Validity and Soundness in the *First Way*

Abstract: We discuss the validity and soundness of arguments that instantiate the formal structure of the first argument in ST 1, Q2, A3:

1. Some things are moving/changing.
2. Nothing moves/changes itself.
3. Whatever is moving/changing is moved/changed.

[4. If one thing moves/changes a second, and the second moves/changes a third, then the first moves/changes the third.]

5. This cannot go on to infinity.
6. (So) There is something that is not moving/changing that moves/changes other things.

We shall begin by attempting to prove that there is a disambiguation of 6 on which it is a logical consequence of 1-5. The proof is in three parts. First, we show, in (classical) first-order predicate calculus, that the selected disambiguation of 6 is a logical consequence of 1-4 if we add the assumption that there are no more than two distinct things. Second, we show, using mathematical induction over proofs in first-order predicate calculus, that, for any $n \geq 2$, the selected disambiguation of 6 is a logical consequence of 1-5 if we add to them the assumption that there are no more than n distinct things. Third, we interpret 5 to mean that, for some n, there are no more than n distinct things.

Since this interpretation of 5 is sub-optimal, we then attempt to prove that a much stronger claim—namely, the same disambiguation applied to:

7. There is something that is not moving/changing that (non-trivially) moves/changes everything else.

--is a logical consequence of 1-4 if we (a) add the further assumption that there is a single *chain* of movement/change or a single *tree* of movement/change to which everything belongs and (b) interpret 5 to mean that this single chain or single tree is, in a suitable sense, finite.

Finally, we note (a) that these conclusions—the selected disambiguations of 6 and 7—fall far short of what Aquinas aims to establish; and (b) that stronger disambiguations of 6 and 7 that he might have hoped to be establishing are not logical consequences of 1-5.

So much for validity. When we turn to soundness, we argue first that it is outright mistaken to think that there is just one movement/change chain and highly implausible—in the light of physics—to suppose that there is just one movement/change tree. Then we consider the premises 1-4, along with the following three claims:

8. Some things are moving/changing some things.
9. If there is something moving/changing everything else, then it is not moving/changing.
10. Whatever is moved/changed is moving/changing.

After noting that we could replace 1 and 3 with 8, 9, and 10, without changing any of our conclusions about validity, we argue (a) 4 and 10 are very plausible, if not analytically true; (b) 1 and 8 are very

plausible; (c) the combination of 2 and 3 is highly implausible; and (d) it is very hard to motivate 9 if you do not already accept one of the stronger claims that Aquinas had been hoping to establish.

1. **Validity**

Among the implicit and explicit assumptions of the *First Way*, there are sentences with the following forms:

1. Something is T-ing (simpliciter): $\exists xTx$
2. If something is T-ing (simpliciter), then there is something that is T-ing it: $\forall x(Tx \rightarrow \exists yTyx)$
3. Nothing is T-ing itself: $\forall x{\sim}Txx$
4. If one thing is T-ing a second, and a second thing is T-ing a third, then the first thing is T-ing the third: $\forall x\forall y\forall z((Txy\&Tyz)\rightarrow Txz)$
5. Whatever has something T-ing it is T-ing (simpliciter): $\forall x\forall y(Txy\rightarrow Ty)$

There are many ways in which T might be interpreted. For example, it might be interpreted in terms of *motion*—something is moving; if something is moving then there is something moving it, etc--or *change*—something is changing; if something is changing then there is something changing it—or *change with respect to R*—something is changing with respect to R; if something is changing with respect to R then something is changing it with respect to R—or *movement from potentiality to actuality*—something is moving from potentiality to actuality; if something is moving from potentiality to actuality, then something is moving it from potentiality to actuality. All that matters for the discussion of the *validity* of the argument is the argumentative form.

In the first part of this paper, I shall take the domain of quantification to be a bunch of things all of which are T-ing (simpliciter) and/or T-ing something and/or being T'd. In brief: I shall take the domain of quantification to be a *T-domain*.

I begin with a proof about one element T-domains. I then go on to prove some things about two-element T-domains. After we have these proofs in hand, we shall regroup.

Proof that, given the *First Way* T-assumptions 1-3, there cannot be a *one-element* T-domain.

1. $\forall x\forall y(x=y)$ (Assumption for reductio: there is no more than one element in the T-domain)
2. $\exists xTx$ (Premise: first T-assumption)
3. $\forall x(Tx\rightarrow\exists yTyx)$ (Premise: second T-assumption)
4. $\forall x{\sim}Txx$ (Premise: third T-assumption)
5. Ta (from 2, existential instantiation)
6. $Ta\rightarrow\exists yTya$ (from 3, universal instantiation)
7. $\exists yTya$ (from 5 and 6, arrow elimination)
8. Tba (from 7, existential instantiation)
9. ${\sim}Taa$ (from 4, universal instantiation)
10. $a\neq b$ (from 8, 9, identity)
11. $a=b$ (from 1, universal instantiation)
12. contradiction (from 10, 11)

Given the *First Way* assumptions 1-3, there cannot be a one-element T-domain.

Clearly, we could simplify this proof by dropping 4 and replacing 3 with:

3*. ∀x(Tx→∃y(x≠y&Tyx)

But when we consider larger domains, we shall need 3 rather than 3*.

I now turn to three proofs concerning two-element T-domains. I shall use part of each proof to simplify the next proof.

(A) Proof that, given the *First Way* T-assumptions 1-4, in a *two-element* T-domain, there is an element that is not T-ing.

1. ∀x∀y∀z((x≠y)→((z=x)v(z=y))) (Premise: there are no more than two elements in the T-domain)
2. ∃xTx (Premise: first T-assumption)
3. ∀x(Tx→∃yTyx) (Premise: T-assumption)
4. ∀x~Txx (Premise: third T-assumption)
5. ∀x∀y∀z((Txy&Tyz)→Txz) (Premise: fourth T-assumption)
6. ∀xTx (Assumption for reductio: everything is T-ing (simpliciter))
7. Ta (from 2, existential instantiation)
8. Ta→∃yTya (from 3, universal instantiation)
9. ∃yTya (from 7 and 8, arrow elimination)
10. Tba (from 9, existential instantiation)
11. Tb (from 6, universal instantiation)
12. Tb→∃yTyb (from 3, by universal instantiation)
13. ∃yTyb (from 11 and 12, arrow elimination)
14. Tcb (from 13, existential instantiation)
15. ~Taa (from 4, universal instantiation)
16. b≠a (from 10 and 15, identity)
17. (Tcb&Tba)→Tca (from 5, universal instantiation)
18. Tcb&Tba (from 10 and 14, conjunction introduction)
19. Tca (from 17 and 18, arrow elimination)
20. ~Tbb (from 4, universal instantiation)
21. b≠c (from 14 and 20, identity)
22. ~Tcc (from 4, universal instantiation)
23. c≠a (from 19 and 22, identity)
24. (b≠a)→((c=a)v(c=b)) (from 1, universal introduction)
25. (c=a)v(c=b))from 16 and 24, arrow elimination)
26. (c≠a)&(c≠b) (from 21 and 23, conjunction introduction)
27. Contradiction (from 25 and 26)

Given the *First Way* T-assumptions 1-4, in any two-element T-domain, there is an element that is not T-ing (simpliciter).

(B) Proof that, given the First Way T-assumptions 1-4, in any two-element T-domain, there is an element that is T-ing something but that is not T-ing (simpliciter).

This time, our set-up replaces 6 with 6*:

6*. ∀x∀y(~TxyvTx)

1. ∀x∀y∀z((x≠y)→((z=x)v(z=y))) (Premise: there are no more than two elements in the T-domain)

2. ∃xTx (Premise: first T-assumption)
3. ∀x(Tx→∃yTyx) (Premise: T-assumption)
4. ∀x~Txx (Premise: third T-assumption)
5. ∀x∀y∀z((Txy&Tyz)→Txz) (Premise: fourth T-assumption)
6. ~∃x(∃yTxy&~Tx) (Assumption for reductio: nothing that is not T-ing T's something))
7. Ta (from 2, existential instantiation)
8. Ta→∃yTya (from 3, universal instantiation)
9. ∃yTya (from 7 and 8, arrow elimination)
10. Tba (from 9, existential instantiation)
11. ∀x~(∃yTxy&~Tx) (from 6, quantifier negation)
12. ~Taa (from 4, universal instantiation)
13. b≠a (from 10 and 12, identity)
14. ~(∃yTby&~Tb) (from 11, universal instantiation)
15. (~∃yTby)v(~Tb)

Given that we now have a disjunction, we must consider both disjuncts:

16. ~∃yTby (15, disjunction elimination)
17. ∀y~Tby (from 16, quantifier negation)
18. ~Tba (from 17, universal instantiation)
19. Contradiction (from 10 and 18)

20. ~~Tb (15, disjunction elimination)
21. Tb (20, double negation elimination)

But now we have everything in this proof that we had to line 11 in the previous proof. So we can just copy over the remaining 16 lines from the previous proof--with appropriate renumbering—to complete this proof. Given the *First Way* T-assumptions 1-4, in any two-element T-domain there is an element that is T-ing something other than itself but that is not T-ing (simpliciter).

(C) Proof given the *First Way* T-assumptions 1-4, in a two-element T-domain, there is an element that is T-ing something and T-ing everything apart from itself but that is not T-ing (simpliciter).

This time, our set up replaces 6* with 6**

6**. ~∃x(~Tx&∃yTxy&∀z((z≠x)→Txz)

1. ∀x∀y∀z((x≠y)→((z=x)v(z=y))) (Premise: there are no more than two elements in the T-domain)
2. ∃xTx (Premise: first T-assumption)
3. ∀x(Tx→∃yTyx) (Premise: T-assumption)
4. ∀x~Txx (Premise: third T-assumption)
5. ∀x∀y∀z((Txy&Tyz)→Txz) (Premise: fourth T-assumption)
6. ~∃x(~Tx&∃yTxy&∀z((z≠x)→Txz) (Assumption for reductio)
7. Ta (from 2, existential instantiation)
8. Ta→∃yTya (from 3, universal instantiation)
9. ∃yTya (from 7 and 8, arrow elimination)
10. Tba (from 9, existential instantiation)
11. ~Taa (from 4, universal instantiation)
12. a≠b (from 10 and 11, identity)
13. ∀x~(~Tx&∃yTxy&∀z(z≠x→Txz) (from 6, quantifier negation)

14. ~(~Tb&∃yTby&∀z(z≠b→Tbz) (from 13, universal instantiation)
15. (~~Tb)v(~∃yTby)v(~∀z(z≠b→Tbz))

Given that we now have a disjunction, we must consider all three disjuncts. From the previous proofs, we know already how to get contradictions from our premises and the first two disjuncts. So we need only consider the third disjunct here:

16. ~∀z(z≠b→Tbz) (from 15, disjunction elimination)
17. ∃z~(z≠b→Tbz) (from 16, quantifier negation)
18. ~(c≠b→Tcb) (from 17, existential instantiation)
19. ~~c≠b (from18, negated arrow elimination)
20. Tcb (from 18, negated arrow elimination)
21. c≠b (from 19, double negation elimination)
22. (Tcb&Tba)→Tca (from 5, universal instantiation)
23. Tcb&Tba (from 10 and 20, conjunction introduction)
24. Tca (from 22 and 23, arrow elimination
25. c≠a (from 11 and 24, identity)
26. (a≠b)→((a=c)v(b=c))(from 1, universal instantiation)
27. (a≠c(&(b≠c) (from 21 and 25, conjunction introduction)
28. (a=c)v(b=c) (from12, 26, arrow elimination)
29. contradiction (from 27 and 28)

Given the *First Way* T-assumptions 1-4, in any two-element T-domain, there is an element that is T-ing something and T-ing everything apart from itself but that is not T-ing (simpliciter).

This completes our discussion of the two-element T-domain. When we move to the three-element T-domain, we get the same set of results. However, when we move to the four-element T-domain, we lose the last of the three results that we proved for the two-element T-domain. It is straightforward to see how the first two proofs for the two-element T-domain extend to corresponding proofs in the four-element T-domain. It is equally straightforward to see why there can be no proof that corresponds to the third proof in the four-element T-domain: there are counter-models in the four-element T-domain.

Consider the following model. There are four elements: a, b, c, and d. Only a and c satisfy T (simpliciter). Only <b,a> and <d,c> satisfy T. We then observe the following:

1. There are four elements, hence no more than four elements.
2. There are two things that are T (simpliciter), hence there is something that is T (simpliciter).
3. For each thing that is T (simpliciter), there is something that Ts it.
4. Nothing Ts itself.
5. It is trivially true that, if one things Ts a second, and the second Ts a third, then the one thing Ts the third.
6. It is not the case that there is something that Ts everything other than itself.

So, in the four-element T-domain, it does *not* follow from our *First Way* T-assumptions that there is something that Ts everything other than itself.

I said above that it is straightforward that the first two proofs for the two-element T-domain extend to corresponding proofs in the four-element T-domain. It is no less straightforward that, for any n∈N, the first two proofs for the two-element T-domain extend to corresponding proofs in the n-element domain. So, it is straightforward to see that there is a proof that, in any finite T-domain, there is an element that is T-ing something other than itself but that is not T-ing (simpliciter). To make this completely rigorous, we would make use of mathematical induction. We would describe how to turn a proof of the result for an n-element domain into a proof of the result for an n+1 element domain.

To illustrate, return to (A), and consider how to extend it to give the same result for a three-element T-domain. We need to add an extra universally quantified variable to 1. And, in order to generate a contradiction, we then need to make one further use of universal instantiation from 3. The rest is mundane. If we have the result for an n-element T-domain, then we can extend it to an n+1-element T-domain in exactly the same way: we add an extra universally quantified variable to the first premise, and make one additional use of universal instantiation from the third premise. Again , the rest is mundane. And something very similar to what is true for (A) is true for (B).

So, making use of (1) some proofs in predicate calculus with identity and (2) a mathematical induction over proofs in predicate calculus with identity, we derive the following conclusion: in any finite T-domain, there is an element that is T-ing something other than itself but that is not T-ing (simpliciter).

But there is another assumption in the *First Way*: “this does not go on to infinity”. If we interpret this to mean that T-domains are finite, then we get a stronger result: in *any* T-domain, there is an element that is T-ing something other than itself but that is not T-ing (simpliciter).

Alas, it is doubtful that Aquinas was committed to the claim that T-domains are finite. If we suppose that T-domains are finite, then we rule out, for example, a scenario in which there are infinitely many two-chains that are unrelated to one another: infinitely many pairs of otherwise T-unrelated T-things in which one thing is T (simpliciter) but is not T-ing anything, and the other thing is not-T (simpliciter) but is T-ing the first thing. Moreover—and this is more relevant—we rule out a scenario in which a thing that is not T-ing (simpliciter) stands at the ‘base’ of an infinite T-chain in which each thing Ts all higher members in the T-chain and which every else in the T-chain is T-ing (simpliciter). (We can think of this as “turtles all the way up”.)

To carry this discussion further, we should turn our attention to the discussion of *T-chains* and *T-trees*. And in order to do this, we need to introduce some new terminology.

Suppose that we have a and b such that Tab, but there is at least one further entity c—distinct from both a and b—such that Tac and Tcb. In this kind of case, a is *distally* T-ing b. Suppose, on the other hand, that we have a and b such that Tab, and there is no further entity c—distinct from both a and b—such that Tac and Tcb. In this kind of case, a is *proximately* T-ing b.

A *T-chain* is a collection of T-entities linked by proximate T-ing, in which (1) no T-entity proximately Ts more than one T-entity; and (2) every entity either proximately Ts exactly one T-entity or is proximately T’d by exactly one T-entity (or both).

A *T-tree* is a collection of T-entities linked by proximate T-ing in which (1) no T-entity is proximately T’d by more than one other T-entity; (2) every T-entity is proximately T’d by at least one entity and/or proximately Ts at least one entity; and (3) for any two T-entities in the collection, there is a unique path of T-ing and being T’d relations that connects them.

Suppose that we take the starting point for the *First Way* to be that there is a T-chain. Then we might think to make something like the following set of assumptions:

1. There is at least one T-chain.

2. In any T-chain, something is T-ing (simpliciter).
3. In any T-chain, if something is T-ing (simpliciter), then there is something that is T-ing it.
4. In any T-chain, nothing Ts itself.
5. In any T-chain, if a first thing Ts a second, and the second thing Ts a third, then the first thing Ts the third.
6. In any T-chain, whatever has something T-ing it is T-ing (simpliciter).

The discussion of finite T-chains goes much the same as the discussion of T-domains went above, except that (1) we have now ruled out one-element T-chains by fiat; and (2) the strong result that we had for two-domains now extends to all finite chains. That is: in all finite chains, there is an element that is T-ing something and T-ing everything apart from itself but that is not T-ing (simpliciter).

Suppose, instead, that we take the starting point for the First Way to be that there is a T-tree. Then we might think to make something like the following set of assumptions:

1. There is at least one T-tree
2. In any T-tree, there is something that is T-ing (simpliciter)
3. In any T-tree, if something is T-ing (simpliciter), then there is something that is T-ing it.
4. In any T-tree, nothing Ts itself.
5. In any T-Tree, if a first thing Ts a second, and the second thing Ts a third, then the first thing Ts the third.
6. In any T-Tree, whatever has something T-ing it is T-ing (simpliciter).

The discussion of finite T-trees goes the same way as the immediately preceding discussion of T-chains. In particular, we have the following conclusion: in all finite trees, there is an element that is T-ing something and T-ing everything apart from itself but that is not T-ing (simpliciter).

With this discussion in hand, we can now return to the question of exactly how to interpret "this does not go on to infinity". To sharpen our discussion, we being by noting that we can think of T-chains and T-trees as "directed" entities. If, in a T-chain or T-tree, we have Tab, then a is "lower" in the T-chain or T-tree than b. Given that T-chains and T-trees have this directness, we can interpret "this does not go on to infinity" in the following way: T-chains and T-trees do not *descend* infinitely. We can leave it as an open question whether T-chains and T-trees can ascend *infinitely*. What matters is that, if T-chains and T-trees do not descend infinitely, then, in all T-chains and T-trees, there is an element that is T-ing something and T-ing everything apart from itself but that is not T-ing (simpliciter).

Collecting the results to this point, we have the following:

a. From our *First Way* T-assumptions and the claim that T-domains are finite, we can establish that in *any* T-domain, there is an element that is T-ing something other than itself but that is not T-ing (simpliciter).

b. From our *First Way* T-assumptions and the claim that T-chains are finite, we can establish that in any T-chain, there is an element that is T-ing something and T-ing everything apart from itself but that is not T-ing (simpliciter).

c. From our *First Way* T-assumptions and the claim that T-trees are finite, we can establish that in any T-tree, there is an element that is T-ing something and T-ing everything apart from itself but that is not T-ing (simpliciter).

In each case, we take our logic to be classical first-order predicate calculus, we employ mathematical induction to establish that the proof holds in all finite cases, and we use an assumption about finitude to rule out relevant infinite cases.

Working with T-domains has the advantage that we make no assumptions about T-chains and T-trees; but it has the disadvantages that (a) it yields a very weak conclusion; and (b) the assumption about infinity is quite strong (and, plausibly, not something that Aquinas would have accepted).

Working with T-chains or T-trees has the advantage that (a) it yields a stronger conclusion and (b) the assumption about infinitely is weaker (and, plausibly, something that Aquinas would have accepted); but it has the disadvantage that it requires us to make assumptions about T-chains and T-trees.

No matter which way we go, the argument does not get us to the conclusion that Aquinas wants to establish. In no case do we get the conclusion that there is an element that is T-ing something, and T-ing everything apart from itself, but that is not T-ing (simpliciter). If we work with T-chains or T-trees, we have no assumption that tells us that there is just one T-chain, or just one T-tree. It is consistent with the full set of *First Way* T-assumptions that there are many T-chains or T-trees. Hence, it is consistent with the full set of *First Way* T-assumptions that, while, within any T-chain or T-tree, there is something that Ts everything apart from itself in that T-chain or T-tree, there is nothing that Ts everything apart from itself. And, if we work with T-domains, we do not get to the conclusion that there is something that Ts everything apart from itself.

It is worth noting that the *validity* of the *First Way* arguments is not affected by the nature of the T-orderings: whether a T-ordering is *accidental* or *per se* makes no difference to the validity of the associated argument. Of course, this is not to say that the *soundness* of the arguments is unaffected by the nature of the T-orderings. Aquinas himself held that, in the absence of revelation, reason could not tell us whether there are infinite accidentally ordered T-chains. However, he insisted, plausibly, that reason tells us that there are no infinitely descending *per se* ordered T-chains.

It is also worth noting, explicitly, that there are relevant stronger claims that clearly do not follow from the premises of the *First Way*. While, from the relevant premise set, it follows that there is mover that is not moved, it does not follow from that premise set that there is a mover that *cannot* be moved. While, from the relevant premise set, it follows that there is a changer that does not change, it does not follow from that premise set that there is a changer that *cannot* be changed. While, from the relevant premise set, it follows that there is something that changes another in respect R that is not itself changed in respect R, it does not follow from that premise set that there is something that changes another in respect R that itself *cannot* be changed in respect R. (In the absence of further assumptions, the move from p to $\Box p$ is simply invalid.)

Perhaps it might be objected to the claim made in the last paragraph that we could formulate an argument with the following premises:

1. Necessarily, something is T-ing: $\Box\exists xTx$
2. Necessarily, if something is T-ing, then there is something that is T-ing it: $\Box\forall x(Tx\rightarrow\exists yCyx)$
3. Necessarily, nothing Ts itself: $\Box\forall x\sim Txx$
4. Necessarily, if one thing Ts a second, and a second thing Ts a third, then the first thing Ts the third: $\Box\forall x\forall y\forall z((Txy\&Tyz)\rightarrow Txz)$
5. Necessarily, for any T-chain that satisfies 1-4, for some n, that T-chain is an n-element T-chain.

However, all that follows from these premises is that, necessarily, in any T-chain, there is an element that does not T. But: (a) from the claim that, necessarily, there is a mover that does not move, it does not follow that there is mover that *cannot* move; (b) from the claim that, necessarily, there is a changer that is not changed, it does not follow that there is a changer that *cannot* be changed; and (c) from the claim that, necessarily, there is something that changes another in respect R that is not itself changed in respect R, it does not follow that there is something that changes other in respect R that itself *cannot* be changed in respect R. (In the absence of further assumptions, the move from $\Box\exists x Ux$ to $\exists x\Box Ux$ is simply invalid.) Of course, while Aquinas himself explicitly accepts 2, 3, and 5, and at least implicitly accepts 4, Aquinas explicitly rejects 1. In his view—as in the view of most traditional theists—it could have been that there was no creation; and, if there were no creation, then nothing would have been moving, or changing, or changing with respect to R, or moving from potentiality to actuality, or etc. But even if he accepted all of 1-5, Aquinas would not *thereby* have sufficient resources to infer to an unmoveable mover, or an unchangeable changer, or etc.

2. **Soundness**

The upshot of our study of the validity of *First Way* arguments is this. (A) We can derive from the *First Way* T-assumptions that in *any* T-domain, there is an element that is T-ing something other than itself but that is not T-ing (simpliciter). (B) We cannot derive from the *First Way* T-assumptions that in any T-domain there is an element that is T-ing something other than itself, T-ing everything other than itself, and not T-ing (simpliciter). (C) We can derive from the *First Way* T-assumptions that in any T-chain or T-tree, there is an element that is T-ing something and T-ing everything apart from itself but that is not T-ing (simpliciter). (D) Only if we add to our *First Way* T-assumptions the further assumption that there is just one T-chain or just one T-tree, we can derive the claim that, in any T-domain, there is an element that is T-ing something and T-ing everything apart from itself but that is not T-ing (simpliciter).

What should we say about the *soundness* of the arguments that we have deemed to be valid? Let us begin by looking at arguments about T-chains and T-trees.

Clearly, it is not plausible to suppose that there is an appropriate interpretation of 'T' on which there is just one T-chain: on any appropriate interpretation of 'T', everywhere we look, there are T-things such that neither is even distally T'd by the other. There are pairs of things in motion, where neither is even distally moving, or being moved by, the other. There are pairs of things that are changing, where neither is even distally changing, or being changed, by the other. Assuming that there is unification of essence and existence, there are pairs of things in which there is unification of essence and existence in which neither is even distally unifying essence and existence in the other. And so on. So adding the assumption that there is just one T-chain to our set of T-assumptions is just to add a falsehood to our premise set.

The case of T-trees is more interesting. It seems to me to be at best an open question whether there is an appropriate interpretation of 'T' on which, *within* the universe, there is one and only one T-tree. If there are horizons in our universe—e.g. particle horizons or event horizons—then, on any appropriate interpretation of 'T', there are T-things in the universe such that, *within* the universe, there is no further T-thing that even distally Ts them both. There are pairs of things in motion in the universe for which there is no third thing in the universe that is even distally moving them both. There are pairs of things that are changing in the universe for which there is no third thing in the universe that is even distally changing them both. Assuming that there is unification of essence and existence, there are pairs of things in the universe in which there is unification of essence and existence but for which there is no third thing in the universe that is even distally unifying essence and existence in both of them. And so on.

Matters may be even worse than the preceding paragraph suggests. If, as some physicists suppose, there is no global now, it is not clear that we can even make sense of the idea that, within the universe, there is a single T-tree. If there is no fact of the matter about what things outside our light cone are doing now, then there is no fact of the matter about what things outside our light cone are doing to one another right now. So, in particular, for any appropriate interpretation of 'T', for T-things outside our light cone, there is no fact of the matter whether they are T-ing (simpliciter), or T-ing others, or being T'd by others right now. The very best that can be said for *First Way* arguments that work with T-trees is that (a) they are hostage to physical theory and (b) current straws in the wind are not blowing in their favour. We do not stick our necks out very far if we claim that adding the assumption that, within the universe there is just one T-tree, to our set of T-assumptions is just to add a falsehood to our premise set.

Perhaps it might be replied that how things are within the universe is irrelevant: what matters is whether, in global causal reality, there is a T-tree. Even if there are many T-trees in the universe, there is something beyond the universe that unites them all into a single T-tree in global causal reality. While, from a certain theistic standpoint, this response might save the claim that there are sound T-tree arguments, it seems that this response undermines any claim that those arguments are proofs. What the arguments are meant to (help to?) establish is that there is a simple, timeless, immutable, impassible Existence untainted by potentiality, "which all call God". But making the assumption that there is something beyond the universe that unites the many T-trees in the universe in a single T-tree in global causal reality goes way beyond what will be allowed by many of those who do not already accept that there is a simple, timeless, immutable, impassible Existence untainted by potentiality. Certainly, no one who accepts naturalism, and no one who is undecided about whether to accept naturalism, consistently accepts the claim that there is something beyond the universe that unites many T-trees in the universe in a single T-tree in global causal reality.

Perhaps it might also be replied that a proponent of the *First Way* can claim that there are interpretations of 'T' on which it is plausible that there are *no* T-trees within the universe. Suppose that there is on-going unification of essence and existence. It is not in the least bit plausible that there are things in the universe that unify essence and existence in other things in the universe. If there is on-going unification of essence and existence in the universe, then the on-going uniters lie outside the universe. Of course, making the requisite assumptions here also goes way beyond what will be allowed by many of those who do not already accept that there is a simple, timeless, immutable, impassible Existence untainted by potentiality. But going this way also has a consequence that is not shared by other interpretations of the *First Way*.

As it stands, our formulation of the *First Way* begins with the following two premises:

1. $\exists xTx$
2. $\forall x(Tx \rightarrow \exists yTyx)$

As readers may readily establish for themselves, it would make no difference to the results that we have established if we had started instead with the following three premises:

3. $\exists x\exists yTxy$
4. $\forall x\forall y(Txy \rightarrow Ty)$

5. $\forall x((\forall y(x \neq y) \rightarrow Txy) \rightarrow \sim Tx)$

4 is the fifth of the assumptions that we listed at the outset. We have not made any use of it to this point. On any standard interpretation of ‘T’, 4 is obviously true; some philosophers may wish to claim that it is analytically true. What is being moved is moving (simpliciter). Whatever is being changed is changing (simpliciter). What is being changed in respect R is changing in respect R (simpliciter). What is being moved from potentiality to actuality is moving from potentiality to actuality (simpliciter). Whatever is being unified (in respect of essence and existence) is unifying (with respect to essence and existence) (simpliciter). And so on.

Aquinas justifies the first premise that he chooses by noting that it is obvious: no one denies that things are moving, or changing, or changing in respect R, or moving from potentiality to actuality, etc. But he could claim that 3 has exactly the same status: no one denies that things are moving things, or that things are changing things, or that things are changing things in respect R, or that things are moving things from potentiality to actuality, and so on. Perhaps *pace* Hume, all of these things are plausibly claimed to be evident to the senses. We can see things moving things; we can see things changing things; and so on. But it is simply *not* true that we can see things unifying essence and existence in things if there are no things in the universe unifying essence and existence in things in the universe. If we want to be able to claim that the first premise in our argument is evident to the senses, then we cannot claim that there are no T-trees within the universe. Of course, you might worry that we also cannot see unifying of essence and existence in things; but Aquinas could at least respond to that claim by observing that we do not see things disappearing out of existence except in circumstances in which we see causes for such disappearance.

We now turn our attention to the premises 1-4 of the *First Way*. In each case, we shall consider the truth of the premise under a range of different interpretations of ‘T’.

As we have just noted, whether we take the first premise to be $\exists xTx$ or $\exists x\exists yTxy$, this premise seems unexceptionable on most interpretations of ‘T’: moving, changing, changing with respect to R, moving from potentiality to actuality, and so on. However, we can certainly suggest interpretations on which this premise is extremely controversial: e.g. ‘unifying in respect of essence and existence’.

If we take the second premise to be $\forall x\forall y(Txy \rightarrow Ty)$, then it seems unexceptionable on all interpretations of ‘T’. However, on some interpretations of ‘T’, it may turn out to be trivially true. For example, if there cannot be unifying in respect of essence and existence, then, under that interpretation, the claim is merely trivially true. On the other hand, if we take the second premise to be $\forall x(Tx \rightarrow \exists yTyx)$, then, unless we allow that things can T themselves, we have a claim that is controversial on many interpretations. Often, when animals are moving (relative to the surface of the earth), there is nothing else that is moving them (relative to the surface of the earth). Often, when animal minds are changing, there is nothing else that is changing them. And so on. However, if we take the second premise to be $\forall x\forall y(Txy \rightarrow Ty)$, then we must have the additional premise $\forall x((\forall y(x \neq y) \rightarrow Txy) \rightarrow \sim Tx)$, and on many interpretations of ‘T’ it is controversial whether this is non-trivially true.

The third premise is $\forall x \sim Txx$. As we have just noted, there are many interpretations on which either this premise or the premise $\forall x(Tx \rightarrow \exists yTyx)$ should be rejected. Consider animals moving on the surface of the earth. If animals can move themselves, then the third premise is false if ‘T’ is taken to be ‘move’. But if animals cannot move themselves, then the premise $\forall x(Tx \rightarrow \exists yTyx)$ is false in cases where there is nothing else that is moving an animal.

The fourth premise is $\forall x \forall y \forall z((Txy \& Tyz) \rightarrow Txz)$. Although this premise is not stated explicitly by Aquinas, it is crucial to his argument. Consider parenthood. There is the one-place relation Px: x is parented. There is also the two-place relation of parenting Pxy: x parents y. There are things that are parented. If something is parented, there is something that parents it. Nothing parents itself. Parenting "cannot go on to infinity". But it is obviously not true that in a P-chain, there is something that is not parented that parents everything else. What is lacking in this case—what is *not* true—is that $\forall x \forall y \forall z((Pxy \& Pyz) \rightarrow Pxz)$. Aquinas' argument should not go through in the case of parenting; it is the falsity of the fourth premise, under this interpretation, that ensures that it does not.

But consider, instead, ancestry. There is the one-place relation Ax: x is ancestored. There is also the two-place relation Axy: x ancestors y. There are things that are ancestored. If something is ancestored, then there is something that ancestors it. Nothing ancestors itself. Ancestoring "cannot go onto infinity". But, unlike in the case of parenting, we do have that $\forall x \forall y \forall z((Axy \& Ayz) \rightarrow Axz)$. And so we can prove—as we ought to be able to do—that, in an A-chain, there is a unique thing that is not ancestored that ancestors everything else in that A-chain.

We are not done yet. Anything that is an ancestor is a parent. So, even though it is not true in a P-chain that there is something that is not parented that parents everything else, it is true in a P-chain that there is a unique thing that (a) parents something and (b) is an ancestor of everything else. Moreover, the result that we have just displayed is perfectly general. Suppose that we have a two-place relation Rxy that relates proximal elements in a finite R-chain. Then there a transitive ancestral relation R'xy that relates all elements in the R-chain. And, because that relation is transitive, we can appeal to it to prove that there is a unique element in the R-chain that is not R'd but that R's everything else in the R-chain. (Also, of course, we have the same result for R-Trees.)

It is customary, in discussions of the First Way, to invoke a distinction between accidental orderings and *per se* orderings. One of the standard illustrations of a *per se* orderings adverts to the *holding up* relation. This relation is clearly transitive: if a is holding up b, and b is holding up c, then a is holding up c. Suppose, for example, that we have a single stack of bricks resting on the ground. Each brick is supported by all of the bricks below it, and supports all of the bricks above it. In this case, the earth is an un-upheld hold-upper: the earth itself prevents other things from falling in its gravitational field, but there is no hold-upper that prevents the earth from falling in its own gravitational field. (Note that there *could not* be a hold-upper that prevents the earth from falling in its own gravitational field since it is impossible for anything to fall in its own gravitational field.)

Perhaps, however, it is a mistake to think that this really is an instance of a *per se* ordering. Suppose that we have a stack of ten bricks, each of which weighs one kilogram. Then, for example, the second brick in the stack is non-derivatively holding up one kilogram of bricks, and the bottom brick in the stack is non-derivatively holding up nine kilograms of bricks. If we suppose that what defines a *per se* ordering is that the relevant "power" is "borrowed from below", then it is not clear that our stack of bricks really does form a *per se* ordering. Moreover, it does not help to point out that, were the bottom brick removed, all of the bricks above it would fall in the earth's gravitational field. For this is non-trivially true for all but the top brick. (It is perhaps also worth noting that, in this case, what we really have is not moving or changing, but rather prevention of moving or prevention of changing.)

Maybe we would do better to consider something like rolling stock on a railway line on a flat plain. If the rolling stock are in motion, then each carriage is being pulled or pushed along by the next. But at one end of the train, there is an engine. And, if that engine stops pulling or pushing the carriage proximate to it, then, before too long, the train will grind to a halt. Perhaps we might suggest that, rather than taking the transitivity assumption as our criterion for admissibility to candidature to instantiate the *First Way* proof, we should take it that there is an additional requirement: in the way that the rolling stock example demonstrates, relevant "power" must be "borrowed" from below.

It is not clear that this suggestion is helpful to Aquinas' cause. Consider another of the standard illustrations of a *per se* ordering: a case in which a walking person is moving a hand that is moving a stick that is moving a ball. In this case, it is plausible that "power" is "borrowed from below": if the person stops walking, then the motion of the other things—the hand, the stick and the ball—will (soon) cease. But, in this case, it is simply not true that, in the relevant sense, there is something that is moving the person. (And, of course, for exactly the same reason, the engine example is also not helpful to Aquinas' cause.)

The challenge to the soundness of the *First Way* that is here emerging is clear. Uncontroversial actual cases of chains or trees in which it is plausible that "power" is "borrowed from below" terminate in this-worldly things: animals, engines, and the like. What Aquinas needs, in order to have a proof, is something that his opponents recognise is a chain or tree in which "power" is "borrowed from below" that does not terminate in this worldly-things. But, for example, by the lights of Aquinas' naturalistic opponents, it is not in the least bit plausible to suppose that there are any such chains or trees. Sure, as we have noted already, some friends of Aquinas have proposed, for example, that we should take him to be adverting to something like "unifying essence and existence". Perhaps Aquinas' opponents can concede that, if there is "unifying of essence and existence", then it is case in which "power" is "borrowed from below". But, for example, it is no secret that Aquinas' naturalistic opponents deny that there is "unifying of essence and existence".

Our finding, concerning the premises of the *First Way*, is as follows: there is no interpretation of 'T' on which all of 1-4 are uncontroversial. On readings that stick close to the text—"moves", "changes", etc.—the first and fourth premises are uncontroversial, but the conjunction of the second and third premises is plainly false. And, on readings that move further from the text—"unifies essence and existence", the first premise is hopelessly controversial, and known to be rejected by many who do not accept that there is a simple, timeless, immutable, impassible Existence untainted by potentiality, "which all call God".

3. **Concluding Remarks**

There is some tidying up that remains to be done.

I have argued that we can prove that it follows from Aquinas' premises that, on an appropriate disambiguation, there is something that is not moving/changing that moves/changes other things. I have not argued that Aquinas was in possession of this proof. The proof relies on two things—(1) an adequate theory of quantifiers and relations, and (2) an adequate understanding of mathematical induction—that we did not have until the middle of the nineteenth century, and hence which Aquinas did not have in the late thirteenth century. Moreover, the ability to clearly make the disambiguation in question also relies on the possession of an adequate theory of quantifiers and relations.

It is no part of my project here to speculate about what Aquinas did and did not understand. It seems very plausible to me that we should take him to have claimed that it is a logical consequence of his premises that there is something that is not moving/changing that moves/changes other things. But I have no view about how he thought about either logical consequence or the content of the ordinary language claim that there is something that is not moving/changing that moves/changes other things.

It is also no part of my project to disparage Aquinas. There really is a claim that follows from his *First Way* premises that shares an ordinary language articulation with a stronger claim for which he wished to offer a proof. That stronger claim, while still not as strong as the claim that he was ultimately seeking to prove—and unlike the weaker claim that actually follows from his premises—is something that contemporary naturalists typically reject. For example: the earth really is an unmoving mover for things that fall in its gravitational field, and a magnet really is an unchanging changer for iron filings that fall into its magnetic field, but, by the lights of contemporary naturalists, there is no simple, timeless, immutable, impassible Existence untainted by potentiality, "which all call God".

Finally, it is no part of my project to criticise other arguments that are, in some sense, 'inspired' by the *First Way*. At least in my idiolect, 'arguments' are individuated by their premises: different premise set, different argument. It is not controversial that the premises of the *main* argument of the text of the *First Way* in the *Summa Theologica* are the premises with which this discussion began. I have said nothing here about the sub-arguments that Aquinas offers on behalf of some of his major premises in the text of the *First Way* in the *Summa Theologica*; and I have also said nothing here about the sub-arguments that Aquinas offers on behalf of those major premises elsewhere in his writings. Moreover, I have said nothing here about the claim that Aquinas knowingly offered invalid arguments in the text of the *First Way* in the *Summa Theologica* because he assumed that his readers would know that 'missing premises', whose addition would make those arguments valid while not rendering any of the given premises redundant, are presented elsewhere in his writings. I think that that claim is massively implausible; but it is beyond the scope of this article to discuss it further. (For defence of this opinion, see Oppy 2018: 109-25.)

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