Countable Fusion Not Yet Proven Guilty: It May Be The Whiteheadian Account of Space Whatdunnit

Graham Oppy

Peter Forrest — in "How Innocent Is Mereology?" Analysis **56.3**, July 1996, pp.127-131 — disputes David Lewis' claim that mereology is ontologically innocent, on the grounds that mereology entails something (Countable Fusion) which is inconsistent with something else (a Whiteheadian account of space) which we do not know *a priori* to be false. However, it seems to me that the inconsistency which Forrest extracts from the conjunction of Countable Fusion with the Whiteheadian account of arises only because the Whiteheadian account of space is formulated in terms which poach on the preserves of mereology. There are formulations of theories which share almost all of the interesting features of the Whiteheadian account of space, but which do not lead to contradiction when wedded to Countable Fusion. Moreover, the other kinds of allegedly objectionable consequences which Forrest extracts from the conjunction of Countable Fusion with the Whiteheadian account of space can be extracted from his Whiteheadian account of space alone. In these circumstances, the oddities should not be taken to redound to the discredit of Countable Fusion.

According to Forrest, the Whiteheadian account of space is based on the following claims:

1. Regions are the fundamental spatial entities.
2. Regions have no parts other than regions, and are parts of nothing other than regions.
3. All regions have the same dimension.
4. Regions may be represented by sets of points in such a way that each representing set contains a sphere (i.e. all the points less than some distance z from some point Z).

5. There are spherical — or at least approximately spherical — regions of arbitrarily small diameter (i.e. for any point X and any positive real number y, there is a region represented by a set of points including all those distance less than y from X and none distance greater than 2y from X).
6. The representation of regions as sets of points preserves volumes (i.e. the volume of a region equals the Lebesgue measure of the corresponding set of points).

I claim that the Whiteheadian has no business writing in 2, given the constraints which are imposed upon regions by 1, 3, 4, 5, and 6. Call things which satisfy 1, 3, 4, 5, and 6 *regions$_w$*. Given that there are regions$_w$, it is (at least partly) up to mereology to tell us what parts and wholes of regions$_w$ there are. The Whiteheadian is free to insist — what is partly distinctive of her position — that points, lines, and surfaces are not parts of regions$_w$. However, what the Whiteheadian cannot do is stipulate that regions$_w$ have no parts other than regions$_w$ if — for example — it turns out that the mereological difference between a region$_w$ and a countable fusion of regions$_w$ need not be a region$_w$. And indeed, if mereology is ontologically innocent, then what Forrest's argument to the alleged inconsistency of Whiteheadian space and Countable Fusion shows is merely that the mereological difference between a region$_w$ and a countable fusion of regions$_w$ need not be a region$_w$ — i.e. regions$_w$ have parts other than regions$_w$, but which are not points, lines, surfaces, etc. It seems to me that this is an easily tolerated — indeed, as we shall later see, desirable! — result, given the other features of Whiteheadian space. (*Given* the rest of the Whiteheadian account, the difference between a region$_w$ and a countable fusion of subregions$_w$ may be something which contains no sphere (ball) of suitable dimension, and yet which counts as being of the same dimension as a region$_w$ simply because of the way it is constructed. Thus, the prime Whiteheadian intuition — that all the spatial things which there are have the same dimension — can be preserved.

We can say, if we like, that there are nothing but regions — but we need to bear in mind that some regions are not $regions_w$: some regions do not contain spheres (balls) of suitable dimension.)

Moreover, if we maintain the insistence that $regions_w$ have no parts other than $regions_w$, and are parts of nothing other than $regions_w$, then we can derive oddities — of a kind which Forrest claims he is not prepared to tolerate — from the Whiteheadian account of space alone. To keep things simple, let us pretend that space is one-dimensional — i.e. we shall consider a Whiteheadian account of the number-line, under the assumption that there are line segments but no points — and let us also suppose that no parts of space are fuzzy. Nothing hangs on this pretence; but it easier to see what is going on in the non-fuzzy one-dimensional case. Consider the line segment (0, 10) — i.e. a line segment of length 10 units. Consider, further, the line segment (0, 1) — a line segment of unit length. We can use our line segment (0, 1) to construct a 'cover' of the line segment (0, 10), following the recipe which Forrest provides for deducing a contradiction from the conjunction of Countable Fusion and the Whiteheadian account of space, as follows: The rational points in the line segment (0, 10) are countable. Let them be the points A_i, I = 0, 1, 2, 3, ... There are line segments R_i such that R_i includes all and only points distance less than $(1/2)^{i+1}$ from A_i. Use our line segment (0, 1) to construct a 'cover' for the R_i. (This gives the sequence of intervals (0, 1/2), (1/2, 1/4), (1/4, 1/8), and so on.) Then note that the R_i provide a 'cover' for the interval (0, 10) in the following sense: there is no <u>interval</u> in — hence, according to the Whiteheadian, no part of — the interval (0, 10) which is not overlapped by at least one of the R_i.

To make this vivid: Suppose you have a stick of length 10 metres, and a stick of length 1 metre. Suppose that you can break the stick of length 1 metre in half infinitely often, and that you can arrange the resulting parts on top of the stick of length 10 metres with perfect accuracy. Then, on the Whiteheadian account of space, you will be able to so arrange the pieces of the stick of length 1 metre that NO part of the stick of length 10 metres is fully uncovered, i.e. fully visible, even to infinitely discriminating vision. Remember: the Whiteheadian account of space says that the only parts of a line segment are line segments. There are no points, nor anything corresponding to kinds of fusions of points other than line segments. So the fact that, in non-Whiteheadian space, there would be many points — and, indeed, entities which are neither points nor line segments — not covered is irrelevant: no part of the 10 metre stick which the Whiteheadian says exists can be seen in its entirety.

This seems to me to be a bad result, at least by Forrest's lights. We can illustrate how bad by copying the example which Forrest uses to 'further illustrate' his case against the conjunction of Countable Fusion and the Whiteheadian account of space. Suppose that line segments can be assigned a colour. Suppose that all line segments are either black or white. Suppose that the line segment (0, 10) is initially all white — i.e. it and all of its sub-segments are entirely white. Suppose that region R_i — and the corresponding part of the interval (0, 10) — is blackened at time $t_i = 1 - (1/2)^i$, so that the t_i converge to time 1. At time 1, there is no region (however small) of the 10 metre stick which is totally white — i.e. white throughout — even though we only blackened regions with a total length of 1 metre. 'From whence did all this blackness come?' we might ask, half expecting to be told that explanations come to an end at some point. But in fact we should be told something quite different, namely that this sudden coming into existence of blackness is indeed explained, since it follows from the Whiteheadian account of space. By parity of reasoning, Forrest should find the explanation so counter-intuitive that he should reject the Whiteheadian account!

(If you don't think that the explanation is quite so counter-intuitive, then try the following thought experiment, beginning where the previous one left off. Over a period of time, remove regions which are entirely black. Continue to do this until there are no regions left. (For definiteness, we may suppose that larger regions are removed earlier, with the further provision that, where there are regions of the same size, they are all removed together. Let regions of measure α — where (by construction) $0<\alpha<10$ — be removed at time $10-\alpha$. Then, at time 10, the process of removal will be complete. This requires removing infinitely many regions — but no matter, Forrest supposes that there can be an infinite number of region fillings; what could be so different about region removals?) We shall end up with: (i) a pile of black regions; and (ii) nothing else. What has happened to all the white? Also, what will now happen if we put the black regions back together again? Will the white magically reappear? Will some other colour magically appear? Or what? In Whiteheadian space, a thing can be almost entirely white even though: (i) it has no parts which are entirely white; and (ii) it has countably many parts which are entirely black. The consequences of this seem to me to be no less disturbing than the results which Forrest finds 'counter-intuitive', and which he charges to the addition of the assumption of countable fusion.)

So: For all that Forrest has shown, mereology may be totally innocent. If it is, and if it turns out *a posteriori* that space is Whiteheadian, then it will turn out that space has some parts — some differences between regions_w and countable fusions of regions_w — which are not regions_w.This result may not be particularly attractive — but if it is not accepted, then space will have to be taken to have other features which are even more bizarre, arguably to the extent of being *a priori* rejectable. In particular, the (at least sometimes) plausible principle that something which is partly F has some part which is entirely F will have to be given up in all cases — including those cases where 'F' is restricted to spatially distributed properties, such as volumes of colours. In such circumstances, it seems to me that it is not absurd to think that there is *a priori* reason to hang on to countable fusion.

One last point. Forrest provides an argument in support of the claim that it follows *a priori* from the innocence of mereology that: if region S is the fusion of countably many regions R_i, then there is no number v such that the volume of every finite fusion of the R_i is less than v but the volume of S is greater than v. "In support of this I note that intuitively we say that the whole is no greater than the sum of the parts. We explicate this as saying that the whole cannot have a volume in excess of the sum of the volume of the parts. If further challenged we would say that this is precisely because there is nothing more to the fusion than the regions fused. That is, we appeal to the innocence of mereology." (130) This argument cannot be any good. For if it were, it would show that the innocence of mereology conflicts with the idea that space can be taken to be the fusion of its points. The volume of a point is zero, as is the volume of a finite fusion of points. Space is a fusion of points. The volume of space can be non-zero. Wherefore, if a region S is the fusion of uncountably many points R_i, there is a number — 0 — such that the volume of every finite fusion of the R_i is no greater than that number but the volume of S is greater than that number. So, if the little argument which Forrest gives were any good, then the detour through Whiteheadian space would be unnecessary — the innocence of mereology would be refuted by considerations concerning standard space alone. (As Peter Forrest pointed out to me, we can use transfinite induction to justify the claim that the sum of uncountably many zeroes is zero. So we can also directly deny the claim that a whole cannot have a volume in excess of the sum of the volume of its parts.)[1]

[1] Thanks to John Bigelow, Peter Forrest, and Daniel Nolan for helpful comments, advice, etc.