A GUIDE FOR THE PERPLEXED

Russell

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continuum

PREFACE

This book is a guide to some of Bertrand Russell's more difficult philosophical works and ideas. Russell's most important and at the same time most difficult work is *Principia Mathematica*, the monumental three-volume opus cowritten with Alfred North Whitehead. These volumes present in elaborate detail his ground-breaking logical analysis of the foundations of mathematics. Written almost entirely in logical notation, it is difficult in the extreme to work through and understand.

Russell wrote an informal guide to *Principia Mathematica*—one without logical symbolism, and, he says, one "offering a minimum of difficulty to the reader." This is his *Introduction to Mathematical Philosophy*. Though concise and beautifully written, it is itself not always easy to understand. This guide's first aim is to help the reader master Russell's informal *Introduction*, then, having mastering that, to understand *Principia Mathematica*. This will enable the reader to also understand Russell's earlier masterpiece on the foundations of mathematics, *Principles of Mathematics*.

Russell also had a larger philosophy—one not just about logic and mathematics, but about the world more broadly, one that sought to understand the nature of the universe and the way that we know it. This philosophy especially includes Russell's metaphysics, his theory of knowledge, and his theory of language, which are the subjects of his following works: "Philosophy of Logical Atomism," *Analysis of Mind*, *Analysis of Matter*, *Inquiry into Meaning and Truth*, and *Human Knowledge*. Because his ideas on these subjects are spread out over many works and evolve over time, we take a different approach in covering them and present each subject as it occurs in Russell's early, middle, and late work. Here, we aim to give the reader a broad understanding of Russell's larger philosophy and to see the evolution of his thought as a whole.

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CHAPTER ONE

Introduction

Bertrand Russell (1872–1970) was arguably the greatest philosopher of the twentieth century and the greatest logician since Aristotle. He wrote original philosophy on dozens of subjects, but his most important work was in logic, mathematical philosophy, and analytic philosophy. Russell is responsible more than anyone else for the creation and development of the modern logic of relations—the single greatest advance in logic since Aristotle. He then used the new logic as the basis of his mathematical philosophy called *logicism*.

Logicism is the view that all mathematical concepts can be defined in terms of logical concepts and that all mathematical truths can be deduced from logical truths to show that mathematics is nothing but logic. In his work on logicism, Russell developed forms of analysis in order to analyze quantifiers in logic and numbers and classes in mathematics, but he was soon using them to analyze points in space, instants of time, matter, mind, morality, knowledge, and language itself in what was the beginning of analytic philosophy.

This first chapter introduces Russell's work in logic, logicism, and analysis, and then introduces his broader inquiries of analytic philosophy in metaphysics, knowledge, and meaning. Subsequent chapters treat each subject in detail. However, all of Russell's technical philosophy revolves around his logicism. Because Russell's mathematical philosophy is the key to the rest of his work, and because it is the most difficult part of it, we begin this chapter with a discussion of logicism, then keep circling back to it, relating it to the rest, until it seems to the reader that it is the easiest thing in the world to understand.

1 Logic and logicism: Basic concepts

Let's start with some basic logical concepts. A sentence is a group of words that express a meaning that is a complete thought. A declarative sentence expresses a meaning that is either true or false. A proposition is the meaning expressed by a declarative sentence such as the true proposition "The earth is round" or the false one "The earth is flat." So propositions are either true or false. The declarative sentences that express them are also said to be true or false.

Subjects and predicates follow. The subject of a proposition is who or what the proposition is about. "The earth is flat" is about the earth. So the earth is the subject of that proposition. The predicate is what is said about, or attributed to, the subject. Here, the proposition attributes flatness to the earth, so "____ is flat" is the predicate. Logicians write predicates using blank spaces, or more usually, variables like x, y, or z to indicate where the subject goes in relation to the predicate. Bertrand Russell called predicates *propositional function*. In this book, we use the terms interchangeably.

The predicate "x is flat" is a *one-place predicate*, because it only has one place where a subject can go—it attributes a property to one thing. Two-place predicates are *relations* like that in "Indiana is flatter than Ohio." Here, the subjects are "Indiana" and "Ohio" and the predicate is "x is flatter than y." (In grammar, the first is the subject and the second is the object; in logic, they are both subjects.) Common two-place relations in mathematics are $x = y$, $x > y$, and $x < y$. There are also three-place relations like that in "Ohio is between Indiana and Pennsylvania," where the predicate is "x is between y and z," which is often used in geometry. There are also four-place relations, and so on.

Before Russell's logic of relations, logic consisted principally of the Aristotelian logic of one-place predicates. This simple logic can analyze sentences that use one-place predicates to attribute properties to objects like "Tom is tall" or "The sky is blue." It can also analyze slightly more complex sentences like "All humans are animals" (if someone is human, that person is an animal) and "Some humans are thoughtful" (at least one person is both human and thoughtful) and from these two sentences infer that "Some animals are thoughtful." You can't get too far with such a simple logic and you certainly can't analyze many mathematical or scientific statements with it.

It was Russell's first great achievement to develop the more powerful logic of relations to describe concepts such as "x is taller than y" used in propositions like "Tom is taller than Bob," which you can't say with a one-place predicate like "x is tall." This allowed Russell to describe propositions containing two-place mathematical relations like $x = y$ or "$x > y$" (needed for arithmetic and algebra), three-place relations like "x is between a and b" (needed for geometry), and the like. With it, all of the concepts of pure mathematics can be expressed, which can't be done with the logic that came before it.

Russell's logic includes set theory. This is because his logic contains predicates and every predicate defines a set. For example, the predicate "x is human" defines the set of all things that can replace the x to make "x is human" a true proposition, namely, the class of humans. The *comprehension axiom* is the assumption that every predicate defines a class. It is an assumption of Russell's logic. Thus, Russell's logic contains sets and a theory of sets, as well as one-place predicates and two-place relations. Russell refers to sets as "classes" and set theory as "the theory of classes." We will use both ways of speaking indifferently and without distinction.

2 The emergence of logicism

After the logic of relations, Russell's greatest achievement is his theory of logicism—the view that mathematics is just logic, so that all mathematical concepts can be defined with logical concepts and all mathematical truths derived from logical truths. Russell's logic and his logicist philosophy were first fully described in his 1903 *Principles of Mathematics*. The actual derivation of mathematics from logic, to prove that all mathematics can be derived from logic, occurs in the three-volume 1910–13 *Principia Mathematica* that Russell wrote with Alfred North Whitehead. Russell also presents logicism simply and informally in the 1919 *Introduction to Mathematical Philosophy*.

Logicism comes down to is this: In the nineteenth century, mathematicians had shown that all of classical mathematics can be defined in terms of, and derived from, arithmetic. Most importantly, Richard Dedekind had shown in 1872 that the real numbers can be defined in terms of rational numbers. Then rational numbers were defined in terms of natural numbers, thus demonstrating that the real numbers can be derived from natural numbers. The next step was taken when Giuseppe Peano, based on work by Dedekind, showed in 1890 that arithmetic can be reduced to five axioms and three undefined terms.

To reduce mathematics to logic, one then simply has to define Peano's three concepts with logical concepts, thus expressing Peano's axioms logically, and then derive the axioms from logical truths, thus showing that Peano's axioms, and all the mathematics based on them, are logical truths. Russell starts by defining natural numbers logically as classes of classes. Specifically, a natural number is the class of all classes containing the same number of things, so that the number 1 is the class of all singletons (classes with one member), 2 is the class of all couples, and so on. With this definition, Russell then defines Peano's other two basic concepts logically and derives Peano's axioms from logic.

Put this way, demonstrating logicism is a seemingly simple task. But Russell and Whitehead soon ran into difficulties, namely, contradictions Russell found in the new logic and set theory. The most famous of these is called *Russell's paradox*. Some sets are members of themselves, others are not. The set of things that are not red is itself not red, so it is a member of itself, but the set of red things is not red, so it is not a member of itself. This allows us to construct the predicate "*x* is not a member of itself," which defines the set of all sets that are not members of themselves. But is the set itself a member of itself? If it is a member of itself, then it isn't. But if it isn't a member of itself, then it is. A contradiction ensues no matter how one answers.

To avoid this and similar paradoxes, Russell's logic, and the logicism based on it, became quite complex, and the ultimate success of this logicism is still a matter of debate. Many believe that it cannot be carried out completely. Others say the final verdict is not yet in. Still others say it can be done. In any case, it is significant and astonishing how much of mathematics Russell and Whitehead demonstrated *can* be reduced to logic. And if one is willing to tolerate a few pesky contradictions here and there, it absolutely can be done.

Russell's original form of logicism, in his 1903 *Principles of Mathematics*, did not attempt to avoid the paradoxes of the new logic, and so did not contain the complex mechanisms Russell later added to his logic to avoid them. It is a straightforward theory, containing all of logicism's basic elements. We present this basic logicism, which we call *naïve logicism*, in Chapter 2. The complex version meant to avoid paradoxes, which occurs in the 1910–13 *Principia Mathematica*, we call *restricted logicism*. We describe that in Chapter 3.

3 Logicism and analysis

As well as founding the logic of relations, developing the theory of logicism, and discovering fundamental contradictions in logic and set theory, Russell more than anyone else founded the twentieth- century movement of analytic philosophy that still dominates philosophy today. Analytic philosophy as practiced by Russell logically analyzes language to say what there is and how we know it. Analysis is a significant part of analytic philosophy and its role in the movement is largely due to Russell. His logical analysis of mathematics is the primary example of analysis.

Notions of analysis vary from one analytic philosopher to another and from one analysis to another by a single philosopher. This last case is true of Russell himself. Most generally, "analysis" for him means beginning with something that is common knowledge and seeking the fundamental concepts and principles it is based on. This is followed by a synthesis that begins with the basic concepts and principles discovered by analysis and uses them to derive the common knowledge with which one began the analysis.

In Russell's own words (*Introduction to Mathematical Philosophy*): "By analyzing we ask . . . what more general ideas and principles can be found, in terms of which what was our starting-point can be defined or deduced" (p. 1). Similarly, in *Principia Mathematica*, he says "There are two opposite tasks which have to be concurrently performed. On the one hand, we have to analyze existing mathematics, with a view to discovering what premises are employed . . . . On the other hand, when we have decided upon our premisses, we have to build up again [i.e., synthesize] as much as may seem necessary of the data previously analyzed" (vol. 1, p. v).

Immanuel Kant uses the same concepts of analysis and synthesis to describe his *Prolegomena to Any Future Metaphysics* and *Critique of Pure Reason*. "I offer here," he says in the *Prolegomena*, "a plan which is sketched out after an analytical method, while the *Critique* itself had to be executed in the synthetical style" (p. 8). In the *Prolegomena* we start with science (mathematics and physics) and by *analysis*, he says, "proceed to the ground of its possibility," that is, to its fundamental concepts, while in the *Critique*, "they [the sciences] must be derived . . . from [the fundamental] concepts" (p. 24).

Russell's *Introduction to Mathematical Philosophy*, an informal introduction to *Principia*'s logicism, is similarly analytic. About it, he says: "Starting from the natural numbers, we have first defined *cardinal number* and shown how to generalize the conception of number, and have then analyzed the conceptions involved in the definition, until we found ourselves dealing with the fundamentals of logic." About synthesis, he says "In a synthetic, deductive treatment these fundamentals [reached by analysis] come first, and the natural numbers [with which the analysis started] are reached only after a long journey" (p. 195).

And *Principia Mathematica* is a synthesis: it begins with the logical fundamentals found by analysis, and from them deductively builds up the mathematics the analysis started with. As Russell says in *Principia* itself, it is "a deductive system" in which "the preliminary labor of analysis does not appear." Instead, it "merely sets forth the outcome of the analysis . . . making deductions from our premisses . . . up to the point where we have proved as much as is true in whatever would ordinarily be taken for granted" (vol. 1, p. v).

Russell's *Introduction to Mathematical Philosophy* is thus to *Principia Mathematica* what Kant's *Prolegomena* is to the *Critique of Pure Reason*—an analysis that takes common knowledge and finds its basic principles, which synthesis then uses to demonstrate the knowledge analyzed. The *Introduction to Mathematical Philosophy* and *Prolegomena* also both informally introduce the subjects presented more rigorously in the synthetic works. But Kant seeks to justify knowledge with the principles uncovered by analysis. Russell does not. For him, the logical ideas analysis uncovers are less certain than the arithmetic it analyzes.

For Russell, what we analyze—arithmetic—is certain and *inductively* justifies the fundamental principles found by analysis when synthesis deduces arithmetic from them. (If synthesis shows that logic implies arithmetic, and arithmetic is true, then logic is *probably* true. The argument is inductive.) Russell does not think arithmetic is made certain by being derived from logic, but that logic is made more certain by arithmetic being derived from it.

As Russell says in *Principia*: "The chief reason in favor of any theory on the principles of mathematics [the justification of the premisses that imply mathematics] must always be inductive, i.e. it must lie in the fact that the theory in question enables us to deduce ordinary mathematics" (vol. 1, p. v). What is found by analysis is less certain than what is analyzed. Russell does not seek certainty from the analysis of mathematics, but an understanding of the reasons, however uncertain, for accepting what we normally take for granted.

"In mathematics," Russell further says, "the greatest degree of self-evidence is usually not to be found quite at the beginning, but at some later point . . . . hence, the early deductions [of *Principia*], until they reach this point, give reasons rather for believing the premisses because true consequences follow from them, than for believing the consequences because they follow from the premisses" (p. v–vi). *Principia* does indeed show that arithmetic follows from logic, which gives us some reason to accept those logical principles as an account of arithmetic's nature.

4 Logical analysis: The theory of descriptions

These concepts of analysis and synthesis may seem vague, but they will get you a long way in understanding Russell's *Introduction to Mathematical Philosophy* and *Principia Mathematica*. At some point, however, to understand Russell's work one must learn his more technical, logical kinds of analysis that are his theory of descriptions and incomplete symbols, his "no-class" theory of classes, his theory of logical types, and his logical constructions.

In the theory of descriptions, Russell analyzes descriptions of objects and classes by translating them into his new logic, where we can see that they do not always mean what they seem to mean in ordinary language. That is, Russell analyzes expressions of ordinary language into more careful logical expressions that are their true meaning. His *Introduction to Mathematical Philosophy* as a whole is the simpler sort of analysis, but within it are several more technical logical analyses using the theory of descriptions.

Russell first published the theory of descriptions in his 1905 article "On Denoting." The theory figures prominently in *Principia Mathematica*, where it is given a fairly clear presentation in the Introduction. Russell clearest exposition of it is in the 1918 "Philosophy of Logical Atomism," and another is in his 1919 *Introduction to Mathematical Philosophy* (Chapter 16), which is the version most people read in college.

For Russell, the theory of descriptions shows that the grammar of ordinary language is often misleading. Using it, sentences containing singular definite descriptions—descriptions of the form "the so-and-so" such as "the author of *Waverly*" in the sentence "Scott was the author of *Waverly*"—are analyzed so that the description does not occur in the logical analysis of the sentence, but is replaced by a predicate.

For example, "the author of *Waverly*" in "Scott was the author of *Waverly*" is replaced with the predicate "*x* wrote *Waverly*" and the sentence becomes "There is exactly one thing *x* such that *x* is Scott, and *x* wrote *Waverly*," or more briefly, "Scott wrote *Waverly*." The description "the author of *Waverly*" no longer occurs in the logical analysis of the sentence. In particular, the word "the" is gone. That is the whole function of the theory of descriptions.

Why analyze a sentence so that the definite description it contains, and especially the word "the," disappears? Notice that "the author of *Waverly*" seems to function like a name and to denote a particular object. However, the expression that replaces it, "*x* wrote *Waverly*," is a predicate, not a name, and by itself it does not denote any such object. Let us pause here to consider this idea that names denote, but predicates do not. It is an important idea to Russell.

The idea that names refer to, or denote, objects should not be controversial. "Napoleon" refers to the commanding French general at the battle of Waterloo, "Einstein" to the man who created the special and general theories of relativity, and so forth. And as Russell points out, names have these references independently of occurring in propositions. Finally, definite descriptions like "the author of *Waverly*" seem to function like names and refer to particular individuals too, just as "Sir Walter Scott" does.

Predicates, on the other hand, do not name, or refer to, objects. For example, the predicate "*x* is red" does not name or denote any particular individual by itself independently of occurring in a proposition. It does not specify which object or objects it might be used to apply to. So a predicate is definitely not a name. Because definite descriptions are not names but are predicates, Russell calls them *incomplete symbols*. They appear to name objects, but they really don't.

By showing that definite descriptions, which appear to be names of objects, really aren't, we can see how sentences containing descriptions can be meaningful without the sentence asserting the existence of what is described. For example, we can see how sentences like "The present king of France rolled the round square down the golden mountain" can be meaningful without asserting that any of these things exist.

This solves a general problem of logic for Russell—how to logically analyze sentences containing definite descriptions true of no objects. More significantly, Russell uses a variation of this theory, called his "no-class" theory of classes, to remove all references to classes in his logic by treating names of classes and descriptions of classes as predicates. Then, since logic, so interpreted, does not assume that sets exist, the Russell paradox of the set of all sets that are not members of themselves cannot occur—as we will see next.

5 Logical analysis: The "no-class" theory of classes

In addition to analyzing singular definite descriptions so that what appear to be names are seen to actually be predicates that do not name anything, Russell sometimes treats proper names the same way, for example, in *Principia Mathematica* (in *14.21). He argues there that words like "Homer" that appear to be proper names are actually concealed definite descriptions like "the author of the Homeric poems." They are then treated like definite descriptions and replaced with predicates. By 1918, in "The Philosophy of Logical Atomism," Russell is using this idea aggressively, insisting that *all* proper names like "Socrates" and "Napoleon" are disguised definite descriptions, but in *Principia*, he only suggests it once.

After singular definite descriptions come plural definite descriptions such as "the inhabitants of London." These too are analyzed so that they are replaced by predicates. "The inhabitants of London" in the sentence "The inhabitants of London are cosmopolitan," seems to name a class of objects, the inhabitants of London. But it is replaced by "x lives in London," which, being a predicate, names no object or objects. The sentence then reads "If anyone lives in London, that person is cosmopolitan."

In the slightly different sentence "The class of people who inhabit London is large," the subject is a description that appears to name a single object, the class of people living in London. Again, we replace the description with a predicate. Similarly, when a symbol stands for a set as its name, we treat it like a disguised definite description, just as "Socrates" is treated as the disguised description "the teacher of Plato." For example, when α = the class of even numbers, we translate "$6 \in \alpha$" ("the number 6 is a member of the class α of even numbers") by replacing α with the predicate "x is divisible by two" and get "6 is divisible by 2." We simply replace the class with the predicate that defines it.

Russell uses these techniques to define classes as predicates in *Principia Mathematica*. This replaces apparent references to classes with predicates that do not refer to classes. Thus, *Principia Mathematica* makes no reference to classes. There are then no classes in his logicist thesis, which ensures that paradoxes of set theory cannot arise in it. So Russell eliminates classes from his logic to prevent paradoxes from arising in it or in the logistic theory based on it. This is Russell's no-class theory of classes. (Though in truth it is a little more complex than this, as we will see in Chapter 3.)

Because Russell defines numbers in terms of classes, the elimination of classes from his logic effects his definition of number. In 1903, Russell defines natural numbers as classes of all classes with the same number of members. At that time, classes are objects for Russell. But when Russell replaces classes with predicates in 1910, he effectively replaces numbers with predicates too. They are then no longer classes and so no longer objects.

For Russell, an incomplete symbol is one that is not a name and does not refer, and definite descriptions are "incomplete symbols" because they are actually predicates and predicates are not names. In *Principia*, descriptions, classes, and numbers are all incomplete symbols. They are additionally, Russell says, all *linguistic* *conveniences* or *logical fictions*. Classes are thus merely symbolic conveniences and not real objects. All this to avoid the paradoxes of set theory which include Russell's own paradox of classes.

For example, in *Principia*, Russell says "The symbols for classes, like those for descriptions, are . . . incomplete symbols . . . merely symbolic or linguistic conveniences, not genuine objects" (p. 75). And in the 1937 Introduction to the second edition of the *Principles of Mathematics*, he says "seeing that cardinal numbers have been defined as classes of classes, they also became 'merely symbolic or linguistic conveniences'" (p. x).

Here is how replacing classes with predicates works for natural numbers. When two classes have the same number of members, the members of those sets correspond to one another in a 1-to-1 relation. Two sets whose members correspond 1-to-1 are called *similar*. A number is thus a class of all the classes that are similar to one another. For example, the number twelve is the class of all classes similar to the class of Apostles.

When classes are replaced with predicates, the class of similar classes is replaced with the relation of similarity itself. And each of those similar classes is replaced by a predicate. So, the relation of similarity is the relation of 1-to-1 correspondence between the objects these predicates apply to. Then, the class of Apostles becomes the predicate "*x* followed Jesus from the beginning" or some such thing. Other classes with 12 members are likewise replaced with predicates that define them. Finally, the fact that there are 12 Apostles is replaced by the fact that the things these predicates apply to can all be put in 1-to-1 correlation with one another. As you can see, the no-class theory of classes quickly becomes very complicated.

Russell himself does not say that names refer in isolation and predicates do not. Rather, he says that names have meaning in isolation and predicates do not and so are not names and do not refer to objects. But Russell's use of "meaning" here really only means that names refer and predicates do not; there is nothing more he is asserting when he uses "meaning" in this context. Elsewhere, it means other things for him. This special use of "meaning" by Russell is quite similar to the use of "bedeutung" (German for "meaning") by Gottlob Frege. Frege, an early logicist, used "bedeutung" in this same special way to mean "reference" while elsewhere using it to mean "meaning" in more standard ways.

6 Logical analysis: The theory of logical types

Though the no-class theory does avoid Russell's paradox of classes, there is a paradox similar to it for predicates that the no-class theory does not eliminate. This of course is because the no-class theory eliminates classes, not predicates. Here is the new paradox: Some predicates are true of themselves, for example, "*x* is a predicate" is itself a predicate. Others are not—for example, "*x* is red" is not red. From this, we can form the predicate "*x* is a predicate that is not true of itself." This predicate is true of some predicates and not of others. But is it true of itself or not? If it is, it isn't, and if it isn't, it is. We thus have a contradiction.

So simply eliminating classes from one's logic and logicism using the no-class theory does not eliminate all self-referential paradoxes from logicism, because similar paradoxes arise in it for predicates. We can try to use something analogous to the no-class theory to eliminate predicates. For example, we might replace predicates with propositions. Unfortunately there are also self-referential paradoxes for propositions. And so on.

Fortunately, Russell has another method for avoiding paradoxes called *the theory of logical types*. Notice that both versions of the Russell paradox result from allowing a set to be a member of itself or a predicate to apply to itself. The many other sorts of self-referential paradoxes similarly arise self-referentially, by allowing sets to be members of themselves, predicates to apply to themselves, propositions to be about themselves, and so forth. The theory of types prevents the paradoxes from arising by banning self-reference.

In the mature "restricted" logicism of *Principia*, then, as well as adopting the no-class theory of classes, Russell adopts the rule that a set cannot be a member of itself and a predicate cannot apply to itself, that is, it cannot take itself as an argument. This rule is the *theory of logical types*. And the theory of logical types is justified by the *vicious circle principle*, which says that any sentence formed by a set taking itself as a member or predicate taking itself as an argument is meaningless. By adopting the rule that is based on this principle, namely, the theory of types, the paradoxes for both sets and predicates do not arise.

The theory of types works like this: If sets cannot meaningfully be members of themselves and predicates cannot meaningfully refer to themselves, we end up with a hierarchy of different types, or levels, of sets or of predicates, their level depending on what types of things *they* can meaningfully take as members or arguments, and on what sets or predicates can meaningfully take *them* as members or arguments.

At the first level in the hierarchy are individuals. This is the zero-order. Then, there are predicates that apply to individuals. These are called *first-order* predicates. Anything we call an object is an individual—cars, people, molecules, mountains, what have you. A first-order predicate is something like "x is brave." It applies to individuals to form propositions like "Nelson Mandela is brave."

Since first-order predicates now cannot apply to themselves, predicates that apply to first-order predicates are called *second-order* predicates. If courage is a first-order property, we must use a second-order property, like "x is an important virtue" to say something about it such as "courage is an important virtue." First-order predicates also cannot take predicates of a higher-order than themselves as arguments. Then there are predicates that apply to second-order predicates—these are third-order predicates. And so on.

Sets are structured similarly with individuals again at the zero-order. Sets that take individuals as members are first-order sets, sets that take sets that take individuals as members are second-order sets, and so on. And propositions about objects are first-order propositions, those about first-order propositions are second-order propositions, and so on.

This is the basic idea. The actual theory of types is a few steps more complicated than this and will be explained in full in Chapter 3. But as you can see, stratifying sets and the things they can take as members, or predicates and the things they can apply to, prevents them from being self-referential, so the paradoxes of logic and set theory cannot arise.

Notice though that *both* the no-class theory of classes *and* the theory of logical types are used to avoid the paradoxes of class theory and logic. Why both methods? First, the no-class theory gets rid of sets by converting them to predicates. But since paradoxes also arise for predicates, the theory of types is needed to stratify predicates and prevent paradoxes for predicates from arising.

There are also predicates that apply to sets, but since the no-class theory transforms these sets into predicates, there is no need to create a separate hierarchy for them. This keeps the theory of types from getting any more complex than it already is. (There are also *philosophical* problems with stratifying predicates that apply to sets. By converting the sets to predicates, the philosophical problems are avoided. These problems will be described in Chapter 3).

Notice that there is still a hierarchy for sets in type theory. Why? Although it is understood that symbols for sets are "really" predicates in *Principia*, the mathematics in it is done using symbols for sets nevertheless. They still need stratifying in order to be used, even though we know they are really predicates. And because there are self-referential paradoxes that arise for propositions, the hierarchy of propositions is included in the theory of types as well.

One last point: notice that the paradoxes for set theory only arise from some sets. But the no-class theory eliminates all sets. This is clearly overkill. Why do it? Answer: As well as needing to avoid the set-theoretic paradoxes, Russell has separate philosophical reasons for wanting to eliminate classes from his logic altogether, for example, to avoid the ancient problem of the one and the many.

Sometimes symbols for sets are treated as representing many things (its members), other times they are treated as representing one thing (the set itself). But it cannot be both. Because of this and other such philosophical puzzles, as well as in order to simplify the theory of types, Russell eliminates *all* classes from his logic using the no-class theory and the idea of logical fictions to define them away.

These, then, are the broad outlines of Russell's mathematical philosophy called logicism. We have seen that Russell uses several different kinds of analysis in his mathematical philosophies. He also applies these methods outside of mathematics to answer philosophical questions about the world at large. We have already seen four varieties of analysis: the general kind that seeks the most basic concepts and principles, the theory of descriptions, the no-class theory of classes, and the theory of logical types. A fifth kind is Russell's analysis of entities with logical constructions, which he uses to analyze physical points, space and time, mental phenomena, matter, and even moral and political concepts. These topics will be introduced in the remainder of this chapter, and discussed at greater length in Chapters 4 through 6.

7 Analysis and metaphysics

Russell's ideas about the nature of reality are often responses to problems in logic, mathematics, and analysis. His views on reality in early work (1900–17) are expressed in *Principles of Mathematics* (1903), "On the Relations of Universals and Particulars" (1912), and "Analytic Realism" (1911). In them, a defense of analysis is part of his view of reality.

Philosophical monists, who were common in England in Russell's time, argue that analyzing the whole of reality into parts is impossible. They feel that the nature of objects is determined by the role they play in larger wholes, and that analyzing wholes into parts leaves out these larger connections. And if the nature of an object lies in the role it plays in a whole, and the nature of that whole lies in the role it plays in some larger whole, reality is ultimately one undivided whole—the plurality we experience is an illusion.

To defend analysis, Russell rejects the monists' arguments and concludes that reality is plural and "atomistic," that is, composed of parts that can be understood independently of their role in the whole. Details about reality in Russell's atomism are reached by analysis of logical principles—it is a *logical* atomism. He believes that logic and grammar reveal the nature of reality. This avoids beliefs about reality not warranted by logic. For example, if reality consists of things that can be analyzed into parts, the parts themselves are either complex and further analyzable or not complex and simple. If they are complex, they presuppose the existence of still simpler entities.

Russell's logical atomism is also based on understanding grammar. Monists assume that the logic of sentences always has a subject-predicate form, where a predicate applies a property to a subject—as in "Socrates is wise" where the predicate "x is wise" applies the property of wisdom to Socrates. If all sentences are really subject-predicate sentences, relations expressed by verbs in sentences like "Socrates is wiser than Plato" must also be properties.

Instead of understanding "Socrates is wiser than Plato" as expressing the relation "x is wiser than y" between Socrates and Plato, monists understand it as saying that "x is wiser than Plato" is a property of Socrates. Treating relations this way makes being wiser than Plato seem like an essential property of Socrates. Treating all relations this way—as essential properties of objects—- makes everything seem interrelated to every other thing as a part of its essential nature. Thus, they can only be understood as parts of wholes. This view takes relations as "internal" (i.e., essential) properties of objects.

With his logic of relations, Russell sees that verbs are not predicates and relations are not properties of things. Rather, relations are entities in their own right, not part of the things related. Relations between things are "external" to the nature of things. They are not facts about the essential nature of the things related. This is the view of "external relations." Complexes of things are thus external relations among things.

Using grammar as a guide, Russell also assumes that entities occur in specific ways in propositions. Some occur only as subjects of propositions. Others occur as relations or properties of propositions but can also occur in other propositions as subjects. Those that can only be subjects he calls "things" or "particulars." Those that can be both subjects and predicates or relations he calls "concepts" or "universals."

Russell also examines logic and grammar to find the basic elements of nature. These include numbers, classes, concepts, properties, propositions, universals, particulars, particles, points, and instants. And as analysis develops, the list of elements changes. The theory of descriptions says descriptions are not names, and soon that "Socrates" is a disguised description and not really a name either. Instead, they are properties or relations. Similarly, the no-classes theory replaces classes with properties, so classes need not be assumed to exist. Both theories are metaphysical: they eliminate the need to assume certain entities, assuming others instead. The theory of types is also metaphysical: it distinguishes these elements into different types of things.

In Russell's middle period (1918–34) logic and metaphysics continue to be linked in works such as "Philosophy of Logical Atomism" (1918), his introduction to Ludwig Wittgenstein's *Tractatus* (1921), *Analysis of Mind* (1921), and *Analysis of Matter* (1927). He now thinks his earlier ideas are mistaken. In 1911, properties and relations are abstract entities, *universals* that can occur in propositions as predicates or as things and subjects, for example, as "Robert is a man" and "*Man* is a concept." He now thinks relations and properties cannot be subjects and that universals are not among the data of experience. We only experience particulars.

Russell uses logical constructions now to show that "mind" and "consciousness" can be defined in other terms and eliminated from psychology's basic vocabulary. He earlier thought that consciousness was something distinct from the abstract and concrete things to which it is related. Now he defines cognitive acts and entities in terms of constituents that are neither mental nor physical, but something "neutral." This is the view of *neutral monism*. *Monism* has different meanings. The neutral monism Russell adopts now says there is one *kind* of thing, not just one thing. Thus, he remains a pluralist and atomist.

His neutral monism asserts that the ultimate constituents, which are all particulars, are of the same "neutral" substance, whether they form objects outside the mind or the mind itself. The neutral stuff includes sensations and images, which are the same, but occur in different contexts: images obey psychological laws of association and cannot have effects for anyone else but the one person. Sensations obey both physical and psychological laws and can have effects on more than one person. The difference between the mental and the physical is thus only a matter of the arrangement of elemental neutral stuff.

By 1920, Russell also has a different view of the nature of logic and mathematics. Rather than viewing them as about the most general features of the world, as he had earlier, he now regards them as merely assertions about symbols. He also begins to respond to advances in physics, specifically, to the theory of relativity and the quantum theory of the atom. He remains committed to particulars as the ultimate neutral stuff but begins to speak of them as "events." Logical techniques are then used to define points of space, instants of time, and matter in terms of neutral events.

Russell does not abandon neutral monism in his late period, from 1935 to 1950, but he focuses on other issues. The main works here are *Inquiry into Meaning and Truth* (1940) and *Human Knowledge* (1948). He is an antiempiricist both in his early and late period. In his late period, he analyzes language to show that though we can explain most general words without assuming universals, we cannot eliminate all universals. For example, we can define "red" in "this is red" without assuming the universal *redness* by replacing "red" with "similar to this." Yet we need at least one universal to define "similar." Thus, particular experienced events alone are not enough to account for the meaning of sentences. Universals are not experienced, but to explain meaning we must assume the existence of at least one.

8 Analysis and the theory of knowledge

Russell's theory of knowledge concerns both empirical and *a priori* knowledge. His early views here occur in "The Philosophical Importance of Mathematical Logic" (1911), "Knowledge by Acquaintance and Knowledge by Description" (1911), *Problems of Philosophy* (1912), and *Our Knowledge of the External World* (1914). Logical and mathematical propositions are thought to be general truths that relate universals existing apart from space and time. These propositions are *a priori*—known independently of experience.

Knowledge in general is consciousness of particular or universal entities known by awareness (direct acquaintance). These are not physical objects, which Russell says we construct, but data of sense, memory, introspection, or logical intuition—patches of color, sounds, feelings, or mind-independent universals like *similar*. We also know about things by description, but then our grasp on them comes from our grasp on names of the things of which we are directly aware.

Throughout his career, Russell's epistemology focuses on verifying the propositions of physics to show how physics as a branch of pure mathematics applies to the world. His view is that physical propositions are not completely verified until terms like "matter" and "instant" are defined by sentences about sense data. The definitions are produced in accord with the theory of descriptions, where phrases apparently naming entities are defined with names for sense data.

By defining physical concepts in terms of sense data, Russell seeks to avoid assuming any more than is necessary about the physical world. That is, he seeks to justify the laws of physics by sense data alone, without having to also assume physical objects that cause our experiences but are not directly experienced and so themselves transcend experience.

In his middle period theory of knowledge of *Analysis of Mind* and *Analysis of Matter*, Russell no longer believes logic and mathematics consist of general truths about the world, though he still thinks knowledge of them is *a priori*. But this is because they are now viewed as definitions, which are uninformative. With empirical knowledge, he no longer thinks we are conscious of particulars and universals or know them by acquaintance. The proper method of philosophy is still to make as few metaphysical assumptions as possible, and neutral monism lets him avoid assuming a nonphysical relation called "awareness." He now defines mental occurrences using logical words, assuming only the particulars of neutral monism.

The construction of minds and objects occurs by gathering particulars together in different ways. At any moment, for example, a star is a class consisting of various sensation-particulars. Your momentary experience of the star, that is, what occurs in you, is a different class of the same particulars. The whole collection of classes over time defines the star, and the whole collection of your experiences of stars and other things defines you.

After constructing mental phenomena in *Analysis of Mind*, Russell returns to the study of matter. This is due to changes he thinks general relativity and quantum theory require. In *Analysis of Matter* (1927), he argues that all experiences—all data—are subjective and determined by a person's standpoint. He now accepts inductive inferences from our experiences to events in the physical world that cause them. He thus gives an account of induction and of scientific reasoning which assumes events continuous with those we perceive and extrapolates from perceived relations to relations among events in physical space-time.

In the 1930s and 1940s, Russell's late period, these themes dominate his discussion of knowledge, especially that of the *a priori* principles that guide scientific reasoning. The principal texts are *Inquiry into Meaning and Truth* (1940) and *Human Knowledge* (1948). The paper "On Verification" (1938) is also important. The postulates are those actually involved when scientists or ordinary people pursue a line of reasoning. Of all possible inferences that might be drawn from the data, what governs the decision to follow one and ignore the others? On his view, it is the presence of *a priori* expectations about the world.

These have a psychological origin. They are caused by experience but not inferred from it and exist as primitive beliefs or habits. For example, if idly watching the path of a cat crossing an empty room, you would be astonished if it winked in and out of sight, or if it should be here and then suddenly somewhere quite different. This is because we bring expectations about continuity and permanence to experience, created by experiencing certain qualities and general patterns in the world, not just our psychology. Our expectations, which, made explicit, are postulates of science, are therefore about the world but known *a priori*, since we bring them to experience. His late period also focuses on "linguistic epistemology," that is, with constructing languages to aid us in discovering what the data are and what we must infer.

9 Analysis and the theory of meaning

In his early period, Russell's theories of meaning are confined to what words and sentences denote. These occur in his early metaphysical works such as the *Principles* (1903). Russell thinks the meaning of a name, verb, or predicate, is the entity it denotes, which may be concrete or abstract, in time and space or outside them. Words that occur as subjects of sentences denote either particulars or universals (things or concepts), while predicates and verbs denote only universals.

Though the things corresponding to words and phrases are their *meanings*, this is not to say that we are aware of them as meanings. Russell explains this with his doctrine of acquaintance with universals. We can be acquainted with a patch of color and not know that it is an instance of the word "yellow." For this, the particular patch is not enough: we need to grasp the universal *yellow*. The understanding of meaning is by way of universals.

The above remarks concern words. Until 1910, the meaning of a sentence is also viewed as a single complex entity—the proposition *aRb* of two objects *a* and *b* with relation *R* to one another. On this view, a sentence has a meaning (the complex entity) even if it is not believed or judged. Eventually, Russell finds this doctrine unacceptable and replaces it with the theory that a sentence has no complete meaning until it is judged or supposed or denied by someone. On this view, *judging* is not a relation between a person and a single entity *aRb*, but a relation between a person and *a*, *R*, and *b*. The proposition is broken into parts and enters into a person's belief, which arranges them in a meaningful way.

There is now no single entity *aRb* that is the meaning of a sentence. There are only sentences, which are incomplete symbols, and the context of belief that gives the sentence a complete meaning. This is another analysis using the theory of descriptions: a sentence "*aRb*" is an incomplete symbol that acquires meaning when judged or believed but is otherwise meaningless. That a person has a belief is a fact, and the entities that constitute the meaning of the sentence are gathered together with the believer in that fact. Just as the theory of descriptions replaces descriptions with predicates, so here it replaces propositions with facts of belief.

This theory requires that a person is acquainted with the things that enter into the belief, for example, with *a*, *R*, and *b*. But acquaintance with this data is not enough to make a judgment. To believe or judge, a person must also be acquainted with the *form* in which things are put together. In this case, he or she must grasp what it means to assert a relation.

In his middle period, Russell's analysis of language and meaning develops well beyond his early views, which hardly constitute a theory of meaning at all. Some texts are "Philosophy of Logical Atomism" (1918), "On Propositions" (1919), *Analysis of Mind* (1921), "Vagueness" (1923), "Logical Atomism" (1924), and "The Meaning of Meaning" (1926). The novelty is the attempt to explain language and meaning in terms of causal relations to the world.

For words, Russell adopts a partly behaviorist account where words are classes of sensations (mouth movements, sounds, etc.) and acquire meaning by association with other sensations of the thing meant. For example, a child experiences certain sensations that are collectively a toy and learns to make certain sounds that are collectively the word "toy." Departing from behaviorism, Russell says the sensations of the toy give rise to images associated both with the toy and the word "toy." The meaning of "toy" and the images are products of cause and effect where the word or image can come to have the effects the original sensations had.

Russell had, in his early period, resisted reliance on images in his theories of meaning, but in his middle period he embraces them. Belief is no longer a relation among things (*a*, *R*, *b*, and a person). Instead, the content of belief consists of images and feelings (acceptance, doubt, etc.). And verbs occur in sentences under new constraints. They now do not name anything (denote no universal) but merely create a structure of words that is the sentence. Just as an egg carton is not a kind of egg but a means of holding eggs in a pattern, verbs are now merely means of creating a spatial (if written) or temporal (if spoken) relation among words in sentence.

Russell's late period work on language occurs in *Inquiry into Meaning and Truth* (1940). There he tries to solve philosophical problems by constructing proto-languages and artificial languages. As before, we have feelings toward images or words. He now builds on this by developing a psychological or causal theory of a hierarchy of languages having logical constraints. In the logically fundamental language, we use single-word sentences for immediate experiences. But our utterances also convey feelings like doubt or certainty toward beliefs, as when we wonder "Is it true that this is sugar?" With this idea, Russell explains the psychological meaning of logical words like "true."

We also find a new analysis of indexical words like "I," "this," and "here." At the same time, he tries to identify a minimum vocabulary for sciences like physics and to identify the kinds of sentences that can serve as premises. Since he is interested in physics and psychology, he asks whether the words and sentences that report the observations of a physicist will also serve in the same way for psychology.

Philosophers besides Russell have pursued their own conceptions of analysis. Russell's friend G. E. Moore, who influenced Russell as well as later philosophers, is an important example. But there is no doubt that Russell is most responsible for founding the movement of analytic philosophy. In the following pages, Russell's contribution to that philosophy is described in greater detail. The next chapter describes Russell's logicism, and the chapter following describes the elaborations he added to it to avoid paradoxes it faced. Following that, in Chapters 4 through 6, we return to the broader doctrines about things, knowledge, and language sketched above.