

My own view is that we won't lose any such right, and my evidence is that we've had, for a while now, a naturalistic explanation for why 'squaring the circle' is absolutely impossible, and we neither haven't nor we will ever, as a result of having the explanation, lose our right to regard squaring the circle as absolutely impossible. In the next section, I'll give this naturalistic explanation, and in Section 6, I'll explain why it hasn't led to us to losing our right to regard squaring the circle as absolutely impossible, thereby explaining why understanding the other side of the boundary doesn't matter.

[5] Nichols uses chimps' cognitive boundedness as an analogy, saying that "when we do the parallel naturalistic project on humans, we're the apes under investigation, and we should try, as far as possible, to achieve a similar distance" (2006, 248). The quasi-realist might pick on this, arguing that given this analogy, the reason we needn't understand the other side of the boundary is as Blackburn says—that we *can't*. That is, Nichols is saying that only an external observer can investigate whatever aspect of our psychology subserves absolute modalizing just as cognitive ethologists do for chimps. But the quasi-realist should be careful here. Nichols clearly says that "when *we* do the parallel naturalistic project on humans", meaning that we are both the investigator and the investigated. Moreover, neuroscience has evolved to such an extent that we now know what we don't know in yesteryears, hence, Nichols saying that "Blackburn surrenders *too quickly*". What neuroscience tells us about our psychology vis-á-vis absolute modalizing is what Nichols laid out, which though entails that we needn't understand the other side of the boundary, excludes why we needn't have the understanding. The last part is what I hope to add in Section 6.

5. Squaring the Circle: A Case for Naturalized Absolute Necessity

To square a circle, one would have to use only a compass and straightedges, and construct for any given circle, a square with the same area as the circle. Some quick clarifications before proceeding. First, it is not that a square of equal area with a circle does not exist. If a circle has a given area, then a square with sides that are each the root of that area, has the same area as the circle. Second, it is not that giving the square of a circle is impossible, since it is *possible* in the above described sense, but it is impossible under the restriction of using only a compass and straightedges. In this second sense, squaring the circle is absolutely impossible, although it isn't absolutely impossible in the first sense. Thus, when I say, 'squaring the circle' in what follows, I mean it exclusively in the restricted second sense of using only a compass and straightedges.

Since the earliest days of mathematics,[6] renowned mathematicians, like Anaxogoras, Euclid, Archimedes, Leonardo da Vinci, Descartes, to mention but a few, laboured in vain to square the circle.[7] But we have had proof, since the late 19th century, that the task is absolutely impossible, thanks to the knowledge that π is a transcendental number, i.e., that π cannot be a root of any algebraic equation. The transcendence of π proved monumental in deciding the absolute impossibility of squaring the circle, because how to determine whether a proposed Euclidean problem is possible has been established. For a proposed Euclidean problem to be possible, each of the data of the problem must be a root of one algebraic equation or other. In the case of squaring the circle, π features as one of the data of the problem, and since π cannot be a root of any algebraic equation, we know that the problem of squaring the circle cannot be determined by Euclidean constructions (Hobson 1913). *Ergo,* squaring the circle is absolutely impossible. It is both nomically and logically (or more precisely, mathematically) impossible that circles can be squared.[8]

Though very brief, this exposition of why squaring the circle is absolutely impossible is apt for our purpose, since it counts, in all relevant senses, as an example that Blackburn shouldn't have any problem accepting as a case of absolute necessity. As we've seen (Section 2), only mathematics and logic cases are absolute necessity for him, and what make squaring the circle absolutely impossible are the mathematical principles that it entails, not that it is a Herculean task that requires superhuman strength and dexterity to be accomplished.[9] Regardless of strength, dexterity, and creativity, no one can square a circle: it is pointless for anyone to try.

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[6] One source (Hobson 1913) traces the first appearance of the problem to the Rhind Mathematical Papyrus (ca. 1650 B.C).
[7] See Fletcher (2007) for a review of the different approaches used over the years for trying to square a circle.
[8] But not epistemically impossible. See, however, Section 6 for why epistemic possibility doesn't count.
[9] Compare the inscription of a heptadecagon (a 17-sided polygon) in a circle. It is seemingly a more difficult task than the inscription of a square (a 4-sided polygon) in a circle, but it has been shown to be constructible by Gauss in 1796, way before

However, despite this explanation for why squaring the circle is absolutely impossible, a quasi-realist might counter that the explanation doesn't rely on any facts about our psychology, and so, it doesn't count as a psychologically naturalistic explanation. But should they make this move, they would be adopting an outdated psychology of mathematical cognition, saying that the only way an epistemology of mathematics can be psychological is for it to rely on some phylogenetic or ontogenetic facts about our psychology, and this is no longer the accepted psychological explanation of mathematical cognition.

Initially, psychological research about mathematics focuses on neuroscientific issues involved in the mathematically relevant behaviour of an individual, i.e., they were elucidated at the individual level: "The typical view in cognitive science is that mathematical cognition should be studied as a purely *individual* achievement" (Hohol and Miłkowski 2019, 2). But this approach has been questioned (e.g., Hutchins 1995). Mathematics seems not to be the achievement of any single individual; rather, "mathematical justificatory practices may be understood in terms of repeatable public procedures that rely on the capacities of cognitive agents to jointly construct, explore and reconfigure representational tokens" (Hohol and Miłkowski 2019, 2). That is, mathematical practice is shared by individuals who can publicly use the same *cognitive artifacts*. Where 'cognitive artifacts' are entities that "maintain, display, or operate upon information in order to serve a representational function and that affect human cognitive performance" (Norman 1991, 17).

Cognitive artifacts that were pivotal in the evolution of mathematical reasoning include diagrams and linguistic formulae (Netz 2003), cognitive metaphors (Núñez and Lakoff 2005), basic spatial capacity (Spelke et al. 2010), and so on.[10] These artifacts enhanced the performance of groups of individuals over time, such that their collective performance and not just the distributed individualistic ones became reified in our evolutionary history, perhaps through the Baldwin effect.[11] After all, it is one of our distinctive characteristics that we are able to modify our environment through the creation of artifacts and transmit the modifications to subsequent generations (Clark 2004). One effect of this transformative process is the availability of "a new kind of cognitive niche whose features and properties complement but do not need to replicate the basic modes of operation and representation of the biological brain" (Clark 2006, 370–1). In this way, highly intellectual endeavour, like mathematics, are now said to not be the achievement of any single individual.

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the latter was proved to be absolutely impossible. The reason the former is not only logically but also nomically possible (i.e., constructible using only compass and straight edges) is that π doesn't feature as one of its data. Otherwise put, in both cases, the challenge isn't about the nature of the tasks, but about the mathematical principles underlying them.

[10] See Heersmink (2013) for a taxonomy of cognitive artifacts.

[11] The Baldwin effect is an evolutionary mechanism which transforms a culturally invented and acquired trait into an instinctive trait by the means of natural selection. See Baldwin (1896) for an etymology, and Godfrey-Smith (2003) for a development.

For instance, even though the transcendence of π was proved by Ferdinand von Lindemann in 1882, it would be myopic to say it was solely his achievement. Had Joseph Liouville not earlier proved that there could be transcendental numbers in 1844, and Charles Hermite not proved that *e* is a transcendental number in 1873, it is highly doubtful that Lindenmann would have been able to prove the transcendence of π. What's more, it is now agreed in most quarters within cognitive science and evolutionary psychology, that individual achievements, like those of Liouville, Hermite, and Lindenmann, would have been unachievable without reliance on some cognitive artifacts. Clark summarizes this point best:

> The project of understanding human thought and reason is easily misconstrued [...] as the project of understanding what is special about the human brain [rather it] requires an even broader perspective: one that targets multiple brains and bodies operating in specially constructed environments replete with *artifacts* (2004, 34, my italics).

Consequently, mathematical cognition (Tylén et al. 2014), and cognition in general (Gureckis and Goldstone 2006) are no longer understood in terms of individual achievements. Cognitive study of mathematical cognition outstrips facts about the psychology of individuals engaged in mathematical practise, including artifacts that have structured the cognitive niche in which the practise thrives.

Under this extended sense, the given explanation for why squaring the circle is absolutely impossible counts as psychological in all relevant senses. It may not rely on any phylogenetic or ontogenetic facts about our psychology, but it certainly relies on some cognitive artifacts that have constrained the construction of the cognitive niche for mathematical practise. If so, then we can explain using psychological facts about us, in the extended sense of cognitive artifacts, why squaring the circle is absolutely impossible. Importantly, no one who understands the explanation now claims as a result of having the understanding that squaring the circle has become contingent for them.

The quasi-realist might object, however, demanding clarification about the modal force of Euclidean axioms qua part of the explanans for why squaring the circle is judged to be absolutely impossible. If it is absolutely necessary—as expected—then I have only pushed Blackburn's concern one step back—to the level of saying we can't naturalistically explain why we take Euclidean axioms to be absolutely necessary. If they don't have modal force at all, then they can't be part of the explanans. This is Blackburn's qualms with truth-conditional analyses of modality, to which Scott Shalkowski (2008) has given a clear response. According to him, both horns of Blackburn's dilemma are misdirected toward *conceptual* analysis, but he clarifies that not all truth-conditional analyses of modality aim to analyse modal concepts. Some take modal concepts as primitive, i.e., concepts for which there are no non-modal equivalents, and for those truth-conditional analyses of modality, both horns of Blackburn's dilemma fail.

The details of Shalkowski's arguments are not important for our purpose; what is, is his conclusion: Euclidean axioms are primitive concepts, and so, despite their absolutely necessary modal status, there are no non-geometrical, a fortiori, non-modal equivalents. Thus, their absolutely necessary modal status does not fall prey to the first horn of Blackburn's dilemma, and so, there is no residual challenge.

However, even though this blocks the quasi-realist from saying Euclidean axioms can't be part of the explanans for why we judge squaring the circle to be absolutely impossible, it leaves open the caveat that, having now admitted Euclidean axioms as (part of) a psychological explanation in the extended sense of cognitive artifacts, we can now vary the judgment with circumstance, i.e., imagine differently. For example, the quasi-realist could say we can now utter this counterfactual:

> *C*: Had π been countable and rational, we would have been able to square the circle.

Simply, *C* captures an imaginable scenario where squaring the circle is logically possible, even though it is nomically impossible. Otherwise put, they may agree that we can explain why we regard squaring the circle as absolutely impossible as I've done here, but add that once we admit the offered explanation as naturalistic, we lose our right to regard squaring the circle as absolutely impossible. I disagree.

To make my case, the argumentative framework would have to differ from what has come before—from psychological to meta-psychological. For whether *C* renders squaring the circle logically possible as the quasi-realist claims, turns on how the notion of 'absolute necessity' is being used in quasi-realism. This is not only because there are different conceptions of absolute necessity, but also because what it takes for each conception to succeed/fail such that a proposition/judgment becomes logically possible differs. I will argue that available conceptions of absolute necessity do not work for *C*.

6. On the Use of Absolute Necessity in Quasi-realism

The quasi-realist says *C* captures an imaginable scenario where squaring the circle is *logically* possible. But why 'logically' it might be asked? Simple: because nomic possibility is ruled out since no one can square a circle, and epistemic possibility, though not ruled out, trivializes things, since the opposite of just about any plausible absolute necessity may be epistemically possible, i.e., true for all we know. This leaves only logical possibility. Hence, Zangwill says: "Blackburn's idea is that when we can 'make something of', explain or understand someone's not believing a true proposition, then we put it down as *logically contingent*" (1988, 145, my italics). Thus, for the quasi-realist to say that *C* renders squaring the circle possible, i.e., for them to insist that it is absolutely necessary that we can't 'make something of' absolutely necessary falsehoods, *C* must render squaring the circle logically possible.

But if so, then the quasi-realist would be saying that absolute necessities are absolutely necessary only to the extent to which our cognitive makeup is necessary. They want to insist that we can't make anything of absolutely necessary falsehoods *come what may*. But if our cognitive makeup isn't necessary in some determinate albeit non-absolute sense, then the 'come what may' qualification drops out, but we've seen (Section 3) that it mustn't. This then explains why the quasi-realist could demand an explanation of why our inferential systems rebel at contradictions from Nichols (Section 4). For according to the quasi-realist, once we can see why our inferential systems so rebel, we lose our right to regard the involved modal judgments as absolutely necessary, i.e., we explain away the necessity of our cognitive makeup, and with it, the necessity of the modal judgments.

If we call the necessity of our cognitive makeup 'psychological necessity', then the quasi-realist is saying that absolute necessity is "at least as strong as" psychological necessity. Where "$\Box_1$ is *at least as strong as* $\Box_2$ if […] '$\Box_1 p$' always entails '$\Box_2 p$'" (Hale 1996, 94). The quasi-realist is saying that the psychological necessity of our inability to 'make something of' absolutely necessary falsehoods always entails the absolute necessity of absolute necessities. More to the point and it bears repeating, once we explain away this psychological necessity, which the quasi-realist is saying my argument in Section 5 inadvertently does, absolute necessities cease to be absolutely necessary: they become logically possible. Hence, *C*.

This way of understanding 'absolute necessity' corresponds to Hale's (2012) 'maximal-absoluteness' conception of absolute necessity. He explains: a proposition *p* is maximal-absolutely necessary if "it implies, and so is at least as strong as, any other comparable kind of necessity" (2012, 121). Thus, the quasi-realist means to say that our inability to make anything of absolutely necessary falsehoods is maximal-absolutely necessary. This, for me, is the conception of absolute necessity at work in quasi-realism, which Blackburn doesn't specify, and which I promised in Section 3 that I will specify. But maximal-absoluteness doesn't afford any room for *C* to render squaring the circle logically possible. Before showing why, let us acknowledge that the quasi-realist may deny that maximal-absoluteness is the conception of absolute necessity at work in quasi-realism.

Hale also gives two other conceptions of absolute necessity: limit- and general counterfactual-absoluteness. A proposition *p* is limit-absolutely necessary if *p* is necessary relative to a set of propositions Φ, no matter the Φ we choose. A proposition *p* is general counterfactual-absolutely necessary if *p* would be the case, no matter what else was the case. Thus, the quasi-realist might say that the conception of absolute necessity at work in quasi-realism isn't maximal-absoluteness, but one of these other alternatives. Fine, but it really doesn't matter whether they are right or I am, for as we will see in what follows, all three conceptions of absolute necessity do not help their case. First, limit-absoluteness.

Even though Blackburn says the absoluteness of absolute necessities partly consists in our inability to understand the other side of the boundary, I suspect that the quasi-realist wouldn't want to say limit-absoluteness is the conception of absolute necessity at work in quasi-realism. For under it, understanding the other side of the boundary becomes irrelevant: squaring the circle might still be absolutely impossible whether we choose how our cognitive makeup actually is, or how it would be were *C* true. This way, we have a first working theory for why Nichols is right to say "we are not obligated to make sense of propositions that are excluded by our imaginative blocks, that an explanation of how the block arises is sufficient": it simply doesn't matter whether we can make sense of the blocked-out propositions. Some absolute necessities would remain absolutely necessary even if we come to understand the other side of the boundary, and we can't decide upfront that squaring the circle isn't part of this category. Limit-absoluteness doesn't help the quasi-realist's case.

Second, the reason I began with limit-absoluteness is that it shares a feature with maximal-absoluteness, which explains why maximal-absoluteness can't also help the quasi-realist's case, but this feature might be obscured if we weren't already familiar with limit-absoluteness. What feature? Hale answers: "limit-absolute and maximal-absolute necessities are always logical—i.e., there can be no *non-logical* absolute necessities in either of these senses of 'absolute'" (2012, 124). Thus, if I am right that the conception of absolute necessity at work in quasi-realism is maximal-absoluteness, then our inability to make anything of absolutely necessary falsehoods is already logically necessary. If so, and since "any kind of alethic possibility [...] conforms to a form of the law of noncontradiction" (Hale 2012, 122), then *C* cannot now render squaring the circle logically possible, since squaring the circle is already logically impossible, given maximal-absoluteness.[12] It can't be both logically possible and impossible that circles can be squared. If so, then both limit- and maximal-absoluteness can't work for the quasi-realist's agenda. A second working theory for why needn't understand the other side of the boundary is then that the quasi-realist can't justify why we should: the demand is self-contradictory.

Third, the quasi-realist might respond that the conception of absolute necessity at work in quasi-realism is rather general counterfactual-absoluteness. For since the reason squaring the circle is absolutely impossible is the mathematical principles underlying it (Section 5), squaring the circle is more

[12] I have silenced a premise here, according to which a judgment can only issue an absolute necessity of its own sort. Hale puts it better: "Evidently [...] the members of Φ themselves will automatically qualify as ø-necessary" (1996, 93). That is, if the judgment that issues a necessity is maximal-absolutely necessary, then the necessities it issues will also be maximal-absolutely necessary. So, if our inability to make anything of absolute necessities is maximal-absolutely necessary, then the necessities the inability issues, are already maximal-absolutely necessary. Since maximal-absolute necessities are always logical, then the necessities issued by the inability are logical necessities. Hence, squaring the circle is already logically impossible if our inability to make anything of it is maximal-absolutely necessary. *Mutatis mutandis* if the judgment is limit-absolutely or general-counterfactually necessary.

appropriately an arithmetical necessity, which may be metaphysical not logical.[13] Following Hale, who says, "Since I think it is very plausible that there are non-logical absolute necessities—for example, arithmetical ones—I think this is a quite strong reason to prefer the generalised counterfactual explanation of absoluteness" (2012, 124), the quasi-realist would then say the conception of absolute necessity at work in quasi-realism is rather general counterfactual-absoluteness. Fair enough, but if they say this, then they run into a brick wall, for under general counterfactual-absoluteness, *C*—the counterfactual that "had π been countable and rational, we would have been able to square the circle"—uncontroversially can't render squaring the circle logically possible.

This is because, given general counterfactual-absoluteness, it would be the case that squaring the circle is absolutely impossible no matter what else was the case, i.e., no matter what else was logically, nomically, and metaphysically possible, i.e., no matter what else was possible in any alethic sense[14]. The quasi-realist should distance themself from general counterfactual-absoluteness if they want *C* to stand any chance of rendering squaring the circle logically possible. A third working theory then for why we needn't understand the other side of the boundary is just *that*—understanding it wouldn't change a thing.

If so, then we are back to a point I made earlier that it really doesn't matter who is right between me and the quasi-realist, concerning the conception of absolute necessity at work in quasi-realism. Either I am right and the conception is maximal-absoluteness, or the quasi-realist is right and it is limit- or general counterfactual-absoluteness. In both cases, we've seen that squaring the circle doesn't come out logically possible. If so, then all three conceptions of absolute necessity can't help the quasi-realist cash out how *C* renders squaring the circle logically possible. However, if, as I said at the beginning of this section, logical possibility is the only sense of possibility available to the quasi-realist, then my argument stands: squaring the circle remains absolutely impossible even though we can now naturalistically explain why we regard it as absolutely impossible. More importantly, we continue to regard it as such despite the failure of *C*.

The problem, as it must have already become clear, is that quasi-realism can't escape being couched in modal terms, as evident in its reliance on 'absolute necessity', which Blackburn doesn't specify. But once specified, as I've done here, quasi-realism becomes problematic. Hale puts it better:

> [Quasi-realism] is, and probably has to be, specified in modal terms (can/can't imagine …). If the possibility referred to here is logical, the theory is circular. If not, of what sort is it? Declaring it to be psychological would avoid direct circularity but threatens to

[13] It isn't clear which preceded the other between logical and metaphysical necessities. Hale (2012) thinks logical necessities precede metaphysical ones. But see Shalkowski (2004) for why and how metaphysical necessities precede logical ones. What I say goes through whichever side of this debate is correct.

[14] Alethic modalities are the modalities of truth, and Blackburn is only concerned with alethic modalities.

> square ill with the hinted normativeness ("our best attitude of comprehension or imagination"). This difficulty is further aggravated if a [quasi-realist] account of non-logical modalities is to be given. Indeed, one wonders whether the [quasi-realist] will be able to preserve the distinctions many of us think we discern between logical and natural or physical necessity, etc (1986, 79; see also O'Connor 2008, 29).[15]

Thus, here is the bridge I have constructed to cover the gap Nichols leaves open. It isn't just that the we shouldn't worry about our inability to understand the other side of the boundary because we needn't be able to understand it as he says, but that we shouldn't worry about understanding it because were we to come to understand it, things won't change, given limit-, maximal-, and general counterfactual-absoluteness.[16]

If so, then the demand that we understand the other side of the boundary loses its dispositive force, since its whole point is to show that once we do, the involved modal commitments become logically possible, and so, no longer absolutely necessary/impossible. But if we can't make much sense of what the quasi-realist means by "the commitments becoming *logically possible*" and "our inability to make anything of absolute necessity is *absolutely necessary*", then why should we be held ransom by the demand. And even when we try to make some sense of what the statements might mean, as I've done here, we end up with a view that understanding the other side isn't as important as the quasi-realist makes it out to be.

7. Conclusion

To summarize: I agree with Blackburn that we can't make anything of absolutely necessary falsehoods in the sense that we can't understand the other side of the boundary. I also agree with Nichols that naturalized modal epistemology isn't equal to 'make something of' in the sense that we needn't be able to understand the other side of the boundary. But unlike Nichols, who left it open whether we lose our right to regard absolute necessities as absolutely necessary once we can naturalistically explain why they are absolutely necessary, a gap that makes his view to fall short of responding satisfactorily to Blackburn, I have shown that we don't lose any such right.

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[15] Plus, as Hale explains, quasi-realism seems unformalizable into purely logical vocabularies, and as we know, it is virtuous for a theory to be simple and elegant. According to him, if [□(2+2=4)] is the formalized version of the sentence, 'we can't imagine how things would be were 2+2≠4', then what should we say about □[□(2+2=4)]? What can't we imagine in this case? Is it that 'we can't imagine our inability to imagine how things would be were 2+2≠4'? If yes, and I see no other option, then there is a problem. I can, and in fact, I am imagining as I write, that I can't imagine how things would be were 2+2≠4. Thus, Hale concludes that the whole formalization "should be ill-formed" (1986, 78; see also Dicken 2010, 203–5). I agree. While this is separate from the content of quasi-realism vis-à-vis absolute necessity, it captures some of its lurking flaws.

[16] There is at least one conception of absolute necessity I haven't considered here—the Kantian-based existence conception, according to which "[a]n absolutely necessary proposition is a proposition whose negation cancels all possibility" (Stang 2016, 127). I set it aside because even the quasi-realist would agree that whether or not we can naturalistically explain why we absolutely modalize doesn't add or remove from the set of all possibility.

Despite having a naturalistic explanation for why we regard squaring the circle as absolutely impossible without having to 'make something of' a world where π is countable and rational, we do not lose our right to regard squaring the circle as absolutely impossible. Importantly, we now know why we don't lose this right: squaring the circle doesn't come out logically possible no matter how we characterize the absolute necessity of our inability to 'make something of' a world π where is countable and rational.

It might do better, however, to moderate the achievement of this paper, in that I have only offered one case where the psychological explanation we have for why we hold a certain judgment as absolutely necessary doesn't threaten the absoluteness of the involved necessity. Fair enough—we don't as yet have similar explanations for many other judgments we take to be absolutely necessary. But if what I have said here is any indication, then we can expect that if and when we do have psychological explanations for our judgments of those other absolute necessities, that the explanations won't come with a forfeiture of the absoluteness of the involved necessity. If so, then following Nichols, we can say convincingly that naturalizing modal epistemology won't leave out absolute necessity.[17]

[17] An earlier version of this paper was presented at an invited talk at Stellenbosch University. I thank the audience of that talk for their insightful questions. I especially thank Oritsegbubemi Oyowe, David Mertens, Monique Whitaker, David Spurrett, Peter Godfrey-Smith, Jessica Leech, Nick Zangwill, and Daniel Nolan for reading earlier versions of this paper and for their valuable comments, without which many aspects of the paper would have remained very obscure and impenetrable.

References

Baldwin, J. 1896. "A New Factor in Evolution." *The American Naturalist* 30: 441–51.

Björnsson, G. 2004. "A Naturalist's Approach to Modal Intuitions." In: *Modal Epistemology*, edited by E. Weber and T. De Mey, 1-24. Brussels: Royal Flemish Academy of Belgium.

Blackburn, S. 1984. *Spreading the Word*. Oxford: Oxford University Press.

_____. 1993. *Essays in Quasi-realism*. Oxford: Oxford University Press.

Clark, A. 2004. "Towards a Science of the Biotechnological Mind." In: *Cognition and Technology: Coexistence, Convergence, and Co-evolution*, edited by B. Gorayska and J. Mey, 25–36. John Benjamins Publishing Company.

_____. 2006. "Language, Embodiment, and the Cognitive Niche." *Trends in Cognitive Sciences* 10(8): 370–74.

Clarke-Doane, J. 2019. "Metaphysical and Absolute Possibility." *Synthese* https://doi.org/10.1007/s11229-019-02093-0

Fletcher, R. 2007. "Squaring the Circle: Marriage of Heaven and Earth." *Nexus Network Journal* 9(1): 119–43.

Dicken, P. 2010. *Constructive Empiricism*. London: Palgrave Macmillan.

Godfrey-Smith, P. 2003. "Between Baldwin Scepticism and Baldwin Boosterism." In: *Evolution and Learning: The Baldwin Effect Reconsidered*, edited by B. H. Weber and D. J. Depew, 53–67. Cambridge: MIT Press.

Gureckis, T., and R. Goldstone. 2006. "Thinking in Groups." *Pragmatics & Cognition* 14(2): 293–311.

Hale, B. 1986. "The Compleat Projectivist." *The Philosophical Quarterly* 36(142): 65–84.

_____. 1996. "Absolute Necessities." *Philosophical Perspectives* 10: 93–117.

_____. 2012. "What is Absolute Necessity?" *Philosophia Scientiæ* 16(2): 117–48.

Heersmink, R. 2013. "A Taxonomy of Cognitive Artifacts: Function, Information, and Categories." *Review of Philosophical Psychology* doi:10.1007/s13164-013-0148-1

Hobson, E. 1913. *"Squaring the Circle": A History of the Problem*. Cambridge: Cambridge University Press.

Hohol, M., and M. Miłkowski. 2019. "Cognitive Artifacts for Geometric Reasoning." *Foundation Science* doi.org/10.1007/s10699-019-09603-w

Hutchings, E. 1995. *Cognition in the Wild*. Cambridge: MIT Press.

Macarthur, D., and H. Price. 2007. "Pragmatism, Quasi-realism and the Global Challenge." In: *The New Pragmatists*, edited by C. Misak, 91–121. Oxford: Oxford University Press.

Netz, R. 2003. *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History*. Cambridge: Cambridge University Press.

Nichols, S. 2004. "Imagining and Believing: The Promise of a Single Code." *Journal of Aesthetics and Art Criticism* 62: 129–39.

_____. 2006. "Imaginative Blocks and Impossibility: An Essay in Modal Psychology." In: *The Architecture of the Imagination*, edited by S. Nichols, 237–56. Oxford: Oxford University Press.

Nichols, S., and S. Stich. 2003. *Mindreading*. Oxford: Oxford University Press.

Nolan, D. 2017a. "Naturalized Modal Epistemology." In: *Modal Epistemology after Rationalism*, edited by B. Fischer and F. Leon, 7–28. Springer.

_____. 2017b. "Methodological Naturalism in Metaethics." In: *The Routledge Handbook of Metaethics*, edited by T. McPherson and D. Plunkett, 659–73. Routledge.

Norman, D. 1991. "Cognitive Artifacts." In: *Designing Interaction: Psychology at the Human-computer Interface*, edited by J. Carroll, 17–38. Cambridge: Cambridge University Press.

Núñez, R., and G. Lakoff. 2005. "The Cognitive Foundations of Mathematics: The Role of Conceptual Metaphor." In: *The Handbook of Mathematical Cognition*, edited by J. I. D. Campbell, 109–24. New York: Psychology Press.

O'Connor, T. 2008. *Theism and Ultimate Explanation*. Blackwell Publishing Ltd.

Omoge, M. 2021. "Imagination, Metaphysical Modality, and Modal Psychology." In: *Epistemic Uses of Imagination*, edited by C. Badura and A. Kind. Routledge.

Papineau, D. 2009. "The Poverty of Analysis." *Proceedings of the Aristotelian Society, Supplementary Volumes* 83: 1–30.

Quine, W. 1960. *Words and Objects*. New York: John Wiley and Sons.

_____. 1969. "Epistemology Naturalized." In: *Ontological Relativity and Other Essays*. New York: Columbia University Press.

Shalkowski, S. 2004. "Logic and Absolute Necessity." *Journal of Philosophy* 101(2): 55–82.

_____. 2008. "Blackburn's Rejection of Modals." *Philosophia Scientiæ* 12(1): 93–106.

Spelke, E., S. Lee, and V. Izard. 2010. "Beyond Core Knowledge: Natural Geometry." *Cognitive Science* 34(5): 863–84.

Stang, N. 2016. *Kant's Modal Metaphysics*. Oxford: Oxford University Press.

Tylén, K., R. Fusaroli, J. Bjørndahl, J. Rączaszek-Leonardi, S. Østergaard, and F. Stjernfelt. 2014. "Diagrammatic Reasoning: Abstraction, Interaction, and Insight." *Pragmatics & Cognition* 22(2): 264–83.

Zangwill, N. 1988. "Fact, Science, and Morality: Essays on A. J. Ayer's Language, Truth, and Reality." *Analytic Philosophy* 29(3): 145–48.